# *Enforcing Wiener Filter in the Iterative Blind Restoration Algorithm*

A. T. Mohammed and A. J. Tawfiq
***Department of Astronomy, College of Science, University of Baghdad.***

**Abstract:**
A new blind restoration algorithm is presented and shows high quality restoration. This is done by enforcing Wiener filtering approach in the Fourier domains of the image and the psf environments.

# تقييد مرشح وينر في تقنية الترميم الأعمى

**علي طالب محمد و أسيل جميل توفيق**
**قسم الفلك / كلية العلوم /جامعة بغداد**

**الخلاصة**
تم عرض تقنيه جديده لترميم الصور البصريه الفلكيه من دون معرفة دالة الأنتشار النقطية. في هذه التقنيه تم فرض مرشح وينر في مجال تحويلة فورير لكل من الصورة ودالة الأنتشار النقطية المفترضة.

## Introduction

The field of image restoration studies methods that can be used to recover an original signal from degraded observations. Many image restoration algorithms have their roots in well-developed areas of mathematics such as estimation theory, linear algebra and numerical analysis. Since most imaging problems are ill-conditioned, a direct deconvolution can be performed only from the ideal case of complete and noise free data.

The Wiener filter and the constrained least square filters are linear techniques for deconvolving blurred and noisy data. The results are derived in terms of the power spectra of the blurred image (noise free) and the noise image.

In classical image restoration, we need to know the blurring function and the degradation phenomenon. Since neither of these two is known exactly, then any technique that is used to restore astronomical images is of limited capability. For better quality deconvolved images, one need to use alternative deblurring algorithms. The maximum entropy techniques (ME) are being increasingly used for the deblurring of images from blurred, noisy and incomplete data. ME techniques have found applications in optical deconvolution, radio astronomy, geophysics, x-ray imaging, and gamma ray imaging.

Hunt introduced the application of a maximum a Posteriori algorithm (MAP) based on Bayesian statistics to deconvolve blurred and noisy data by modeling the noise as a Gaussian additive film grain noise.

Deconvolution is sometimes used to refer to the entire field of image restoration. It is, however, more common to refer to deconvolution as techniques that invert the blurring process in a deterministic way. Often these techniques ignore the effects of noise on the inverse procedure. In applications where the knowledge about the degradations is very limited, blind deconvolution techniques can be applied to produce both an estimate of the object as well as an estimate of the (translation- invariant) degradation.

Most image restoration methods require a priori knowledge about the point spread functions. Blind image restoration algorithms, however, do not require knowledge about the point spread function. Instead they aim to restore the original image from the degrade image and estimate the point spread function.

Blind iterative deconvolution techniques (BID) combine iterative constrained with blind deconvolution. Essentially, it consists of using very limited information about the image, like positivity and image size, to iteratively arrive at a deconvolved image of the object, starting from a blind guess of either the object or both the convolving function.

The iterative loop is repeated enforcing image-domain and Fourier domain constraints until two images are found that produce the input image when convolved together.

Christou, *et. al*., (1995) [1], presented an application of an iterative deconvolution algorithm to speckle interferometric data. This blind deconvolution algorithm permits the recovery of the target distribution when the point spread function is either unknown or poorly known.

Schulz, *et. al*., (1997) [2], studied the process of the blind deconvolution when the precise form of the blurs is unknown in order to improve the resolving power of the ground based optical telescopes.

A method for performing blind deconvolutions on degraded images and data has been developed by Caron, et. al., (2001) [3]. The technique uses a power law relation applied to the Fourier transform of the degraded data to extract a filter function. This filter function closely resembles the point spread function of the system and can be used to restore and enhance higher frequency contents. The process is noniterative and requires only that the point spread function be space invariant and the transfer function be real.

**Image Restoration**

Image restoration methods are used to improve the appearance of an image by application of a restoration process that uses a mathematical model for image degradation. Examples of the types of degradation include blurring caused by motion or atmospheric disturbance, geometric distortion caused by imperfect lenses, superimposed interference patterns caused by mechanical systems, and noise from electronic sources. It is assumed that the degradation model is known or can be estimated. The idea is to model the degradation process and then apply the inverse process to restore the original image. In general, image restoration is more of an art than a science; the restoration process relies on the experience of the individual to model degradation process successfully [4].

Blurring of the image by the point spread function quantitatively decreasing the accuracy of the measurements performed. The goal of image restoration is to invert the degradations that impose on the image. This requires an accurate model of the image formation.

