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A Hyperbolic Potential Field Model for Designing an Einzel Lens of Low Aberrations

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Abstract:
An analytical model in the form of a hyperbolic function has been suggested for the axial potential distribution of an electrostatic einzel lens. With the aid of this hyperbolic model the relative optical parameters have been computed and investigated in detail as a function of the electrodes voltage ratio for various trajectories of an accelerated charged-particles beam. The electrodes voltage ratio covered a wide range where the lens may be operated at accelerating and decelerating modes. The results have shown that the proposed hyperbolic field has the advantages of producing low aberrations under various magnification conditions and operational modes. The electrodes profile and their three-dimensional diagram have been determined which showed the possibility of being practically realized.

1. Introduction:

The most commonly used einzel (unipotential) lenses have three electrodes. The distinctive feature of einzel lenses is that they have the same constant potential U_1 at both the object and the image side, the central electrode is at a different potential U_2. Therefore, they are used when only focusing is required but the beam energy must be retained. Einzel lenses are symmetrical with respect to the center of the lens for both of its foci. Hence, they are frequently called symmetrical lenses [1]. The symmetrical lens is usually used in cathode ray tubes, and many other electron optical devices.The einzel lens has two modes of operation relative to the central electrode voltage U_2. The two modes are called accelerating (accel) mode when $U_2>U_1$ and the decelerating (decel) mode when $U_2<U_1$. The accel mode aberration is lower than that of the decel mode, but the accel mode requires substantially high central electrode voltage [2].

2. Axial potential distribution:

It is possible to fit the axial potential distribution of a lens to some simple function, with sufficient accuracy, and then study the optical properties. According to Plass [3] and Cosslett [4] it was indicated by Scherzer that in the symmetrical einzel lens the axial field could be represented by an expression of the following form:

$$U(z) = V_o + Ae^{-bz^2} \qquad (1)$$

Where V_O, A and b are constants. Scherzer's work assumed a weak lens [i.e. $V_O>>A$] and a symmetrical lens [i.e. $U(-z)=U(z)$]. Plass [3], however, has considered the following simple distribution for the axial field:

$$U(z) = V_o(1 + \frac{1}{2}e^{-z^2/2}) \qquad (2)$$

A three-electrode electrostatic einzel lens has been investigated by Kanaya and Baba [5] whose axial potential distribution is given by the following equation:

$$U(z/w) = U(z_c)\exp[K_o \tan^{-1}(\frac{z}{w})^n] \qquad (3)$$

where $U(z_c)$ is the central electrode voltage, n is an integer representing the degree of sharpness of the potential distribution, K_O is a constant parameter indicating a positive or negative field depending on the voltage ratio, and w is the full width at half maximum (FWHM) of the axial field. In the present work a more simple analytic expression that would describe the axial potential distribution of einzel lenses with electron-optically acceptable aberrations has been proposed. The following expression is suggested to represent the potential distribution along the optical axis of an einzel lens:

$$U(z) = a \pm \sigma \operatorname{sech}(z) \qquad (4)$$

where a is a constant affecting the value of the ratio of the voltages applied on the central electrode and the outer electrodes which have equal voltages. The positive (+) sign denotes for an accelerating mode of operation and the negative (-) sign denotes for the decelerating mode. The coefficient σ of sech(z) has the unit of volts to match that of a and U(z). The choice of the expression given in equation (4) is justified since it produces an axial potential distribution resembling that given in the literature (see for example, Szilagyi [6]; Hawkes and Kasper [7]). Furthermore, to the best of the authors' knowledge, this expression has not been previously proposed to represent the axial field of symmetrical einzel lenses.

