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The Calculation of the Charge Density Distributions and the Longitudinal Form Factors of ^{10}B Nucleus by Using the Occupation Numbers of the States

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ABSTRACT:

The charge density distributions of ^{10}B nucleus are calculated using the harmonic oscillator wave functions. Elastic and inelastic electron scattering longitudinal form factors have been calculated for the similar parity states of ^{10}B nucleus where a core of ^{4}He is assumed and the remaining particles are distributed over $1p_{3/2}$ and $1p_{1/2}$ orbits which form the model space.

Core-polarization effects are taken into account. Core-polarization effects are calculated by using Tassie model and gives good agreement with the measured data.

حساب توزيعات كثافة الشحنة وعوامل التشكيل الطولية لنواة ^{10}B وباستخدام حالات أعداد الملئ

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الخلاصة:

حُسبت توزيعات كثافة الشحنة لنواة ^{10}B باستخدام الدوال الموجية للمتذبذب التوافقي. إن عوامل التشكل الطولية للأستطاره المرنة وغير المرنة حُسبت للحالات المتشابهة التناظر لنواة ^{10}B حيث تم افتراض نواة ^{4}He بمثابة قلب مغلق والجسيمات (النيوكليونات) المتبقية تتوزع على القشرتين الفرعيتين $1P_{1/2}$, $1P_{3/2}$ والتي تمثل أنموذج الفضاء.

في هذا البحث أخذ بنظر الاعتبار تأثير استقطاب القلب المغلق حيث إن هذا التأثير حسب باستخدام أنموذج .Tassie. لقد تم الحصول على نتائج متوافقة مع النتائج العملية.

INTRODUCTION:

The charge density distribution (CDD), transition densities and form factors are fundamental characteristics of a nucleus and can be determined experimentally from the scattering of high-energy electrons by the nucleus. The larger the momentum transferred to the nucleus, the more accurate the information obtained in experiments of this type [1].

Several theoretical and experimental groups have devoted their works on studying the electron scattering from 1p-shell nuclei [2,3]. Comparisons between calculated and measured longitudinal electron scattering form factors have long been used as stringent tests of models of nuclear structure. Shell-model within a restricted model space succeeded in describing static properties of nuclei.

The aims of the present work are calculating the inelastic electron scattering form factors for ^{10}B nucleus depending on the ground state charge density distributions. In the present work the CDD's of the 1s-1p shell nuclei are calculated by means of the harmonic oscillator wave functions on the assumption that occupation numbers of the states in a real nucleus differ from the predications of the simple shell-model. These numbers can be determined from the comparison between the calculated and experimental charge density distribution.

THEORY:

The general theory for electron scattering from nuclei is given in many refs.[4,5,6,7]. The total form factor $F_J(q,\theta)$ of a given multipolarity J is the sum of longitudinal F^L and transverse F^T terms:

$$F_J^2(q,\theta)=\left(\frac{q_\mu}{q}\right)^4\left|F_J^L(q)\right|^2+\left[\frac{q^2{}_\mu}{2q}+\tan^2(\theta/2)\right]\left|F_J^T(q)\right|^2 \qquad (2.1)$$

where q is the momentum transfer, q_μ is the four momentum transfer and θ is the scattering angle. The longitudinal form factor can be given by:

$$\left|F_J^L(q)\right|^2=\frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle J_f\left\|\hat{L}_J(q)\right\|J_i\right\rangle\right|^2\left|F_{c.m}(q)\right|^2\left|F_{f.s}(q)\right|^2 \qquad (2.2)$$

where $\hat{L}_J(q)$ is the Coulomb multipole operator.