By imposing two types of distortions on the image, a deterministic blurring by the point spread function and a stochastic distortion by noise, we can therefore formulate the goal of image restoration as the reconstruction of the original sample from that of the distorted image[5].

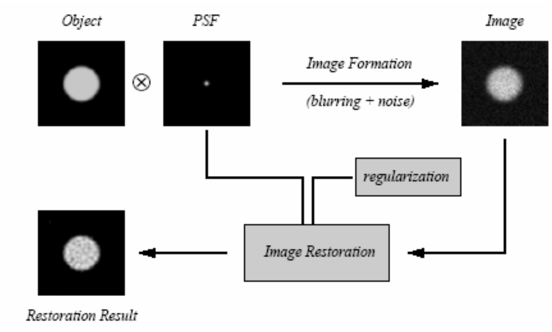



***Fig.(1): Diagram of a general image restoration procedure***

The degradation process model consists of two parts, the degradation function and the noise function. The general model in the spatial domain follows:

$$g(x,y) = psf(x,y) \otimes f(x,y) + n(x,y) \quad (1)$$

where $n(x,y)$ is an additive noise function.

Because convolution in the spatial domain is equivalent to multiplication in the frequency domain, the frequency domain model of Eq. (1) is given by:

$$G(u,v) = T(u,v)F(u,v) + N(u,v) \quad (2)$$

where,

$G(u,v)$ =Fourier transform of the degraded image, $g(x,y)$.

$T(u,v)$ =Fourier transform of the blurring function, $psf(x,y)$.

$F(u,v)$ =Fourier transform of the original image, $f(x,y)$.

$N(u,v)$ =Fourier transform of the additive noise function, $n(x,y)$.

The inverse filter uses the foregoing model, with the added assumption of no noise $(N(u,v) = 0)$. If this is the case, the Fourier transform of the degraded image is:

$$G(u,v) = T(u,v)F(u,v) \quad (3)$$

So, the Fourier transform of the original image can be found as:

$$F(u,v) = \frac{G(u,v)}{T(u,v)} = G(u,v)\frac{1}{T(u,v)} \quad (4)$$

To find the original image, we take the inverse Fourier transform of $F(u,v)$:

$$\hat{f}(x,y) = F^{-1}[F(u,v)] = F^{-1}\left[\frac{G(u,v)}{T(u,v)}\right] \quad (5)$$

where $\hat{f}(x,y)$ is the restored image and $F^{-1}[\ ]$ represents the inverse Fourier transform operator.

The Wiener filter, also called a minimum mean square estimator, alleviates some of the difficulties inherent in inverse filtering by attempting to model the error; the average absolute error is mathematically minimized, as shown below:

$$\left|f(x,y) - \hat{f}(x,y)\right| \cong \min$$

The Wiener filter is given by:

$$R_W(u,v) = \frac{T^{\bullet}(u,v)}{\left|T(u,v)\right|^2 + \left[\frac{S_n(u,v)}{S_f(u,v)}\right]} \quad (6)$$

where $T^{\bullet}(u,v)$ = complex conjugate of $T(u,v)$

$S_n(u,v) = \left|N(u,v)\right|^2$ = power spectrum of the noise

$S_f(u,v) = \left|F(u,v)\right|^2$ =power spectrum of the original image

If we assume that the power noise term $S_n(u,v)$ is zero, this equation reduces to an inverse filter since $\left|T(u,v)\right|^2 = T^{\bullet}(u,v)T(u,v)$. The Wiener filter is applied by multiplying Eq. (6) with the Fourier transform of the degraded image, and the restored image is obtained by taking the inverse Fourier transform of the result,

$$\hat{f}(x,y) = F^{-1}\left|\hat{F}(u,v)\right| = F^{-1}[R_W(u,v)G(u,v)] \quad (7)$$

As the noise term increases, the denominator of the Wiener filter increases, leading to decrease the value of $R_W(u,v)$.

If the psf is known in advance, many restoration algorithms, such as a Wiener filter and a generalized inverse filter, are proposed and the restoration method is almost established.

On the other hand, several restoration algorithms without using information about the psf are also proposed. This kind of the algorithm restores an original image by simultaneously estimating an image and a psf, and it is called a blind deconvolution. These algorithms iteratively estimate both an image and a psf by using a priori constraints such as the non-negative property of an image and a psf, the finiteness of the support area of the object in an image, or the symmetric shape of a psf [6].

