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Transverse Magnetic Form Factor for $^{13}C(e,e)\,^{13}C$ with Core-Polarization Effects

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Abstract:

Elastic magnetic M1 electron scattering form factor has been calculated for the ground state J^{π},T=1/2$^-$,1/2 of ^{13}C. The single-particle model is used with harmonic oscillator wave function. The core-polarization effects are calculated in the first-order perturbation theory including excitations up to 5ħω, using the modified surface delta interaction (MSDI) as a residual interaction. No parameters are introduced in this work. The data are reasonably explained up to q~2.5fm^{-1} .

عوامل التشكل المغناطيسية المستعرضة لنواة الكاربون–١٣ باستخدام تأثير استقطاب القلب

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الخلاصة:

حسب عامل التشكل للاستطارة الإلكترونية المرنة للحالة الارضية لنــواة الكــاربون ^{13}C والمعلمــة بـــــ J^{π}, T=1/2$^-$,1/2. استخدم أنموذج الجسيم المنفرد مع الدالة الموجية للمتذبــذب التــوافقي. حســبت تــأثيرات استقطاب القلب من خلال نظرية الاضطراب للمرتبةالاولىمتضمنة التهيجات لغاية 5ħω ، باســتخدام تفاعــل دلتا السطحي المطور (MSDI) كتفاعل متبقي. معاملات الحالات المعتمدة لم تستخدم فىهــذاالعمل. فســرت البيانات العملية بصورة جيدة لقيم الزخم المنتقل q~2.5fm^{-1}.

Introduction

The scattering of high energy electrons from nuclei is a good way to extract information on both nuclear structure and properties of bound nucleons.

The elastic M1 form factor of ^{13}C has been measured by Hicks *et al.* [1] to a maximum momentum transfer of 3.29 fm^{-1}. Their investigation includes theoretical attempts to explain the experimental data using 1p-shell model, with and without one-pion exchange currents. Donnely and Sick [2] noted the particular importance of $(2p)^2$ matrix element by including this term with the modest one-body density matrix $|\psi|$=0.04, the data could be fitted adequately up to q ≅ 3 fm^{-1}.

K. Singham [3] calculated the transverse form factors for the ground and 11.15MeV states of ^{13}C using the configuration mixing within the 1p-shell.

Sato *et al*. [4] studied the electron scattering on ^{12}C and ^{13}C by the nuclear shell model to investigate M1 and E2 nuclear form factors. The effects of core-polarization and pion-exchange current were taken into account. The diffraction minimum shifted to $q \cong 1.2 fm^{-1}$.

Hicks *et al*. [5] measured the M1 form factors to $q \cong 4.6 fm^{-1}$. A comparison was made with theoretical form factors which are calculated by using harmonic-oscillator and Woods-Saxon wave functions.

Amos *et al*. [6] analyzed the form factors for the ground state $(1/2^- 1/2)$ and $(3/2^- 1/2)$ state in ^{13}C and ^{13}N, respectively.

The upper limit of q at 5.08 fm^{-1} is measured in elastic M1 form factor of ^{13}C by Miskimen *et al*. [7]. Their results show that the elastic magnetic form factor continues to decline at q = 5.08 fm^{-1} . Their calculation within 1p-shell space succeeds in fitting the data only to $q \cong 2$ fm^{-1} as previously noted by Singham [3].

Cheon and Jeong [8] performed calculations with the modified nucleon form factors, using effective parameter. Their results succeeded in reproducing the experimental data at $q \cong 2.5$ fm^{-1} .

Ismatov *et al*. [9] obtained good description for the elastic M1 form factor of ^{13}C. Translation Invariant Shell Model (TISM) were used.

The present work aims to study the effects of core-polarization on the elastic magnetic electron scattering from ^{13}C, where the experimental data are available for large momentum transfer.

The single-particle wave function of the harmonic oscillator (HO) potential are used with size parameter b = 1.628 fm chosen to reproduce the measured root mean square (rms) The single-particle form factor of the magnetic operator is defined by [11]:

$$\left|F_J^{\eta}(q)\right|^2 = \frac{4\pi}{Z^2 (2j_b+1)} \left|\left\langle n_a l_a j_a \left\| \hat{T}_J^{\eta}(q, t_z) \right\| n_b l_b j_b \right\rangle\right|^2 \times \left|F_{c.m}(q)\right|^2 \left|F_{f.s}(q)\right|^2 \qquad (1)$$

center of mass which is inherent in the fixed-center shell-model formulation. The convential harmonic-oscillator approximation for this correction is used to give [11,12]:

where < ‖ ‖ > is the single-particle matrix element.