The multipolarity J in the above equation is restricted by the momentum selection rule:

$$\left|J_i-J_f\right|\le J\le J_i+J_f \qquad (2.3)$$

and the parity selection rule:

$$\pi_i\pi_f=(-1)^J \qquad (2.4)$$

for Coulomb (C) multipoles

$F_{c.m}(q)$ is the center of mass correction given by [8]:

$$F_{c.m}(q)=\exp(q^2b^2/4A) \qquad (2.5)$$

$F_{f.s}(q)$ is the finite size correction takes the form:

$$F_{f.s}(q)=\exp(-0.43q^2/4) \qquad (2.6)$$

where b is the harmonic oscillator size parameter and A is the mass number of the nucleus. In isospin formalism the form factor takes the form [9]:

$$F_J^2(q)=\frac{4\pi}{Z^2(2J_i+1)}\sum_{T=0,1}(-1)^{T_f-T_z}\begin{pmatrix}T_f & T & T_i\\ -T_z & 0 & T_z\end{pmatrix}\left|\left\langle J_fT_f\left\|\hat{L}_{JT}(q)\right\|J_iT_I\right\rangle\right|^2\left|F_{c.m}(q)\right|^2\left|F_{f.s}(q)\right|^2 \qquad (2.7)$$

where :

$\left|T_i-T_f\right|\le T\le T_i+T_f$

$T_z=\dfrac{Z-N}{2}$ and the bracket $\begin{pmatrix}\cdots\\ \cdots\end{pmatrix}$ is the 3j-symbol.

THE CHARGE DENSITY DISTRIBUTION:

The charge density distribution (CDD) can be evaluated by means of the wave functions of a harmonic oscillator. The density distribution of a system of A point nucleons can be expressed by using the one body density operator $\hat{\rho}^{(1)}(\bar{r})$ which is given by means of the Dirac delta function as[10]:

$$\hat{\rho}^{(1)}(\bar{r})=\sum_{i=1}^{A}\delta(\bar{r}-\bar{r}_i) \qquad (3.1)$$

The charge density distribution for closed shell nuclei with Z=N are given by:

$$\rho(r)=2\sum_{nl}\left(\frac{2l+1}{4\pi}\right).\left|R_{nl}(r)\right|^2 \qquad (3.2)$$

In general:

$$\rho_o(r)=\frac{1}{4\pi}\sum_{nl\in I}2(2l+1)\left|R_{nl}(r)\right|^2+\frac{1}{4\pi}\sum_{nl\notin I}N_p\left|R_{nl}(r)\right|^2 \qquad (3.3)$$

in unit of electronic charge.

Where:
I ≡Closed orbit
$N_p \equiv$ Number of protons in the unfilled orbits.
n is the principal quantum number, l orbital angular momentum quantum number and $R_{nl}(r)$ is the radial part of harmonic oscillator wave functions, i.e., single-particle wave function of the harmonic oscillator potential(nucleon moves in an average potential of all other nucleons)and is given by[11]:

$$R_{nl}(r) = \frac{1}{(2l+1)!!}\left\{\frac{2^{l-2+3}(2n+2l-1)!!}{b^3\sqrt{\pi}(n-1)!}\right\}^{1/2} \left(\frac{r}{b}\right)^l e^{-r^2/b^2}\,{}_1F_1(1-n,l+\frac{3}{2};\frac{r^2}{b^2}) \tag{3.4}$$

${}_1F_1(1-n,l+\frac{3}{2};\frac{r^2}{b^2})$ is the confluent hypergeometric series and is given by:

$${}_1F_1(1-n,l+\frac{3}{2};\frac{r^2}{b^2}) = \sum_{k=0}^{n-1}(-1)^k\frac{(n-1)!2^k}{(n-k-1)!k!}\frac{(2l+1)!!}{(2l+2k+1)!!}\left(\frac{r}{b}\right)^{2k} \tag{3.5}$$

In the simple shell model, the 1p-shell nuclei are assumed that there is a core of filled 1s (^{4}He) and the proton numbers in 1p shell is Z-2. Here, Z is the number of protons in the nucleus. By using equation (3.3)and (3.4), one can obtain:

$$\rho_o(r) = \frac{2e^{-r^2/b^2}}{b^3\pi^{3/2}}\left\{1+(Z-2)/3\left(\frac{r}{b}\right)^2\right\} \tag{3.6}$$

The mean square radii (MRS) of the nuclei can be obtained by using[12]:

$$\langle r^2\rangle = \frac{4\pi}{Z}\int_0^\infty \rho_o(r)\, r^4\, dr \tag{3.7}$$