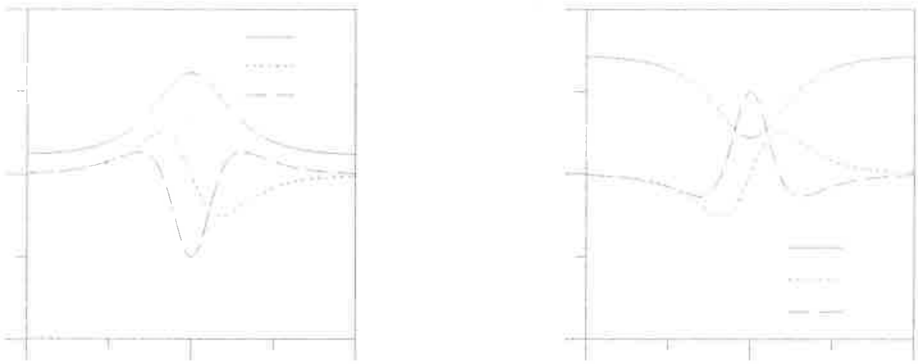
The suggested axial field distribution given in equation (4) for an einzel lens is shown in figure (1) with its first and second derivatives. Figure (1a) shows the axial field distribution of an einzel lens whose central electrode holds the highest voltage. The distribution in figure (1b) is for an einzel lens where the voltage applied on the central electrode is lower than that applied on the two outer electrodes. Since the potential distribution U(z) is constant at the boundaries then its first derivative $U'(z)$ is zero. This indicates that there is no electric field outside the lens i.e. there is a field-free region away from the lens terminals where the trajectory of the charged particles beam is a straight line due to the absence of any force acting on it. Since the second derivative $U''(z)$ of the axial potential has two inflection points, thus the einzel lens would have three electrodes because the number of electrodes is greater than the number of inflection points by one [6].

The shape of the electrodes forming a specific electrostatic lens has been determined using the following truncated electrode construction formula [8]

$$u(r, z) = U(0, z) - r^2 U''(0, z) / 4 \qquad (5)$$

where u(r,z) is the off-axis potential, U(0,z) is the axial potential function whose number of inflection points are counted.. The radial displacement r approaches infinity at each inflection point where $U''(0, z) = 0$. Equation (5) can be written in the following form:

$$r = 2\{[U(z) - u(r, z)] / U''(z)\}^{1/2} \qquad (6)$$



Figure(1): The axial potential distribution U(z) and its first and second derivatives $U'(z)$ and $U''(z)$ respectively of an einzel lens (a) accelerating mode (b) decelerating mode.

If the value of the second derivative $U''(z)$ is negative between two inflection points or between the start/end point and the nearest inflection point, then the electrode potential which is slightly higher than the maximum value of the axial potential in this interval is chosen. If the second derivative is positive, the chosen electrode potential must be lower than the minimum value of the axial potential [6]. The profile and the three-dimensional diagram of the three electrodes forming an electrostatic einzel lens is shown in figure (2). The lens is symmetrical about its center in addition to the rotational symmetry. Thus the lens would appear identical whether viewed from upstream or downstream. In the 3-D diagram a section of the lens has been cut off in order to show the internal shape of the electrodes. The lens geometry is independent of the mode of operation and the magnification conditions. The central electrode is in the form of a disk with central hole of radius equivalent to 0.002L to allow passage for the accelerated charged particles beam, where L is the length of the lens field. The surface of the central electrode is inclined with the optical axis at an angle of about 66° in the region of its inner hole. This result is in excellent agreement with the approximation of Pierce derived electrode contours where the electrode shaping makes an angle of 67.5° with the beam axis [9]. The two outer electrodes are geometrically identical having the shape of a hat, and both have an inner hole of radius equal to that of the central electrode.

3. The trajectory of the charged particles:

The trajectory of the charged particles was determined with the aid of the Runge-Kutta method and the well-known paraxial ray equation