The $F_{c.m}(q)$ is the center of mass correction. It divides out the form factor due to the spurious motion of the

$$F_{c.m}(q) = e^{q^2 b^2 / 4A} \qquad (2)$$

where b is the harmonic-oscillator size parameter and A is the nucleus mass .

The $F_{f.s}(q)$ is the correction factor which takes into account the finite size of the nucleus and given by [12]:

$$F_{f.s}(q) = e^{-0.43 q^2 / 4} \qquad (3)$$

of the matrix elements reduced in both angular momentum and isospin spaces: Using the Wigner-Eckart theorem, the form factor can be written in terms

$$F_J^2(q) = \frac{4\pi}{Z^2 (2j_b+1)} \left| \sum_{T=0,1} (-1)^{T_a - T_{a_z}} \begin{pmatrix} T_a & T & T_b \\ -T_{a_z} & M_T & T_{b_z} \end{pmatrix} \times \left\langle n_a l_a j_a \left\| \left| \hat{T}_{JT}^{\eta}(q) \right\| \right| n_b l_b j_b \right\rangle \right|^2 \left| F_{c.m}(q) \right|^2 \left| F_{f.s}(q) \right|^2 \qquad (4)$$

The core-polarization effects is included through the first-order perturbation theory as[13]:-

$$\left\langle n_a l_a j_a \left| \hat{T}^{\eta} \frac{Q}{E - H^{(0)}} V^{MSDI} \right| n_b l_b j_b \right\rangle + \left\langle n_a l_a j_a \left| V^{MSDI} \frac{Q}{E - H^{(0)}} \hat{T}^{\eta} \right| n_b l_b j_b \right\rangle \qquad (5)$$

these terms are evaluated as [13]:

$$\left\langle n_a l_a j_a \left| \hat{T}^{\eta} \frac{Q}{E - H^{(0)}} V^{MSDI} \right| n_b l_b j_b \right\rangle =$$

$$\sum_{ph} \left\langle h \left| \hat{T}^{\eta} \right| p \right\rangle \times \frac{1}{e_b - e_a - e_p + e_h} \left\langle a\, p \left| V^{MSDI} \right| bh \right\rangle \tag{6}$$

$$\left\langle n_a l_a j_a \left| V^{MSDI} \frac{Q}{E - H^{(0)}} \hat{T}^{\eta} \right| n_b l_b j_b \right\rangle =$$

$$\sum_{ph} \left\langle ah \left| V^{MSDI} \right| bp \right\rangle \times \frac{1}{e_a - e_b - e_p + e_h} \left\langle p \left| \hat{T}^{\eta} \right| h \right\rangle \tag{7}$$

spaces, and taking care of the proper normalization of the angular momentum coupled two-particle states, with concise Green symbols to denote the quantum numbers, one can obtains from equation (6) the expression, where the summation covers all possible particle-hole states, and e_i is the single-particle energy.

Using Wigner-Eckart theorem to reduce the single-particle matrix element in both spin and isospin,

$$\left\langle n_a l_a j_a \left\| \hat{T}^{\eta} \frac{Q}{E - H^{(0)}} V^{MSDI} \right\| n_b l_b j_b \right\rangle =$$

$$\sum_{\alpha_1 \alpha_2 \Gamma} \frac{(-1)^{b+\alpha_2+\Gamma}}{e_b - e_a - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma + 1) \begin{Bmatrix} a & b & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix}$$

$$\times \left\langle a\,\alpha_1 \left| V^{MSDI} \right| b\alpha_2 \right\rangle_{\Gamma} \left\langle \alpha_2 \left\| \hat{T}^{\eta} \right\| \alpha_1 \right\rangle \times \sqrt{(1 + \delta\alpha_1 a)(1 + \delta\alpha_2 b)} \tag{8}$$

