# Determination of Skip Entry Trajectories for Space Vehicles at Circular and Super Circular Speeds 

Waleed I. Yaseen<br>Department of Astronomy, College of Science, University of Baghdad


#### Abstract

The study of entry and reentry dynamics for space vehicles is very important, particularly for manned vehicles and vehicles which is carry important devices and which can be used again. There are three types for entry dynamic, ballistics entry, glide entry and skip entry. The skip entry is used in this work for describing entry dynamics and determining trajectory. The inertia coordinate system is used to derive equations of motion and determines initial condition for skip entry. The velocity and drag force for entry vehicle, where generate it during entry into earth's atmosphere are calculated in this work. Also the deceleration during descending and determining entry angles, velocities ratio and altitude ratio have been studied. The circular velocity and super circular velocity are used in this work as initial values. From results we noted the skip entry type has longer flight time compared with other types, and the velocity vehicle is lower at high layer for earth atmosphere, where the density of air is very low. Therefore the skip entry is suitable for space shuttle during entry from space.


## Keywords

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\begin{gathered}
\text { تحدبد مسار ات الاخول القافز للمركبات الفضـائية عند السر ع الدائرية وفوق الدائرية }
\end{gathered}
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الخلاصة
إن در اسة ديناميكا الاخول وإعادة الدخول للمركبات الفضائية من الدراسات المهمة جدا، وخصوصا للمركبات المأهولة و المركبات التي تحمل أجهزة مهمة ومكلفة والتي يمكن إعادة استعمالها مرة ثانية. هناك ثلاث انوا واع لاع لايناميك الالخول ، الاخول البالستي، والاخول الانز لاقي، والدخول التقافز وسنستعمل في هذا البحث الاخول القافز لوصف ديناميكا الالخول وتحديد المسار لهذا النوع وحساب السرع وقوة الإعاقة التي يسبيها الغلاف الغازي على المركبة أثناء الاخول. وسيتم في هذا البحث استخدام منظومة الإحداثيات القصورية لاشتقاق المعادلات وتحديد اللشروط الابتائية لهنا النوع. وتمت درُسة الثتاقل خلال النزول وتحديد زو ايا الاخول الخاصة والسرع النسبية والارتفاع النسبي لهذا النوع. وسنأخذ في هذا العمل السر عة الائرية و السر عة فوق الدائرية كقيم دخول ابتائئة. ومن النتائج النتي حصلنا عليها من هذا العمل نلاحظ ان هذا النوع من الاخول يمتلك زمن طير ان طويل مقارنة مع الانواع الاخرى وسر عة المركبة تكون واطئة في الطبقات العليا للغلاف الغازي الارضي. لذلك يكون الاخول القافز مناسبا للمكوك الفضائي خلال دخوله للغلاف.

## Introduction

The atmospheric entry or reentry are important problem, and arises for any vehicle which approaches a planetary atmosphere and for which physical recovery or survival is desired at the planetary surface, and it is very important
and dangers for manned vehicles. This includes a series of possibilities ranging from simple sounding rockets to manned vehicles returning from interplanetary trips [1].
The heating and deceleration accompanying atmospheric entry bring
about severe design problems; the presence of a planetary atmosphere is advantageous in that it acts as force breaking to reduce a space vehicle's velocity to safe landing speed. [1]. There are three types of reentry trajectories,(a) ballistic entry, (b)glide entry and (c) skip entry shown in Fig.(1). The skip entry type is investigated in this work. The skip trajectory is creating up of ballistic phases connected by skipping phases. The principle is that the vehicle develops lift with minimum drag during the skip entry. The flight path resembles a stone piece skipping over a water pond. During the skipping phase large aerodynamic forces are experienced. They tend to be so large when the reentry angle is steep that for most practical purposes it is permissible to neglect the gravity force and to treat such a skipping phase as an impact. The skip vehicle with a lift-to-drag ratio between 1 and 4 appears to be more efficient than both the ballistic and glide types in converting high velocity into long range [2].
The aerodynamics force are acted on the vehicle during entry into the atmosphere by skip trajectory .This force are made between a vehicle and the air. Caused by their relative motion, such as the wind blowing across the surface of the earth. Typical aerodynamic forces are those acting on a vehicle in subsonic flight. These particular forces are: firstly, the lift force perpendicular to the flight path and concerned with the sustentation of a vehicle. Secondly, the drag force or resistance acting to the rear along the flight path and thirdly the thrust from the power plant necessary to propel or sustain a vehicle, these forces are illustrated in Fig. (2) [3].


