Collective C2 transitions in ^{32}S with higher – energy configurations

Raad. A. Radhi*, Nadia. M .Adeeb and Abdal-Sattar. K. Hashim*****

**University of Baghdad/College of Science/Department of Physics (raadradhi@yahoo.com)*
***University of Baghdad/College of Science/Department of Physics (nadiaaadeeeb@yahoo.com)*
****University of Karbala/College of Science/Department of Physics (abuyaser67@yahoo.com)*

Abstract

Collective C2 transitions in ^{32}S are discussed for higher energy configurations by comparing the calculations of transition strength B(CJ↓)with the experimental data. These configurations are taken into account through a microscopic theory including excitations from the core orbits and the model space orbits with $n\hbar\omega$ excitations.

Excitations up to n=10 are considered. However n=6 seems to be large enough for a sufficient convergence. The calculations include the lowest seven 2^+0 states of ^{32}S.

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إنتقالات رباعي القطب الكهربائي لنواة ^{32}S باستخدام التشكيلات ذات الطاقات العليا

رعد عبد الكريم راضي*، نادية محمد أديب* وعبد الستار كريم هاشم
**قسم الفيزياء – كلية العلوم – جامعة بغداد*
*** قسم الفيزياء – كلية العلوم – جامعة كربلاء*

الخلاصة

تمت مناقشة مجموعة انتقالات C2 للمراتب العليا للطاقة لنواة الكبريت ^{32}S من خلال حساب شدة الانتقالات الكهربائية ومقارنتها مع القيم العملية. حيث تم الأخذ بنظر الأعتبار اعتماد النظرية المجهرية والمتضمنة الأنتقالات من مدارات القلب ومدارات فضاء الأنموذج لمدى $n\hbar\omega$ من المستويات المثارة.

كما تم اعتماد المستويات المثارة لغاية ($n = 10$) في حين ان($n = 6$) اظهرت تقاربا كافيا للحسابات المتضمنة المستويات السبعة 2^+0 من نواة الكبريت.

Introduction

Comparisons between the calculated and the measured transition strength rates have long been used as a stringent test of models of the nuclear structure. In the nuclear shell model, the sd shell is an interesting region for nuclear structure investigation by inelastic electron scattering. Experimental data, such as transition rates in the sd-shell region, cannot be explained by the simple shell model, when few nucleons are allowed to be distributed over the sd-shell orbits, outside the closed ^{16}O core. Inadequacies in the shell-model wave functions are revealed by the need to scale the matrix elements of the one body operators by state-and-mass-independent effective charges to match the experimental data [1]. The effective charges may yield the same reduced transition probabilities yet differ substantially in the radial dependence of transition matrix elements. It is clear that either the *q* dependence of effective charges or a large model space must be considered explicitly [2]. A quite successful alternative is to be considered the excitation of particles from the core, usually taken to be ^{16}O, in the case of sd-shell nuclei. These particle–hole (p–h) excitations of the ^{16}O core are referred to be as the core polarization (cp).

A microscopic model has been used to study the C2 and C4 longitudinal form factors of the stable even–even *N*=*Z* sd-shell nuclei [3]. Their results gave a remarkably good agreement with the measured data.

Same model was employed to calculate the C2 form factors for the first two 2^+ states in the open shell nuclei ^{22}Ne,^{26}Mg and ^{30}Si, but with more realistic interaction for the core – polarization effects [4].

Shell model has long been recognized that electric quadrupole (C2) excitations have highly collective properties [5]. In the simple shell model of ^{32}S, the inert core is the nucleus ^{16}O with sixteen distributed active (8p+8n) in the sd-shell single particle orbits ($1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$). The calculation of this type of model space interaction based on a new version of the USD (universal sd) which is labeled as USDB interaction for the sd-shell model space [6].

The collective properties can be supplemented to the usual shell model treatment by allowing to excitations from the core and model space orbits into higher orbits. The conventional approach to supplying this added ingredient to shell model wave functions is to redefine the properties of the valence nucleons from those exhibited by actual nucleons in free space to model-effective values [2]. Also this can be treated by connecting the ground state to the *J* -multipole $n\hbar\omega$ giant resonances [2], where the shape of the transition densities for these excitations is given by Tassie [7].

The aim of the present work is to describe the collective properties such as the transition strengths for the lowest seven excited 2+ states in ^{32}S at excitation energies *Ex* = 2.160, 4.38, 5.45, 6.67, 8.58, 8.78 and 9.357 MeV. The calculations are performed with model space wave functions including core-polarization effects calculated in a perturbation approach including excitations up to $10\hbar\omega$.