The contribution from equa.(7) can be evaluated similarly, one finds [13]:

$$\left\langle n_a l_a j_a \left\| V^{MSDI} \frac{Q}{E - H^{(0)}} \hat{T}^{\eta} \right\| n_b l_b j_b \right\rangle =$$

$$-\sum_{\alpha_1 \alpha_2 \Gamma} \frac{(-1)^{b+\alpha_1+\Gamma}}{e_b - e_a - e_{\alpha_2} + e_{\alpha_1}} (2\Gamma + 1) \begin{Bmatrix} a & b & \Lambda \\ \alpha_1 & \alpha_2 & \Gamma \end{Bmatrix}$$

$$\cdots \times \left\langle a\,\alpha_2 \left| V^{MSDI} \right| b\alpha_1 \right\rangle_{\Gamma} \left\langle \alpha_1 \left\| \hat{T}^{\eta} \right\| \alpha_2 \right\rangle \times \sqrt{(1 + \delta_{a\alpha_2})(1 + \delta_{\alpha_1 b})} \tag{9}$$

elements which are given in equations (8) and (9).

The total form factor is calculated from equation (4) by replacing the single-particle matrix element by following expression, to include the core-polarization effects:

The core-polarization form factor can be evaluated by replacing the single-particle matrix element in equation (4) with the two matrix

$$\left\langle n_a l_a j_a \left\| \hat{T}^{\eta}_{JT}(q) \right\| n_b l_b j_b \right\rangle + \left\langle n_a l_a j_a \left\| \hat{T}^{\eta} \frac{Q}{E - H^{(0)}} V^{MSDI} \right\| n_b l_b j_b \right\rangle + \left\langle n_a l_a j_a \left\| V^{MSDI} \frac{Q}{E - H^{(0)}} \hat{T}^{\eta} \right\| n_b l_b j_b \right\rangle \tag{10}$$

where the Modified Surface Delta Interaction (MSDI) can be expressed as[13]:

$$V^{MSDI}(1,2) = -4\pi A'_T \, \delta\left(r(1) - r(2) \right) \delta \left(r(1) - R_0 \right) + B'\left(\vec{\tau}(1).\vec{\tau}(2) \right) + C' \tag{11}$$

where A'_T, B`, and C` are strength regarded as free parameters that must be determined from experimental spectra.

Results and Discussion

The elastic M1 scattering has greatly enhanced our understanding of single-particle aspects of nuclei. The single-particle prediction is shown in Fig. (1) as a dashed curve. This description is unsatisfactory, especially at high momentum transfer and fails to locate the diffraction minimum. The contribution of the core-polarization is indicated by the dotted curve. The inclusion of the core-polarization effects shifts the location of the diffraction minimum to $q \cong 1.04\ fm^{-1}$ as shown by the solid curve, in good agreement with the measured data [1,5,7] at this region of q. Also, the M1 form factor is suppressed at the second maximum and gives a good description of the experimental data up to $q \cong 2.3\ fm^{-1}$. The calculated form factor underpredicts the data at higher momentum transfer. This deviation cannot be attributed to non-nucleonic effects, since meson-exchange currents (MEC) contribution has been included recently [14] and showed minor contribution in this region of q. The suppression of the form factor was introduced empirically by Amos *et al*.[6] by applying a suppression factor to the isovector spin amplitude, while effective g-factors were introduced recently [14] to reproduce the measured magnetic moment ($\mu_{exp.}$=0.702nm) [15]. In the present work, the suppression is due to the contribution of core-polarization effect, where a microscopic calculation is performed and given a reasonable value of magnetic moment (μ = 0.46nm).

The core-polarization effects introduced through different procedure are further illustrated in Fig. (2). The present calculations are indicated as a solid curve and compared with the recent work [14] as introduced by dashed curve, which incorporated the reduced size parameter (b=1.54fm). The higher q-data are still underestimated and therefore the ^{13}C elastic magnetic form factor needs more understanding.

Conclusion

For large momentum transfer values no calculation reproduces the data variation,the high q data ($q>4fm^{-1}$) electron scattering form factor is most unusual. But the inclusion of the core-polarization effects give agreement with the experimental data at the low momentum transfer. The high momentum transfer enhancements could be attributed to the second-order core-polarization effects.

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