Substituting the equation (3.6) into equation (3.7), one can obtain

$$\langle r^2\rangle = b^2.(5-4/Z)/2 \tag{3.8}$$

When 1p-shell occupied by (Z-2-α) protons where α is the occupation number of higher shells and resulted from the nucleon-nucleon interaction in the model space and core-polarization effect [13].

$$\alpha = \alpha_1 + \alpha_2 \tag{3.9}$$

α_1 is the occupation number of $2s_{1/2}$ state.
α_2 is the occupation number of ($1d_{5/2} or 1d_{3/2}$) state.
Then:

$$\rho_o(r) = \frac{2e^{-r^2/b^2}}{b^3\pi^{3/2}}\left\{1+\frac{3\alpha_1}{4}+\frac{Z-2-\alpha-3\alpha_1}{3}\left(\frac{r}{b}\right)^2+\frac{3\alpha_1+2\alpha}{15}\left(\frac{r}{b}\right)^4\right\} \tag{3.10}$$

So:

$$\langle r^2\rangle = \frac{(4\alpha+10Z-8).b^2}{4Z} \tag{3.11}$$

The parameter α_1 is determined from the central CDD, $\rho_o(r=0)$, of equation (3.10), i.e.:

$$\alpha_1 = \frac{2\rho_o(0)b^3\pi^{3/2}-4}{3} \tag{3.12}$$

The value of $\rho(0)$ can be taken from the experiments. The harmonic oscillator size parameter b is obtained by introducing the experimental MSR of considered nuclei into equation(3.8). The parameter α is determined from equation(3.11)by:

$$\alpha = \frac{Z}{b^2}\langle r^2\rangle - \frac{5}{2}Z + 2 \tag{3.13}$$

So:

$$\alpha_2 = \alpha - \alpha_1 \tag{3.14}$$

CORE-POLARIZATION EFFECT:

TASSIE MODEL:

The many particle reduced matrix elements consist of two parts, one is the model space (ms) matrix element and the other is the core polarization matrix element.

$$\left\langle f \left\| \hat{L}_J(\tau_Z, q) \right\| i \right\rangle = \left\langle f \left\| \hat{L}_J^{ms}(\tau_Z, q) \right\| i \right\rangle + \left\langle f \left\| \hat{L}_J^{core}(\tau_Z, q) \right\| i \right\rangle \quad (4.1)$$

Where the model space matrix elements are given by:

$$\left\langle f \left\| \hat{L}_J^{ms}(\tau_Z, q) \right\| i \right\rangle = \sum_{j j'} OBDM(i, f, J, j, j', \tau_z) \left\langle j \left\| \hat{L}_J(\tau_z, q) \right\| j' \right\rangle \quad (4.2)$$

Where $OBDM(i, f, J, j, j', \tau_z)$ is the one body density matrix and is given by [14]:

$$OBDM(i, f, J, j, j', \tau_z) = \frac{\left\langle f \left\| \left[a_j^+(\tau_z) \times \tilde{a}_j(\tau_z) \right]^{(J)} \right\| i \right\rangle}{\sqrt{2J+1}} \quad (4.3)$$

The isospin associated with the shell model wave functions is a good quantum number, so it is convenient to calculate the OBDM in terms of the isospin-reduced matrix elements:

$$OBDM(\tau_z) = (-1)^{T_f - T_z} \begin{pmatrix} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \sqrt{2} \frac{OBDM(\Delta T = 0)}{2} \tau_z (-1)^{T_f - T_z} \begin{pmatrix} T_f & 1 & T_i \\ -T_Z & 0 & T_i \end{pmatrix} \sqrt{6} \frac{OBDM(\Delta T = 1)}{2} \quad (4.4)$$

Where $T_z = \frac{Z-N}{2}$ and the OBDM (ΔT) is defined as:

$$OBDM(i, f, J, j, j', \tau_z) = \frac{\left\langle f \left\| \left[a_j^+(\tau_z) \times \tilde{a}_j(\tau_z) \right]^{(J, \Delta T)} \right\| i \right\rangle}{\sqrt{2J+1}\sqrt{2\Delta T+1}} \quad (4.5)$$