Theory

The core-polarization effects on the form factor are based on a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation. The reduced matrix elements of the electron scattering operator $\hat{T}_\Lambda$ are expressed as a sum of the model space (MS) contribution and the core polarization (CP) contribution, as follows:

$$\left\langle \Gamma_f \left\| \hat{T}_\Lambda \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}_\Lambda^\eta \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta\hat{T}_\Lambda^\eta \right\| \Gamma_i \right\rangle_{CP} \quad (1)$$

The MS matrix element is expressed as the sum of the product of the elements of the one-body density matrix (OBDM) $X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)$ times the single-particle matrix elements, and is given by

$$\left\langle \Gamma_f \left\| \hat{T}_\Lambda \right\| \Gamma_i \right\rangle_{MS} = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta) \left\langle \alpha \left\| \hat{T}_\Lambda \right\| \beta \right\rangle \quad (2)$$

Where α and β labeled as a single-particle states (isospin is included) for the shell model space. The states $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model space wave functions. The model space is defined by the sd-shell orbits, $1d_{5/2}$, $2s_{1/2}$ *and* $1d_{3/2}$.Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e. $\Gamma_i \equiv J_iT_i$, $\Gamma_f \equiv J_fT_f$ and $\Lambda \equiv JT$. The CP term can be expressed in terms of the two-body residual interaction V_{res} and the single-particle operator $\hat{T}_\Lambda$ [8]:

$$\left\langle \Gamma_f \left\| \delta\hat{T}_\Lambda \right\| \Gamma_i \right\rangle_{CP} = \left\langle \Gamma_f \left\| \hat{T}_\Lambda \frac{Q}{E_i - H_0} V_{res} \right\| \Gamma_i \right\rangle + \left\langle \Gamma_f \left\| V_{res} \frac{Q}{E_f - H_0} \hat{T}_\Lambda \right\| \Gamma_i \right\rangle \quad (3)$$

The operator *Q* is the projection operator onto the space outside the model space. The two CP terms can be expressed in terms of the matrix element of the two-body residual interaction V_{res} and the single-particle operator $\hat{T}_\Lambda$:

$$\left\langle \Gamma_f \left\| \delta \hat{T}_\Lambda \right\| \Gamma_i \right\rangle_{CP} = \sum_{\alpha,\beta} X^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha,\beta) \left\langle \alpha \left\| \delta \hat{T}_\Lambda \right\| \beta \right\rangle, \qquad (4)$$

Where the single-particle core-polarization term is given by [8]:

$$\left\langle \alpha \left\| \delta \hat{T}_\Lambda \right\| \beta \right\rangle = \sum_{\alpha_1, \alpha_2, \Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{e_\beta - e_\alpha - e_{\alpha_1} + e_{\alpha_2}}$$

$$(2\Gamma+1) \times \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix}$$

$$\times \sqrt{(1+\delta_{\alpha_1 \alpha})(1+\delta_{\alpha_2 \beta})}$$

$$\times \left\langle \alpha_1 \alpha \middle| V_{res} \middle| \alpha_2 \beta \right\rangle_\Gamma \times \left\langle \alpha_2 \left\| \hat{T}_\Lambda \right\| \alpha_1 \right\rangle$$

+terms with $\alpha_1$and α_2 exchanged with an over all minus sign, (5)

Where the index α_1 runs over particle states and α_2 over hole states, with respect to the chosen core. The single-particle energies (e) are given using the harmonic oscillator potential [8]

$$e_{nlj} = (2n + l - \tfrac{1}{2})\hbar\omega +$$

$$\begin{cases} -\frac{1}{2}(l+1)\langle f(r) \rangle_{nl} \ for\ j = l - \frac{1}{2}, \\ \frac{1}{2} l \langle f(r) \rangle_{nl} \quad for\ j = l + \frac{1}{2}, \end{cases} \qquad (6)$$

with: $\langle f(r) \rangle_{n\ell} \approx -20A^{-2/3}$ and

$,\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$which are taken from [1].