Blind deconvolution was developed mainly for restoration of images in astronomy. This is because the image of an astronomical body (a star) should be a point in principle and this information is very useful as a priori constraint of the estimation [7].

In many instances, the degraded observation, $g(x,y)$ , can be modeled as the two-dimensional convolution of the true image, $f(x,y)$, and the point spread function of a linear shift-invariant system [8]. In some applications, the point spread function $psf(x,y)$ is known explicitly priori to the restoration process, and the recovery of $f(x,y)$ is known as the classical linear image restoration problem. However, there are numerous situations in which the point spread function is not explicitly known, and the true image, $f(x,y)$, must be identified directly from the observed image, $g(x,y)$, using partial or no information about the true image and the point spread function.

Blind deconvolutions restore the higher spatial frequency components of the degraded data with little or no a priori knowledge of the degradation. Blind deconvolution techniques can be either iterative or noniterative. Iterative techniques generally require a significant amount of computation, and their implementation can be difficult. The noniterative, does not require information about the degradation, is implemented comparatively easily, and can be applied to many types of data. This algorithm does require the degradation to be approximately space invariant.

**Simulations**

The convolution g(x) of two functions, f(x) and psf(x), can be expressed mathematically by the integral equation,

$$g(x) = \int_{-\infty}^{+\infty} f(x_1)\, psf\,(x - x_1)\, dx_1 \qquad (8)$$

The iterative blind deconvolution algorithm (Fig.(2)) is demonstrated by[9] as follows:

Starting with complete, although possibly noisy, knowledge of the convolution function g(x), the present technique uses some general priori information concerning the function f(x) and psf(x) (for example, the functions may be known to be nonnegative everywhere) and attempts to deconvolve the two functions. The basic deconvolution method consists of the following steps. First, a nonnegative valued initial estimate $\tilde{f}_o(x)$ is input into the iterative scheme. This function is Fourier transformed to yield $\tilde{F}_o(u)$, which is then inverted to form an inverse filter and multiplied by $G(u)$ to form a first estimate of the second function's spectrum $T_o(u)$.

This estimated Fourier spectrum is inverse transformed to give $psf_o(x)$. The image domain constraint of nonnegativity is now imposed by putting to zero all points of the function $ps\tilde{f}_o(x)$ is consequently formed that is Fourier transformed to give the spectrum $\tilde{T}_o(u)$. This is inverted to form another inverse filter and multiplied by $G(u)$ to give the next spectrum estimate $F_1(u)$.

A single iterative loop is completed by inverse Fourier transforming $F_1(u)$ to give $f_1(x)$ and by constraining this function to be non-negative, yielding the next function estimate $\tilde{f}_1(x)$. This algorithm is of a limited capabilities because it is used a new estimate of the optical transfer function, T(u,v) *and* using inverse filtering.

Now our approach is to enforce Wiener filters and three constraint conditions in both image and psf domains as shown below see Fig. (3).