So, the model space matrix elements can be written as:

$$\left\langle f \left\| \hat{L}_J^{ms}(\tau_Z, q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \rho_{J,\tau_z}^{ms}(i, f, r) \quad (4.6)$$

Where $\rho_{J,\tau_z}^{ms}(i, f, r)$ is the model space transition density, given by:

$$\rho_{J,\tau_z}^{ms}(i, f, r) = \sum_{jj'}^{ms} OBDM(i, f, J, j, j', \tau_z) \left\langle j \| Y_J \| j' \right\rangle R_{nl}(r).R_{n'l'}(r) \quad (4.7)$$

where $j_J(qr)$ is the spherical Bessel function, Y_J is the spherical harmonics and $e_i = (1 + \tau_z(i))/2, \tau_z$ is the analog of the Pauli matrices into isospin space. The sum extends over the orbits of the model space. The core-polarization matrix elements in equation (4.1) can be written as:

$$\left\langle f \left\| \hat{L}_J^{core}(\tau_Z, q) \right\| i \right\rangle = \int_0^\infty dr\, r^2 j_J(qr) \rho_J^{core}{}_J(i, f, r) \quad (4.8)$$

According to Tassie model [15], the core transition density is given by:

$$\rho^{ms}{}_{Jt_Z}(i, f, r) = N \frac{1}{2} (1 + \tau_Z) r^{J-1} \frac{d}{dr} \rho_{o,\tau_z}(i, f, r) \quad (4.9)$$

Where N is proportionality constant and $\rho_{o,\tau_z}(i, f, r)$ is the ground state charge density given by:

$$\rho_{0,P}(i, f, r) = \sum_j OBDM(i, f, J, j, j', \tau_z) \left\langle j \| Y_0 \| j \right\rangle \left| R_{nl}(r) \right|^2 \quad (4.10)$$

The total transition density is defined as:

$$\rho_{J,\tau_z}(i,f,r) = e_i \rho^{ms}_{J,\tau_z}(i,f,r) + \rho^{core}_{J,\tau_z}(i,f,r) \qquad (4.11)$$

The Coulomb form factor for this model becomes:

$$F(q) = \sqrt{\frac{4\pi}{(2J_i+1)}}\frac{1}{Z}\left\{\int_0^\infty r^2 j_J(qr)\rho^{ms}_{J,\tau_z}(i,f,r)dr + N\int_0^\infty dr\, r^2 j_J(qr) r^{J-1}\frac{d\rho_o(i,f,r)}{dr}\right\} \times F_{c.m}(q)\times F_{f.s}(q) \qquad (4.12)$$

The form factor at the photon point q=k, is related to the reduced transition strength B(CJ) by the following equation:

$$B(CJ) = \frac{[(2J+1)!!]^2}{4\pi}\frac{Z^2e^2}{k^{2J}}\left|F^L_J(k)\right|^2 \qquad (4.13)$$

So, the proportionality constant N can be determined from the form factor evaluated at q=k, and can be shown to be equal to:

$$N = \frac{\int_0^\infty dr\, r^{J+2}\rho^{ms}_{J,t_z} - \sqrt{(2J_i+1)B(cJ)}}{(2J+1)\int_0^\infty dr\, r^{2J}\rho_o(i,f,r)} \qquad (4.14)$$

RESULTS AND DISCUSSIONS:

Our results for the parameters α,α_1,α_2 and the experimental values of $\langle r^2\rangle^{1/2}$ and $\rho(0)$ of ^{10}B nucleus are presented in table (1). We evaluate α, α_1 and α_2 from equation (3.13),(3.12)and (3.14), respectively. In all of the following diagrams, the longitudinal C2 form factors are plotted vs.q while the experimental values are plotted vs.q_{eff} .