The single-particle matrix elements reduced in both spin and isospin are written in terms of the single-particle matrix elements reduced in spin only [8]

$$\left\langle \alpha_2 \left\| \hat{T}_\Lambda \right\| \alpha_1 \right\rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \left\langle j_2 \left\| \hat{T}_{Jt_z} \right\| j_1 \right\rangle \qquad (7)$$

with

$$I_T(t_z) = \begin{cases} 1 & for T = 0 \\ (-1)^{1/2 - t_z} & for T = 1 \end{cases} \qquad (8)$$

where t_z = 1/2 for a proton and −1/2 for a neutron.

Core-polarization effects are taken into consideration through 1p–1h excitations from the core orbits into higher orbits and excitations are also considered from model space orbits into higher orbits. All excitations are considered with up to 10ħω excitations. For the residual two-body interaction V_{res}, the M3Y interaction of Bertsch et al. [9] is adopted. The form of the potentials defined in Eqs. (1)–(3) in Ref. [9]. The parameters of 'Elliot' are used which are given in Table.1 of the mentioned reference. A transformation between *LS* and *jj* coupling states is needed, which makes use of the Talmi–Moshinsky transformation brackets [10] in order to obtain the two-body shell model matrix elements starting from the relative two-body matrix elements.

The reduced single-particle matrix element of the Coulomb operator is given by [11]

$$\left\langle j_2 \left\| \hat{T}_J \right\| j_1 \right\rangle = \int_0^\infty dr r^2 j_J(qr) \left\langle j_2 \left\| Y_J \right\| j_1 \right\rangle R_{n_1 \ell_1}(r) R_{n_2 \ell_2}(r) \qquad (9)$$ W

here $j_J(qr)$ is the spherical Bessel function and $R_{n\ell}(r)$ is the single-particle radial wave function. Electron scattering longitudinal (Coulomb) form factor involving angular momentum J and momentum transfer q, between initial and final nuclear shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ are [12]

$$\left| F_{CJ}(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \sum_{T=0,1} (-1)^{T_f - T_{zf}} \begin{pmatrix} T_f & T & T_i \\ -T_{Zf} & M_T & T_{Zi} \end{pmatrix} \left\langle \Gamma_f \left\| T^{\eta}_{J,T}(q) \right\| \Gamma_i \right\rangle \right|^2$$

$$\times \left| F_{c.m}(q) \right|^2 \times \left| F_{f.s}(q) \right|^2 \qquad (10)$$

Where T_z is given by $T_z = (Z - N)/2$. The nucleon finite size (fs) form factor is $F_{fs}(q) = e^{-(0.43q^2/4)}$ and $F_{cm}(q) = e^{(q^2 b^2/4A)}$ is the correction for the lack of translational invariance in the shell model (center of mass correction), where A is the mass number and b is the harmonic oscillator size parameter.

The reduced electromagnetic transition probabilities B(EJ) may be expressed in terms of the electron scattering form factors evaluated at $q=k$, where $k=E_x/\hbar c$, E_x is the excitation energy of the state [8] Thus:

$$B(EJ)=\frac{Z^2e^2}{4\pi}\left[\frac{(2J+1)!!}{k^J}\right]\left\|F_{CJ}(k)\right|^2 \qquad (11)$$

the relation between B(EJ↑)and B(EJ↓) which is given by

$$B(EJ\uparrow)=\frac{(2J_f+1)}{(2J_i+1)}B(EJ\downarrow) \qquad (12)$$

The reduced transition strength B (E2) in Weisskopf unit (W.u) which is given by [8]

$$M_w^2(E2)=\frac{16.8\ B(E2)}{A^{4/3}e^2.fm^4} \qquad (13)$$

In the present work, we have taken the $B(EJ\uparrow)$ and $B(EJ\downarrow)$ in eq.(12) as B(C2↑) and B(C2↓) for calculating of the transition strengths in W.u for all seven transition from $J^{\pi}T=0_1^+0$ to $J^{\pi}T=2_i^+0$, where $B(0^+0\rightarrow 2^+0)=5B(2_i^+0\rightarrow 0^+0)$ i.e. B(C2↑)= 5B(C2↓).

The measured Transition strength rates to these states are available from Ref [13]. The radial wave functions for the single-particle matrix elements were calculated with the harmonic oscillator (HO) potential. The oscillator length parameter *b* = 2.001 fm was chosen to reproduce the measured root mean square charge radius. The reduced transition strength B(C2↑) and B(C2↓) are tabulated in [Table- 1] for better comparison with measured values.

Results and discussion

Calculations are presented for the lowest seven excited 2^+ states with excitation energies 2.160, 4.83, 5.45, 6.67, 8.58, 8.87 and 9.357 MeV in the ^{32}S.