$$r'' + r'\frac{U'}{2U} + r\frac{U''}{4U} = 0 \qquad (7)$$

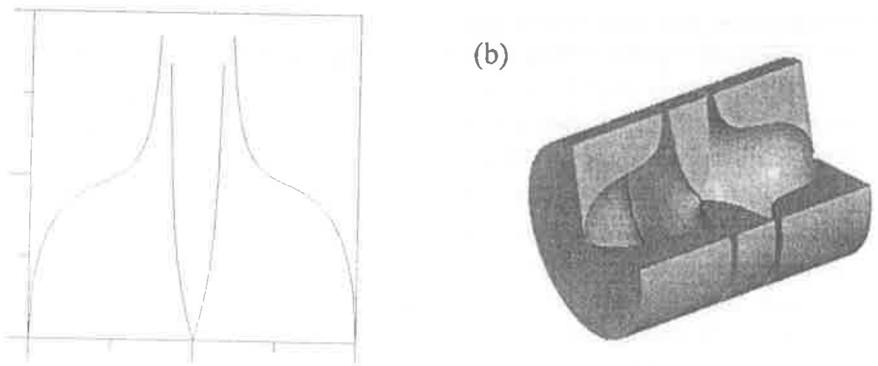
The trajectory of an ion beam traversing the axial field of figure(1) under infinite magnification condition is shown in figure (3) for both the accelerating ($U_2>U_1$) and decelerating ($U_2<U_1$) modes of operation of the einzel lens. In this mode of operation the beam emerges parallel to the optical axis. However, it is seen that the beam trajectory at the lens center is much closer to the optical axis when $U_2>U_1$ than that when $U_2<U_1$. This can be attributed to the fact that in the decelerating mode the lens acts as a series of three lenses, namely from left (object side) to the right (image side), a diverging, a converging and a diverging lens. Since the charged particles are slowed down in the central electrode region, the converging action predominates. However, the trajectory spreads out from the axis initially before entering the central electrode region [10]. For this reason, the spherical aberration coefficient of a lens with $U_2>U_1$ is always less than that of the corresponding lens with $U_2<U_1$ [11].

The electrodes voltage ratio U_2/U_1 can be varied by varying the value of the constant a which appears in equation (4). It should be mentioned that at certain values of a (and hence at certain values of the electrodes voltage ratio) the lens will operate under telescopic magnification condition as shown in figure (4).

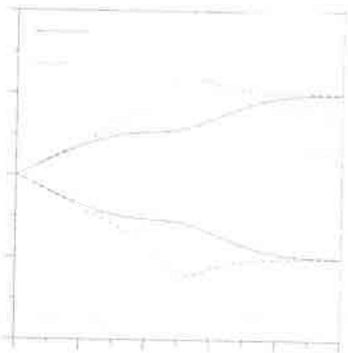
An important criterion of a telescopic lens is that a crossover must always exist inside such lens.

(a)

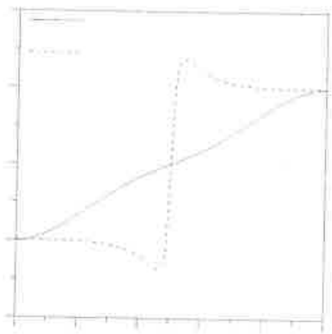
(b)



Figure(2): (a) The profile of the three–electrode einzel lens.
(b) A three-dimensional diagram of the three- electrode einzel lens.



Figure(3): The trajectory along the optical axis of the einzel lens under infinite magnification condition.



Figure(4): Telescopic beam trajectory of three-electrode einzel lens.

4. First and third order optical properties:

In general the focal length (f) of a lens at various values of the voltage ratio is determined from the gradient of the beam trajectory at the point where the charged particles enter or emerge from the lens field region. The object-and image-side focal lengths f_o and f_i respectively have been computed from the following equations:

$$f_o = \frac{r(z_i)}{r'(z_o)} \tag{8}$$

and

$$f_i = \frac{r(z_o)}{r'(z_i)} \tag{9}$$

Since under infinite magnification condition the beam leaves the einzel lens parallel to the optical axis, thus the image-side focal length f_i is infinite. Therefore, in this case the variation of the object-side focal length f_o will be taken into account and it will be normalized in terms of the field length L. The relative object-side focal length f_o/L depends on the electrode voltage ratio as shown in figure (5) for both modes of operation. It is seen that f_o/L has a minimum value at which the lens will have its maximum refractive power. In this case the beam will enter the lens at the point of intersection of lens field boundary with the optical axis. For instance, in the decelerating mode $(f_o/L)_{min}=0.11$ at $U_2/U_1 = 0.03$ and in the accelerating mode $(f_o/L)_{min}=0.351$ at $U_2/U_1 =14.43$. This suggests that relatively small focal length and hence high magnification can be achieved depending on the axial extension of the lens. Any reduction in the lens axial length is, of course, limited by the voltage ratio at which no electrical breakdown would occur. It is seen that as U_2/U_1 approaches unity the relative focal length goes to infinite and the beam trajectory becomes telescopic. The charged-particles beam also traverses a telescopic path as U_2/U_1 exceeds 50 in the accelerating mode. The general form of the focal length variation with the electrodes voltage ratio is similar to that given in various references [1,10,12].

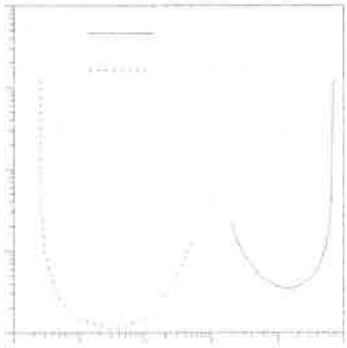
The spherical aberration coefficient C_{so} and the chromatic aberration coefficient C_{co} referred to the object side have been computed from the following formula [13,14]:

$$C_{so}=\frac{U_o^{-1/2}}{16r_o'^4}\int_{zo}^{zi}\left(\frac{5}{4}\left(\frac{U'}{U}\right)^2+\frac{5}{24}\left(\frac{U}{U}\right)^4+\frac{14}{3}\left(\frac{U}{U}\right)^3\frac{r'}{r}-\frac{3}{2}\left(\frac{U}{U}\right)^2\frac{r'^2}{r^2}\right)\sqrt{U}r^4dz \tag{10}$$

$$C_{co}=\frac{\sqrt{U_o}}{r_o'^2}\int_{zo}^{zi}\left(\frac{1}{2}\left(\frac{U'}{U}\right)r'+\frac{1}{4}\left(\frac{U''}{U}\right)r\right)\frac{r}{\sqrt{U}}dz \tag{11}$$

where U=U(z) is the axial potential, the primes denote derivative with respect to z, and $U_o = U(z_o)$ is the potential at the object where $z = z_o$.

The relative spherical and chromatic aberration coefficients C_s/f_o and C_c/f_o respectively in the object side at accelerating and decelerating modes are shown in figure (6) as a function of the electrodes voltage ratio U_2/U_1 under infinite magnification condition. It is seen that C_s/f_o has two minima of 0.8 and 7.35 at U_2/U_1=14.43 (accel), and 0.0663 (decel) respectively. The interesting feature is that in the accelerating mode the minimum of C_s/f_o which is less than unity occurs at the same voltage ratio where f_o/L is minimum (see figure 5). This result suggests that an electrostatic einzel lens of excellent focal properties may be achieved. When the beam trajectory is telescopic C_s/f_o is infinite. Thus this mode of operation is not recommended as far as the spherical aberration is concerned.



Figure(5): The relative object-side focal length of three-electrode einzel lens as a function of the electrodes voltage ratio operated under infinite magnification condition.

In the accelerating mode the minimum of C_c/f_o is 0.47 at $U_2/U_1 = 12.82$; a value which is electron-optically highly acceptable for an electrostatic lens in addition to its appearance within the region of $(C_s/f_o)_{min}$ and $(f_o/L)_{min}$. It is worth mentioning that the behavior of the aberration coefficients with increasing voltage ratio is similar to that in the published literature (see for example references 6 and 15). Considering figure (6) one may deduce from it that under infinite magnification condition it is preferable to operate an einzel lens at the accelerating mode ($U_2 > U_1$) since the relative aberration coefficients are smaller than those at the decelerating mode ($U_2 < U_1$). Table (1) shows a comparison between the values of the relative aberration coefficients for a three-electrode einzel lens achieved in the present work and those given in the literature under similar magnification conditions. From the table it is found that at the constant voltage ratio of 5, the proposed axial potential distribution [equation (4)] gives highly favorable and electron optically acceptable values for the aberration as far as electrostatic lenses are concerned