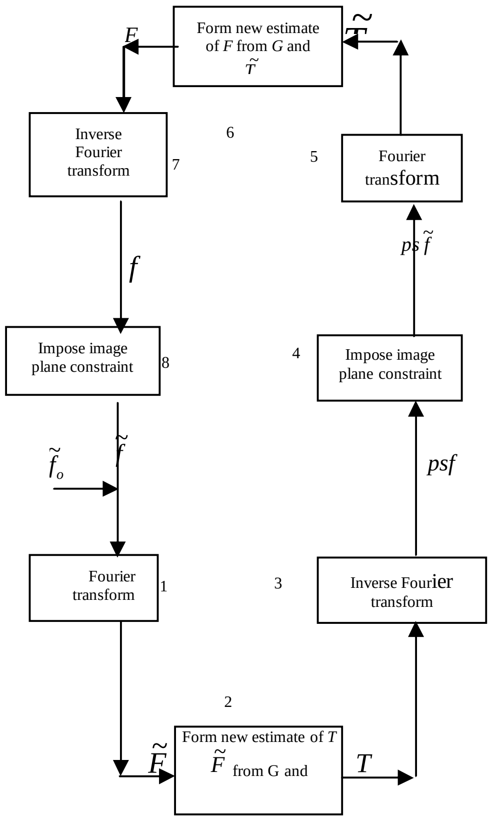



***Fig. (2): General blind deconvolution algorithm***

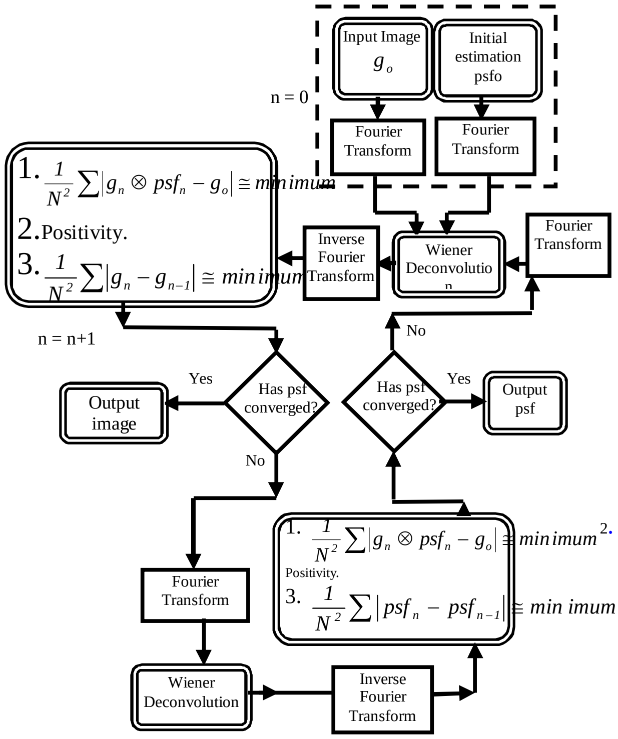



***Fig. (3): Flow-chart of the new adaptive blind deconvolution algorithm.***

In the image domain:

1. Residual error =

$$\frac{1}{N^2}\sum|g_n \otimes psf_n - g_o| \cong \text{minimum} \qquad (9)$$

2. Positivity (intensity is always positive).

3. $$\frac{1}{N^2}\sum|g_n - g_{n-1}| \cong \text{min imum} \qquad (10)$$

In the psf domain:
1. Eq. (9) is also used in this domain.
2. Positivity (intensity is always positive).

3. $$\frac{1}{N^2}\sum|psf_n - psf_{n-1}| \cong \text{min imum} \qquad (11)$$

The flow-chart of this adaptive algorithm is demonstrated in Fig. (3). To implement this algorithm, the following computer simulations are carried out

1- The image of the Saturn as shown in Fig. (4a) is taken to be the original object, f(x, y).

2- This image is then convolved with Gaussian blurring function of HWHM = 2.35. The result is a blurred image of the Saturn, i.e $f(x,y)\otimes psf(x,y)$. This result is then normalized to one at maximum.
3- A normalized random noise, n(x,y), is then added to step 2 following the equation below:

$$g(x,y)=f(x,y)\otimes psf(x,y)+Q\ \ n(x,y)$$

where Q is a parameter takes values between 0 and 1.
In this simulation, Q is taken to be 0.3. The result is a degraded image of Saturn, g(x,y) with (SNR= 30.24) as shown in Fig.(4b). It should be pointed out here that $SNR=\sigma_s/\sigma_n$, where $\sigma_s\ \&\ \sigma_n$ are the standard deviations of the signal and the noise respectively. This image is taken to be as an input image $g_o(x,y)$. The initial estimate of the psf is taken to be a Gaussian psf with HWHM = 3.5. The above algorithm Fig.(3) is then executed by enforcing inverse and Wiener filter respectively. The final estimate of the actual psf and blind restored image using inverse filter are shown in Fig.(4c &d). The results of Wiener filter are shown in Fig.(4e & f). The central lines of Fig. (4a,b,d,e) along the x-axis and y-axis are shown in Fig. (5) and Fig. (6) respectively. The restored lines in Fig. (5d & 6d)in the case of Wiener filter are much smoother and sharper than that of the inverse filter