We will discuss the core-polarization effects on the transition strength for the seven 2^+ states in the large basis shell model. The core-polarization effect on the transition strength is based on microscopic theory, which combines shell-model wave functions and configurations with higher energy. These higher configurations can be calculated by perturbation theory as described in [3]. We adopt the USDB interaction [6] to generate the zero-order (sd-model space) matrix elements, using the shell model code OXBASH [14]. The effects of virtual excitations of nucleons from 1s and 1p core orbits into higher allowed orbits and also from the 2s1d orbits into higher allowed orbits are considered up to $10\hbar\omega$ excitations, where $6\hbar\omega$ is large enough for a sufficient convergence in the Coulomb matrix elements. The M3Y interactions of Bertsch *et al* [9] between the core nucleons and the valence nucleons are assumed.

The sd-shell model space (without core polarization) failed to describe the data for the transition strengths [Table 1] for the seven excited 2^+states with excitation energies 2.160, 4.83, 5.45, 6.67, 8.58, 8.87 and 9.357 MeV, respectively. The Inclusion of core polarization effects gives a remarkable improvement in the reduced transition strength both with introducing effective charges (δe=0.35) and with M3Y interaction with differ effective charges as given in Table 1. Also, an excellent overlap between the core polarization with effective charges and M3Y interaction for B(C2↓) values.

1. The state 2_1^+ at 2.160 MeV

In the sd-model space, the predicted B(C2↓) value is 3.16 W.u which is a factor of about one-third of the experimental value 10±1 W.u and one-half of the cluster value 7.27 W.u.[13]

The inclusion of core polarization with introducing effective charge δe=0.35 enhances the reduced transition strength. The calculated B(C2↓) value is 9.15 W.u. This value is very close to that of experimental value and overestimates the cluster value.

When higher energy configurations (M3Y interaction) are included, the reduced transition strength B(C2↓) becomes 8.47W.u

The calculated effective charge at the photon point is δe=0.318.

2. The state 2_2^+ at 4.38 MeV

In the sd-model space, the predicted B(C2↓) value is 0.786 W.u which is a factor of about one-half of the experimental value 1.4±0.2 W.u and overestimates of the cluster value.[13]

The inclusion of core polarization with introducing effective charge δe=0.35 enhances the reduced transition strength. The calculated B(C2↓) value is 2.27 W.u . When higher energy configurations (M3Y interaction) are included, the reduced transition strength B(C2↓) becomes 2.39W.u

These values overestimate the experimental value and the cluster value. The calculated effective charge at the photon point is δe=0.372.

3. The state 2_3^+ at 5.45 MeV

The sd-model space fails to describe the transition strength data as shown in table 1. The predicted B(E2↓) value is 0.0776 W.u which are a factor of about one- half the experimental value 0.12±0.03 W.u and still far from cluster value 1.44 W.u.[13]

Both the inclusion of core polarization with effective charge and higher energy configuration (hec) increases B(C2↓) markedly. The predicted transition rate B(C2↓) becomes 0.224 W.u(cp)and 0.290 W.u(hec), respectively. These values still close both other but overestimate the experimental value 0.12±0.03 W.u.[13]

The calculated effective charge at the photon point is δe=0.467.

4. The state 2_4^+ at 6.67 MeV

In sd-model space, the predicted B(C2↓) value is 0.0166W.u.

Core polarization effects with introducing effective charge enhance the B(C2↓) and bring the calculated value close to the core polarization(M3Y) value. The B(C2↓)value is 0.048 W.u with effective charge δe=0.35 and 0.061W.u with higher energy configurations. These values underestimate the cluster value 1.06 W.u. [13]

The calculated effective charge at the photon point is δe=0.463.

5. The state 2_5^+ at 8.58 MeV

The sd shell model without core polarization effects failed to predict the transition strength for this state where the calculated value is 0.00086W.u, while the experimental value is 0.8±0.3W.u.[13]

Core polarization effects which are included through giving the protons and neutrons in the model space effective charges, different from those of free proton and neutron, enhance the reduced transition strength, and describe the data better than those of the bare charges. However, the values 0.00249W.u (cp with effective charge) and 0.00374W.u (with M3Y interaction) are still underestimated in the experimental value 0.8±0.3W.u The reduced transition probabilities *B*(C2↑) and *B*(C2↓) are shown in Table 1, using different values of effective charges for the transition to this state.