The spherical, chromatic, and total disks diameter d_{so}, d_{co}, and d_{to} respectively, referred to the object side, have been computed from the following formulae (see for example reference 16):

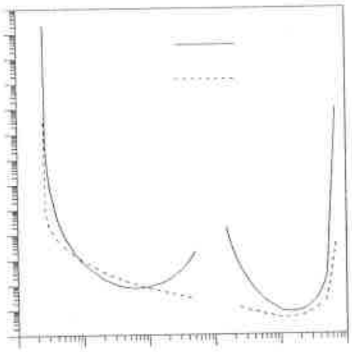
$$d_{so} = 0.5\, C_{so}\, \alpha_o^{\,3} \qquad (11)$$

$$d_{co} = C_{co}\, \alpha_o\, \Delta E/E_o \qquad (12)$$

$$d_t = (d_{so}^{\,2} + d_{co}^{\,2})^{1/2} \qquad (13)$$

where α is the semi-aperture angle at the object side, ΔE is the energy spread, and E_o is the ion energy at the object-side.

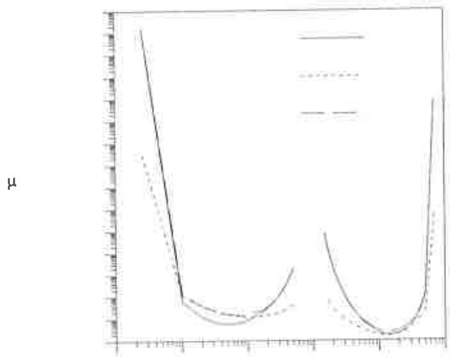
Figure (7) shows the aberration disks diameter as a function of the voltage ratio U_2/U_1. For the decelerating mode the total spot size disk d_t nearly equals to d_s since the value of d_c is negligible in comparison with the corresponding d_s.



Figure(6): The relative spherical and chromatic aberration coefficients as a function of electrodes voltage ratio for three-electrode *einzel lens under infinite magnification condition*

Table (1) Comparison between the results of the present work and the published data under infinite magnification condition.

References	U_2/U_1	C_s/f	C_c/f
Present work	5.0	3.302	0.705
	0.066	7.35	9.87
Szilagyi [6]	5.0	5.6	0.65
	0.06	8.4	high
El – Kareh and Sturans [10]	5.0	8.85	3.99



Figure(7): The aberration disks as a function of electrodes voltage ratio *of three-electrode einzel lens under infinite magnification condition.*

In the published literature the values of α and $\Delta E/E$ are usually taken to be 5 mrad and $2x10^{-5}$ respectively; in this case the minimum values of the aberration disks are as follows:

$(d_s)_{min}$ = 0.175, 0.58 μm at U_2/U_l=14.43 , 0.05 respectively.
$(d_c)_{min}$ =0.165, 1.21 μm at U_2/U_l = 12.82, 0.09 , respectively.
d_t = 0.240, 1.42 μm at U_2/U_l = 14.43, 0.084, respectively.

It should be mentioned that $(d_s)_{min}$ occurs at U_2/U_l =14.43, the ratio at which both C_s and f_o approach their minimum values simultaneously.

5. Conclusions:

It appears from the present investigation that it is possible to design an electrostatic einzel lens with small aberrations operated under infinite magnification condition. The hyperbolic function that has been put forward to represent the axial potential distribution introduced an einzel lens of favorable performance as far as the relative aberration coefficients are concerned. In particular, low chromatic aberrations have been exhibited by the suggested einzel lens under infinite magnification condition and different modes of operation. The present work has shown that the electrodes potential ratio is a crucial parameter since it governs the mode of operation and hence the path of the accelerated beam traversing the axial field. The suggested einzel lens has shown the possibility of being operated for telescopic ray trajectory at a specific value of the electrodes potential ratio. The electrodes geometry and configuration that have been determined from the suggested hyperbolic function are independent of the beam trajectory and mode of operation of the einzel lens.

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