The dashed lines give the results obtained using the 1p-shell model space wave functions. The results of core-polarization effects (CP) are shown by the cross symbols. The results including 1p and CP are shown by the solid lines. The size parameter b is taken to be 1.67fm [16] to get the single-particle wave functions of the harmonic oscillator potential. The calculations of this work for the inelastic longitudinal C2 form factor, covered the states (J^π,T): $(1^+,0)$ at E_x=0.718 MeV, $(1^+,0)^*$ at E_x=2.154 MeV, $(2^+,0)$ at E_x=3.587 MeV,$(3^+,0)^*$ at E_x=4.774 MeV, $(2^+,0)^*$ at E_x=5.920 MeV and $(4^+,0)$ at E_x=6.025 MeV. The ground state of ^{10}B nucleus is a T=0 state. The experimental values are shown as circles. The calculations for the C2 isoscalar transition from the ground state 3^+0 to the 1^+0 state at 0.718MeV are shown in Fig.(1). The data are well described by the 1p-shell for 2.4 $fm^{-1} > q >$1 fm^{-1}. The low-q data are underestimated. The inclusion of core-polarization effect enhances the form factor. This enhancement brings the total theoretical results of the longitudinal C2 form factor very close to the experimental data. The B(C2) values are given in table(2) and taked from [17]. The experimental data are taken from Ref.[17].

For the isoscalar transition (1^+0) at E_x=2.154MeV, the result of CP effect increases the longitudinal C2 form factor component by a factor of about 2.0 over the 1p-shell calculations, making the total theoretical form factor closer to the measured data as shown in Fig.(1). However it was pointed out by Warburton et al [18] that admixtures of two lowest p-shell states for both 1^+, T=0 and 2^+,T=0 states give much better agreement with the electromagnetic properties of these states. Kurath[19] showed that for the (8-16)POT interaction which is used in this work a 13% admixture by intensity would lower the transition strength for the 0.718MeV state and raise it for the 2.154MeV state. The experimental data are taken from Ref.[17].

The inelastic form factors for the transition 2^+0 at E_x=3.587MeV are shown in Fig.(1) where the CP effects increases the longitudinal C2 form factor components by a very small amount, making the 1p-shell model calculations closer to the experimental values. The experimental values are taken from Ref.[17].

The CP effect results for the transition 3^+0 state at 4.774MeV are very close to the experimental data at 1.8 $fm^{-1} > q >$0.6 fm^{-1}. On the other hand the 1p-shell model calculations are shifted from the experimental data from Ref.[17] at $q \geq$ 0.6 fm^{-1}. So, the total form factors as shown in Fig.(2) are shifted from the experimental data. The comparisons are especially difficult at the photon point where most calculations give reduced transition probabilities B(C2↑) of 0.4-0.6 $e^2 fm^4$, values that are at least 10 times the small upper limit of 0.04 $e^2 fm^4$ deduced from the observed ground state radiative width [20,21].

The 1p-shell model calculations for the transition 2^+0 state at E_x=5.920MeV are very close to the experimental data. The CP effect decreases the 1p-shell model space calculations by a small amount to make the total theoretical form factor in well agreement with the experimental values of Ref.[17] as shown in Fig.(2). The improvement in the description of the form factors for the states considered so far is also reflected in the longitudinal form factor for 4^+0(E_x=6.025MeV)state, as shown in Fig.(2). While the CP effect calculations raise the 1p-shell model space calculation by a factor of 2.0 making the total theoretical longitudinal C2 form factor agrees with the experimental values for q < 1fm^{-1}, but not good at higher q. The experimental values are taken from Ref. [17] as circles and Ref.[16]as squares.

Table (1)

$\rho_o(r)\, e.fm^{-3}$	$\langle r^2 \rangle^{1/2}$ fm	α	α_1	α_2
0.009	2.45	0.2614123	0.2236630	0.03774926

Table (2)
The experimental values of B (C2)

$J_i^\pi\ T$	$J_f^\pi\ T$	**B(C2)** $e^2.fm^4$	E_X **(MeV)**
3^+0	1^+0	1.7 ± 0.3	0.718
3^+0	1^+0	0.4 ± 0.1	2.154
3^+0	2^+0	0.6 ± 0.1	3.587
3^+0	3^+0	>0.04	4.774
3^+0	2^+0	0.17 ± 0.05	5.920
3^+0	4^+0	17.4 ± 0.7	6.025