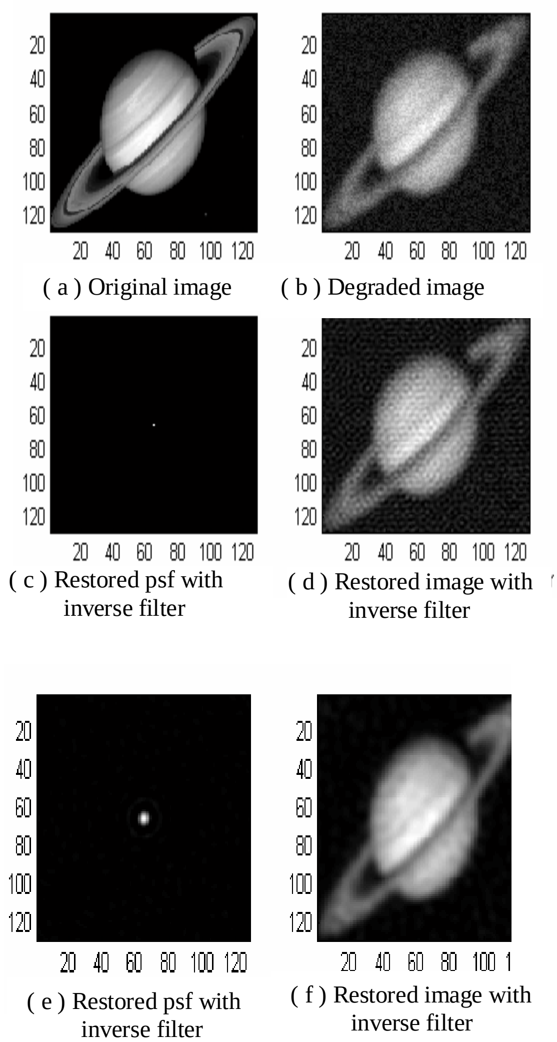



***Fig.(4): Blind Restoration***

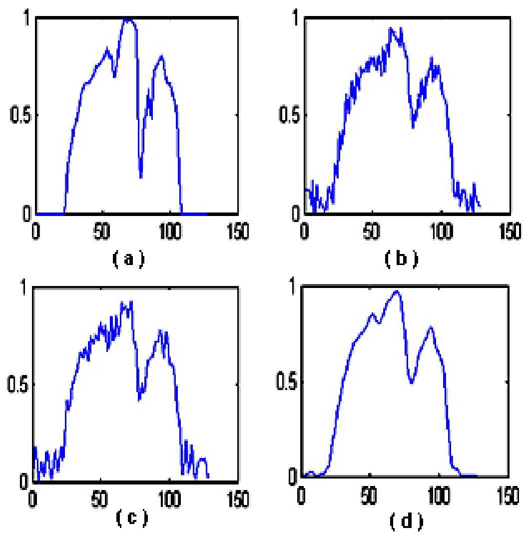



***Fig. (5): Line plots through the x-center of Fig. (4a,b,d,e) respectively.***

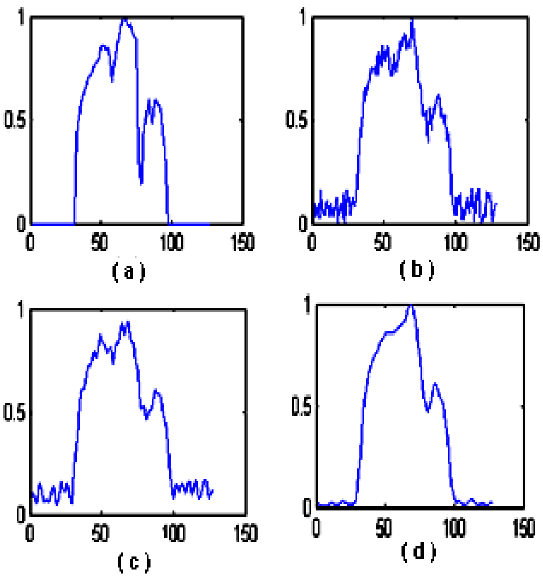
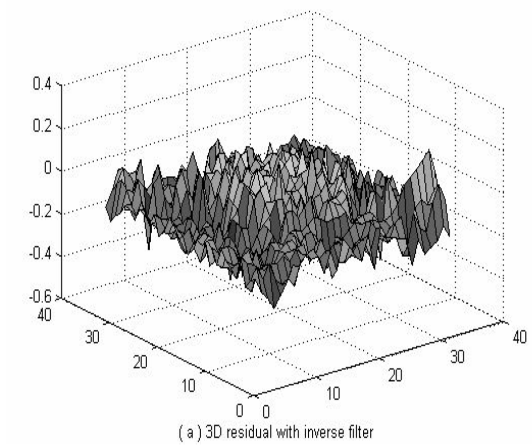



***Fig. (6): Line plots through the y-center of Fig. (4a,b,d,e).***

The plots shows very clearly that the psf that estimated by Wiener filter is very close to that of the original actual psf that used in the convolution process that generates Fig. (4b).