Fig. (1): Types of re-entry trajectories, including (a) ballistic, (b) glide, and (c) skip [2]

## Entry Dynamics Equations

The equations of motion of a vehicle entering a planet's atmosphere have been derived by a nonrotating two dimensional inertial coordinate system, and are employed with its origin at the center of the earth or planet. The gravitational field during the atmospheric entry portion is variation with altitude [4].
The simplified geometry of a skip entry is illustrated in Fig. (2). Here the vehicle encounters the outer atmosphere with a velocity Vo, and exits at a flight path angle, $\theta$, with a lower velocity (due to atmospheric drag), V.


Fig. (2): simplified geometry of a skip entry [4]
From Fig. (2) we get:
$-D+m g \sin \theta=m \frac{d V}{d t}=\frac{m}{2} \frac{d V^{2}}{d s}$
$L-m g \cos \theta+m V\left(\frac{V \cos \theta}{R o}\right)=-m V \frac{d \theta}{d t}$
Where, (m) mass of vehicle, ( $\theta$ ) angle of entry, (t) flight time, (s) range and Ro radius of planet.
The drag force and lift force are assumed to have the following standard form [4]:
$L=\frac{1}{2} C_{L} \rho V^{2} A$
$D=\frac{1}{2} C_{D} \rho V^{2} A$
Where (D) is drag force ( L ) lift force, $\left(\mathrm{C}_{\mathrm{L}}\right)$ lift coefficient, $\left(\mathrm{C}_{\mathrm{D}}\right)$ drag coefficient, ( $\rho$ ) density of atmosphere, (A) cross sectional frontal area [5].
From Newton's inverse-square law of gravity we have the acceleration due to force of gravity (g):
$g=g o\left(\frac{R o}{R o+y}\right)$
Where go is the acceleration due to force of gravity at sea level and (y) is the altitude of the vehicle [6].
By using the kinematics relation we get:

$$
\frac{d y}{d s}=-\sin \theta
$$

Substituting for Eq. (5) from Eq. (1) and Eq. (2) and rearranging terms, one obtains [4]:

$$
\begin{aligned}
& \frac{d \cos \theta}{d \rho}+\left(\frac{1}{\beta R o}\right) \frac{\cos \theta}{\rho}\left(\frac{g R o}{V^{2}}-1\right)=\frac{1}{2}\left(\frac{L}{D}\right)\left(\frac{C_{D} A}{m \beta}\right) \\
& \frac{d\left(\frac{V^{2}}{g R o}\right)}{d \rho}+\left(\frac{C_{D} A}{m \beta}\right) \frac{\left(\frac{V^{2}}{g R o}\right)}{\sin \theta}=\left(\frac{2}{\beta R o}\right) \frac{1}{\rho}
\end{aligned}
$$

Where $\beta$ is constant planetary. Eq. (6) and Eq. (7) are the exact equations of motions of entry into planetary atmosphere.
From these two equations the second order solution of entry dynamics may be writing [4]:

$$
\begin{equation*}
\cos \theta=\frac{\cos \theta_{f}+\frac{1}{2}\left(\frac{L}{D}\right)\left(\frac{C_{D} A}{m \beta}\right) \rho\left(1-\frac{\rho_{f}}{\rho}\right)}{1+\left(\frac{1}{\beta R o}\right)\left(\frac{g R o}{V^{2}}-1\right)\left(1-\frac{\rho_{f}}{\rho}\right)} \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\ln \left[\frac{\frac{V_{2}}{g R o}}{\frac{V_{f}^{2}}{g R o}}\right]=\frac{\left(\frac{C_{D} A}{m \beta}\right)\left(\theta-\theta_{f}\right)}{\frac{1}{2}\left(\frac{L}{D}\right)\left(\frac{C_{D} A}{m \beta}\right)-\left(\frac{1}{\beta R o}\right) \frac{\cos \theta}{\rho}\left(\frac{g R o}{V^{2}}-1\right)} \tag{9}
\end{equation*}
$$

## Atmospheric Model

In this model we have used the balance forces theory for a vertical column of air for calculate the variations of parameters of atmospheric of the earth (density, pressure, temperature and molecular weight) with altitude from sea level up to 200 km . The density equation of air is important parameter in skip entry dynamics and we get it from equation:
$\rho=\rho_{o} e^{-\beta_{y}}$
( $\rho_{o}$ ) is density air at sea level [7].