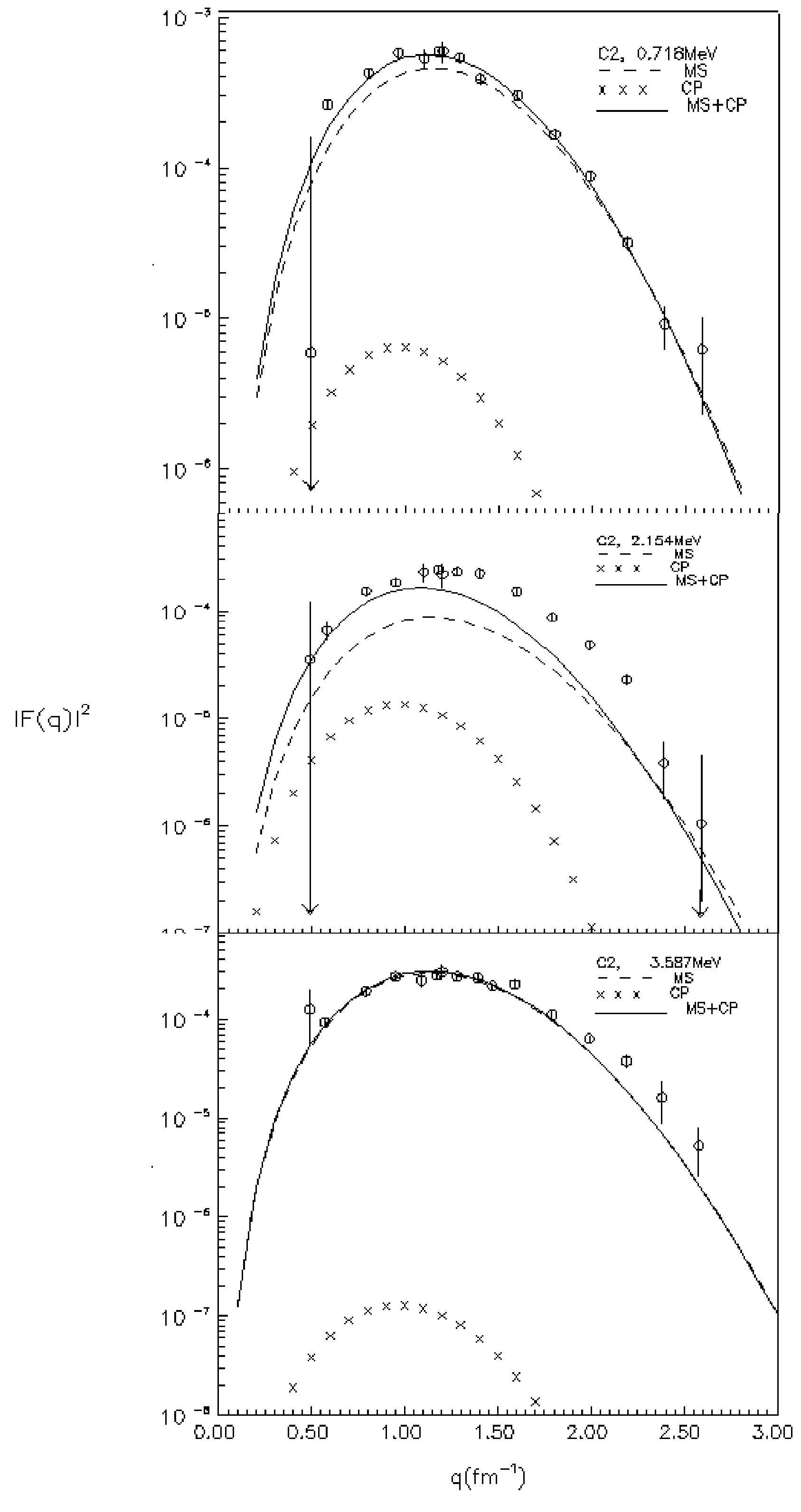

Figure (1) Inelastic longitudinal C2 form factors. The dashed curve is the model space (MS) calculation and the cross symbol is the core-polarization effect (CP) calculation. The solid curve is the total form factor (MS and CP).The experimental values are shown as circles and taken from Ref.[17]

