The Electro-Excitation Form Factors of ^{7}Li Nucleus with Exact Center of Mass Correction

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Abstract

The inelastic longitudinal electron scattering form factors are calculated for the low-lying excited states of ^{7}Li {the first excited state $J^\pi T = \frac{1}{2}^- \frac{1}{2}$ (0.478 MeV) and the second excited state $J^\pi T = \frac{7}{2}^- \frac{1}{2}$ (4.63 MeV)}. The exact value of the center of mass correction in the translation invariant shell model (TISM) has been included and gives good results. A higher 2p-shell configuration enhances the form factors for high q-values and resolves many discrepancies with the experiments. The data are well described when the core polarization (CP) effects are included through effective nucleon charge. The results are compared with other theoretical models.

Keyword: ^{7}Li inelastic electron scattering form factors calculated with exact center of mass correction. CP is included through effective charge model.

Keywords
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عوامل التشكل للاستثارة الالكترونية لنواة ^{7}Li مع التصحيح الدقيق لمركز الكتلة

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الخلاصة:

حسبت عوامل التشكل الطوليةغير المرنة للأستطارة الألكترونية للمستويات المنخفضة لحالات **النواة** ^{7}Li (الحالة المتهيجة الأولى $J^\pi T = \frac{1}{2}^- \frac{1}{2}$ (0.478MeV) والحالة المتهيجة الثانية $J^\pi T = \frac{7}{2}^- \frac{1}{2}$ (4.63MeV)). أدخلت القيمة الحقيقية لتصحيح مركز الكتلة مع فضاء إنموذج الأغلفة غيرالمعتمد على حركة مركز الكتلة (TISM) معطية نتائج جيدة. أدى مساهمات المدارات العالية كالغلاف-2p الى زيادة عوامل التشكل لقيم الزخوم المنتقلة العالية وحل العديد من التباينات مع التجارب. البيانات تم حسابها ثانية بشكل جيد عندما أدخل تأثير أستقطاب القلب من خلال الشحنة الفعالة للنوية. قورنت النتائج الحالية مع نتائج نماذج أخرى.

Introduction

The scattering of electrons from nuclei gives the most precise information about nuclear size and charge distribution, since it is sensitive to the spatial dependence of the charge and current densities. In the electron scattering, the target is probed through the well-understood electromagnetic interaction. Since the interaction is relatively weak, the measurement can be made on the target nucleus without greatly disturbing its structure [1].

Bumiller et. al. (1972) [2] measured the elastic longitudinal form factor of ^{6}Li and ^{7}Li at momentum transfer $q \leq 1.0$ fm^{-1}. Lichtenstadt et al. (1983) [3] measured the transverse form factors for ground and

0.478 MeV states of ^{7}Li. A good agreement between the data and their results were obtained. Their calculations were based on Cohen-Kurath (C-K) [4] shell model amplitudes, using oscillator parameter of value 1.65 fm. Weller (1985) [5] measured the electric ground state properties of ^{7}Li. Comparison with data from other experiments and with other theoretical predictions were made. The theoretical calculations tend to underestimate the actual transition probability B(C2, $\frac{3}{2}^{-} \rightarrow \frac{1}{2}^{-}$) value ($8.3 \pm 0.5$ $e^{2}.fm^{4}$).

Lichtenstadt et. al. (1989) [6] measured the longitudinal and transverse electromagnetic form factors of ^{7}Li ground-state doublet (the $J^{\pi} = \frac{3}{2}^{-}$ ground state and the E_x = 478 KeV, $J^{\pi} = \frac{1}{2}^{-}$ first excited state) by electron scattering up to momentum transfers of 4.2 fm^{-1} and 4.5 fm^{-1}, respectively. The electromagnetic form factors of 4.63 MeV excitation ($J^{\pi} = \frac{7}{2}^{-}$) in ^{7}Li were measured by Lichtenstadt et. al. (1990) [7] over momentum transfer range $0.8 \leq q \leq 4.2$ fm^{-1}. Comparison with the form factors of the ground-state doubled indicates that high multipoles may make significant contributions to the transverse form factors, but not in the longitudinal ones. Wolter et. al. (1990) [8] studied the electromagnetic structure of 1p-shell nuclei. Their calculations included the extended $(0+2)\hbar\omega$ model space , and the effective nucleon charges. They obtained the values (e_p=1.19e and e_n= 0.06e) by fitting the electric quadrupole moments calculated in $2\hbar\omega$ space to the experimental values. The electron scattering form factors of 1p-shell nuclei have been performed by Booten (1992) [9] in 1p-shell model as well as in the extended $(0+2)\hbar\omega$ model space, the effects of meson exchange current were included to the transverse form factor. The longitudinal form factors were calculated for only ^{7}Li nucleus, and reasonably well-reproduced up to q ~3.2 fm^{-1} in the enlarged model space. Karataglidis et. al. (1997) [10] used $(0+2+4)\ \hbar\omega$ wave functions in the analysis of the elastic and inelastic electron scattering form factors in ^{6}Li and ^{7}Li nuclei. For the longitudinal inelastic electron scattering form factors, none of their results made within all shell model space were able to reproduce the data at low momentum transfer, but high q-data were well reproduced with multi-$\hbar\omega$ model. In (1999) Mihaila and Heisienberg [11] proposed a many-body expansion for the computation of the charge form factor in center of mass system. They applied their formalism to the case of the harmonic oscillator shell model, where an exact solution exists.