The calculated effective charge at the photon point is δe=0.541.

6. The state 2_6^+ at 8.78MeV

The sd-model space without introducing effective charges fails to describe the reduced transition strength B(C2↓) for this state as shown in [Table 1].The predicted B(C2↓) value is 0.00651W.u which is a factor about between one–third and one-sixth of the experimental value 0.026±0.009 W.u.[13]

The inclusion of core polarization effects which are included through giving the proton and neutron effective charges, also, inclusion higher energy configuration effects, enhance the transition strength, and describe the data better than those of the bare charges. The calculated B(C2↓) are 0.0188W.u and 0.01645 W.u for the core polarization(δe=0.35) and higher energy configuration, respectively.

The calculated effective charge at the photon point is δe=0.295. The reduced transition strength calculated with the model space with constant effective charges agrees with the calculated with the inclusion of higher energy configuration (hec).

7. The state 2_7^+ at 9.357 MeV

In the sd-model space, The predicted B(C2↓) value is 0.00148W.u
Core polarization effects which are included through giving the protons and neutrons in the model space effective charges, different from those of free proton and neutron, enhance the reduced transition strength, and describe the data better than those of the bare charges. Inclusion of higher energy configuration (hec) with M3Y interaction increases B(C2↓) markedly. The predicted transition rate B(C2↓) becomes 0.00429 W.u(cp)and 0.00331W.u(hec), respectively.

The sd-model space predicted B(C2↓) value is 0.00148W.u which is a factor about between one–third and one-half of the calculated core polarization (δe=0.35) 0.00429W.u and hec 0.00331W.u, respectively.

The calculated effective charge at the photon point is δe=0.247.

Conclusions

The sd-shell model space (without core polarization) failed to describe the data for the transition strengths [Table-1] for the seven excited 2^+states with excitation energies.

Core polarization effects are essential in the calculation of transition strength. The inclusion of core polarization effects gives a remarkable improvement in the transition strength and describes the data well for the first and sixth 2^+ states. The structure of the low-lying states of ^{32}S has long been a problem in nuclear physics and needs a lot of theoretical efforts to overcome this discrepancy.

The inclusion of core polarization effects with introducing constant effective charges shows a good agreement with the higher energy configurations (hec) for all the 2_i^+ states.

$0_1^+0 \rightarrow 2_i^+0$	δe	B(C2↑) $e^2.fm^4$	B(C2↑) W.u	B(C2↓) $e^2.fm^4$	B(C2↓) W.u	Exp. B(C2↓) W.u	Cluster B(C2↓) W.u	E_x(MeV) Exp.	E_x(MeV) The.
0ħω	0	95.742	15.832	19.148	3.166				
CP	0.35	276.694	45.755	55.338	9.151	10±1	7.27	2.230	2.160
M3Y	0.318	256.253	42.375	51.250	8.475				
0ħω	0.0	23.779	3.932	4.755	.786				
CP	0.35	68.723	11.364	13.744	2.272	1.4±0.2	0	4.282	4.38
M3Y	0.372	72.327	11.960	14.565	2.392				
0ħω	0	2.351	0.388	0.470	0.077				
CP	0.35	6.796	1.123	1.359	0.224	0.12±0.03	1.44	5.549	5.45
M3Y	0.467	8.796	1.454	1.759	0.290				
0ħω	0	0.5045	0.0834	0.10091	.01669				
CP	0.35	1.458	.24112	.2916	.04822	--------	1.06	6.666	6.67
M3Y	0.463	1.87153	.30948	.37431	.06190				

0ħω	0	.02608	.00432	.00522	.00086				
CP	0.35	.07537	.01246	.01507	.00249	0.8±0.3	--------	9.464	8.58
M3Y	0.541	.11304	.01869	.02261	.00374				
0ħω	0	.19669	.03253	.03934	.00651				
CP	0.35	.56845	.09400	.11369	.01880	0.026±0.009	--------	9.712	8.78
M3Y	0.295	.49726	.08223	.09945	.01645				
0ħω	0	.04484	.00742	.00897	.00148				
CP	0.35	.12959	.02143	.02592	.00429	--------	--------	------	9.357
M3Y	0.247	.10009	.01655	.02002	.00331				

Table (1): The calculated values of the reduced transition probabilities B(C2) (in unit $e^2.fm^4$ and W.u) compared with the experimental and cluster values. [13]

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