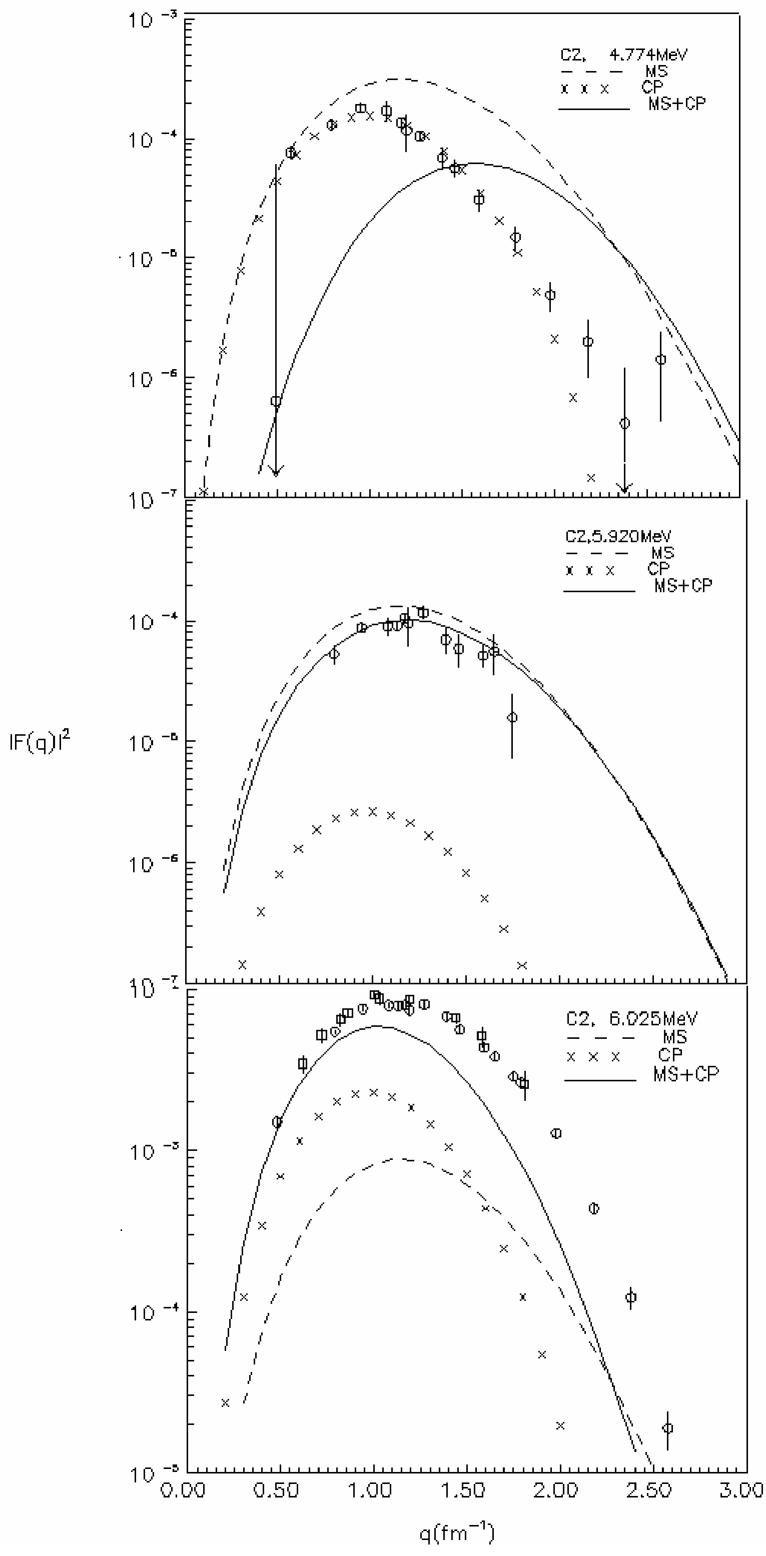

Figure (2): Inelastic longitudinal C2 form factors. The dashed curve is the model space (MS) calculation and the cross symbol is the core-polarization effect (CP) calculation. The solid curve is the total form factor (MS and CP).The experimental values are shown as circles and taken from Ref.[17]. In the last graph, the experimental values as squares are taken from Ref.[16]

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