The original image of Saturn as shown in Fig. (4a) and the restored blind image of the modified algorithm are then normalized to one at maximum and the two-dimensional error is then calculated by direct subtraction. The perspective error demonstrated in Fig. (8). The results indicates that the error with Wiener filter is much more smoother than that of inverse filter.



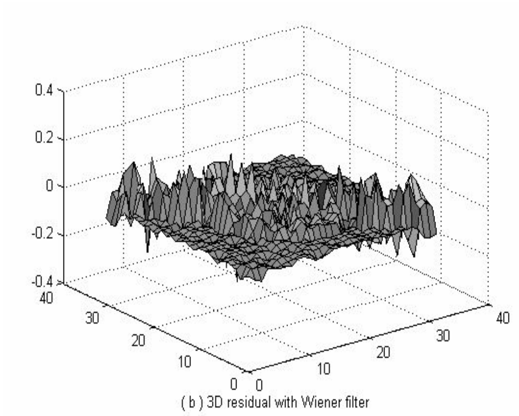




***Fig. (8): Perspective plot of the residual between restored and original images.***

t should be pointed out here that the iterations is stopped until there is no significant change is the residual errors that given by Eq. (11).

he values of SNR parameters of the Wiener filter in the adaptive algorithm are taken according to the following figures:

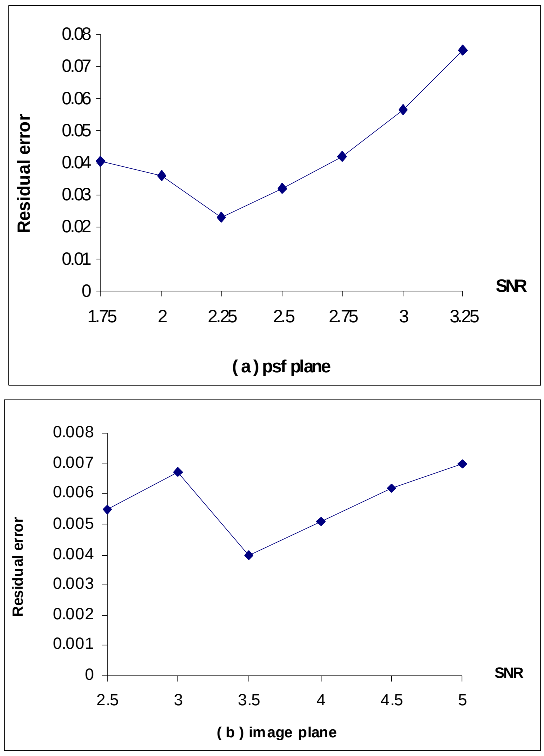



***Fig. (9): Residual error as a function of SNR parameter of the Wiener filter:***
***(a) he psf plane. (b) he image plane.***

For blind restoration, the value of SNR of the Wiener filter in the psf environment is taken when $\frac{1}{N^2}\sum\left|(psf_k - psf_{k-1})\right| \cong \min$.

The following figure shows very clearly that at SNR=2.25 the curve is so smoothed and there is no sudden changes or peaks that appear with other values

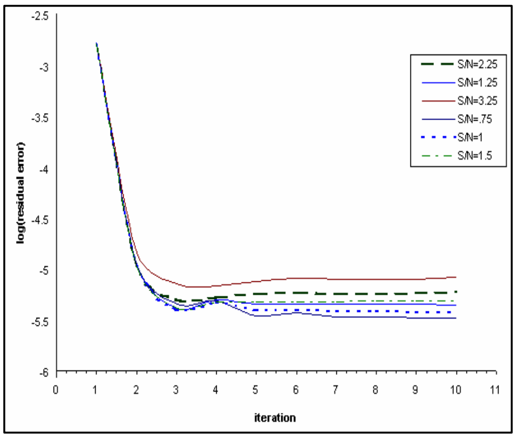



***Fig. (10): Residual error as a function of iterations for psf.***

**Conclusions**

The conclusions that could be drawn from this study are:

1. The residual error that obtained from the subtraction of restored image and original undegraded image is much less in the case of Wiener filter.
2. The psf that estimated by Wiener filter is much closer to the actual psf that used in the convolution process.
3. The equatorial cloud built is much sharper in the case of Wiener filter.

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