## Skip entry dynamics

The aerodynamic lift is obviously the predominant force for skipping phase.
(5)Where the difference between gravitational force and centrifugal force compared with the lift force (L) is negligible. From Eq. (2) we get [4]:
$m g \cos \theta-m V^{2}\left(\frac{\cos \theta}{R o}\right)=m V^{2}\left(\frac{\cos \theta}{R o}\right)\left(\frac{g R o}{V^{2}}-1\right) \ll L$
Since $m V^{2}$ is large quantity, $\left(g R o / V^{2}-1\right)$ must be small or zero. With this simplification and $\left(\rho_{f} / \rho\right)$ as a negligible
(7quantity, Eq. (8) and Eq. (9) are respectively simplified to [4]:

$$
\begin{align*}
& \cos \theta-\cos \theta_{f}=\frac{1}{2}\left(\frac{C_{L} A}{m \beta}\right) \rho \\
& =\frac{1}{2}\left(\frac{C_{L} A}{m \beta}\right) \rho_{o} e^{-\beta y} \tag{12}
\end{align*}
$$

$$
\begin{equation*}
\ln \left[\frac{V}{V_{f}}\right]^{2}=\frac{2}{\left(\frac{L}{D}\right)}\left(\theta-\theta_{f}\right) \tag{13}
\end{equation*}
$$

Eq. (12) and Eq. (13) are the first order solutions for skipping entry.
The maximum deceleration due to the action of aerodynamic force is determined primarily by the velocity and angle of entry into the atmosphere, and its essentially independent of vehicle size, weight, and shape, although the altitude at which maximum deceleration occurs is not.
Equation of deceleration is[4]:
$d e c=-\frac{d V}{d t}$

## Results and discussion

Eq. $(3,4)$ and Eq. $(12-14)$ and may be are solved by using $\mathrm{C}++$ language and use Rang Kuta method. Chart (1) is illustrated the programming for these equations. Initial conditions for skip entry illustrated in Table (1). These initial conditions are include, velocity V , altitude y in start entry, cross section area for vehicle entry A, entry angle $\theta$, mass of vehicle m , drag coefficient $C_{D}$, lift coefficient $C_{L}$, and density of air $\rho$.

Table (1): Initial condition for skip entry.

| Initial <br> parameters | Value |
| :---: | :---: |
| Velocity V | $11280.48 \mathrm{~m} / \mathrm{s}$ |
| $\theta$ | 12 deg |
| $\rho$ | $1.225 \mathrm{~kg} / \mathrm{m}^{3}$ |
| Ro | 6378 km |
| y | 200 km |
| CDW | 3.2 |
| BR | 900 |
| $\mathrm{~L} / \mathrm{D}$ | 1 |
| g | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ |
| d | 1.5 m |
| $\mathrm{C}_{\mathrm{L}}$ | 2 |
| $\mathrm{C}_{\mathrm{D}}$ | 0.5 |



Chart (1): flow chart of skip entry dynamics.
The following figures are results of skip entry dynamics when run of the program.


Fig. (3): Variation of altitude with velocity ratio for various initial velocities.

Fig. (3) is illustrated the effect of initial velocity (circular and super circular velocity) on the variation of velocity ratio (velocity vehicle to circular velocity) during penetration of the earth's atmosphere from space. The similar behavior for curve velocity ratio is seen for two cases with shifted to lower region of velocity ratio. From this figure we noted, the velocity decreasing occur at lower altitude because the density of air is increasing at this altitudes.
Also when velocity ratio is varied with entry angle for skip entry we noted that similar curves result, with relative displacements for various values of initial velocity, and velocity ratio increasing with increase entry angle, are shown in Fig. (4).


Fig. (4): Variation of entry angle with velocity ratio for various initial velocities.


Fig. (5): Variation of altitude with entry angle for various initial velocities.

The variation of entry angle with altitude is shown in Fig. (5). The entry angle decrease at low altitude and it
becomes negative when altitude equal to zero. The various initial velocities are not effect on the curve behavior for entry angle.

The variation of deceleration with altitude as given by Eq. (13) is shown in Fig. (6) for various values of initial velocity (circular and super circular velocity). In each case it we noted that similar curves result with relative displacements for various values of initial velocity. The deceleration is dependent on density of air and square velocity therefore maximum deceleration is occurring, and because dependent it on square velocity, therefore when initial velocity is increases, also the maximum deceleration is increase. Also the entry angle is effect on the maximum deceleration, and when entry angle becomes negative the deceleration is began decreases to minimum value, this result shown in Fig. (7).


Fig. (6): Variation of altitude with deceleration for various initial


Fig. (7): Variation of entry angle with deceleration for various initial velocities.


Fig. (8): Variation of deceleration with velocity ratio for various initial velocities

Fig. (8) is illustrated effecting of velocity ratio on deceleration with various initial velocities.
Fig. (9) is illustrated the lift and drag force during skip entry, where the lift force larger than drag force. From this figure the lift and drag force in start increases to maximum value at increasing velocity ratio after that it decreases to lower value with velocity ratio increase.


Fig. (9): Variation of lift and drag force with velocity ratio.

From this figures we noted the skip entry kind have longer flight time compared with other kinds, and the velocity vehicle is lower at high layer for earth atmosphere, where the density of air is very low. Therefore the skip entry is suitable for space shuttle during entry from space.

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