Radhi et. al. (2001) [12] studied the CP effects on the longitudinal form factors of 1p-shell nuclei. The modified surface-delta interaction (MSDI) was adopted as a residual interaction. Their results described the data very well in both the transition strengths and momentum transfer dependence. Very recently, Radhi et. al. (2009) [13] have studied the electro-excitations, for p-shell nuclei, especially ^{9}Be, using large-basis shall model wave functions. They found that excitations up to $6\hbar\omega$ were enough for sufficient convergence. These excitations were found to be essential in obtaining a reasonable description of the data.

In the present work the effect of the center-of-mass correction on the longitudinal form factors is investigated. The exact center-of-mass correction of Mihaila and Heisenberg [11] has been adopted to generate the longitudinal form factors in the Born approximation picture. The center-of-mass correction that was used in other previous works was also taken into account for comparison.

Theory

The longitudinal form factor for a given multipolarity J and momentum transfer $\vec{q}$ is expressed as [9]:

$$|F_J^{coul}(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left|\left\langle J_f \left\| \hat{T}_J^{coul}(q) \right\| J_i \right\rangle\right|^2$$

$$\times |F_{c.m}(q)|^2 \times |F_{f.s}(q)|^2 \text{ ------------- } (2.1)$$

where is the finite size of the nucleon, and $F_{f.s} = e^{\frac{-0.43q^2}{4}}$ $F_{c.m.} = e^{\frac{q^2b^2}{4A}}$ is the center of mass correction [14, 15]. The reduced matrix elements of the longitudinal electron scattering operator $\hat{T}^{coul}$ is expressed as a sum of the one body density matrix (OBDM) $\chi_{J_iJ_f}^{JT}(\alpha,\beta)$ times the single-particle matrix elements and given by:

$$\left\langle J_f \left\| \hat{T}_J^{coul} \right\| J_i \right\rangle = \sum_{\alpha\beta} \chi_{J_iJ_f}^{J}(\alpha,\beta) \left\langle \alpha \left\| \hat{T}_J^{coul} \right\| \beta \right\rangle \text{ ------- } (2.2)$$

where α and β label single-particle states (isospin is included) for the model space.

The exact value of the center of mass correction $F_{exa}(\vec{q})$ in the translation invariant shell model TISM is given by [11]:-

$$F_{exa}(\vec{q}) = F_{c.m.}(\vec{q})\, F_{\text{int}}(\vec{q}) \text{ ------- } (2.3)$$

where F_{int} is for the internal motion; equation (2.1) becomes:-

$$|F_J^{coul}(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left|\left\langle J_f \left\| \hat{T}_J^{coul}(q) \right\| J_i \right\rangle\right|^2$$

$$\times |F_{exa}(q)|^2 \times |F_{f.s}(q)|^2 \text{ --------------(2.4)}$$

When the 1p-shell model space is extended to include the 2p-shell model space, the wave functions of the initial (i) and final (f) states will be written as:

$$|i\rangle = \delta|i\,(1p)\rangle + \sqrt{1-\delta^2}\,|i\,(2p)\rangle \text{ ----(2.5)}$$

$$|f\rangle = \gamma|f\,(1p)\rangle + \sqrt{1-\gamma^2}\,|f\,(2p)\rangle \text{ ----2.6)}$$

where δ and γ are mixing parameters. Since the C-K interaction depends on the angular parts only, the same OBDM are used for both 1p and 2p shells. The reduced transition probability is given by [16]:

$$B(CJ) = \frac{[(2J+1)!!]^2}{4\pi} \frac{Z^2}{k^{2J}} |F_J^{co}(q=k)|^2$$

--------(2.7)

where $q = k = \frac{E_x}{\hbar c}$

Results and Discussion

1. <u>The 0.478 MeV (1/2$^-$ 1/2) State</u>

For the coulomb transition to the $J^\pi T = \frac{1}{2}^- \frac{1}{2}$ (0.478 MeV) first excited state, only the C2 multipole is allowed. The calculated longitudinal form factors with size parameter b_{rms}=1.74fm [2] and with bare nucleon charges are shown in Fig.(1). The 1p-shell results with and without the exact value of c.m. correction (red and blue dashed curves respectively), fail to reproduce the experimental data of Lichtenstadt et al. [6] (circles) at all regions of momentum transfer. The second diffraction maximum is measured around q= 3.3 fm^{-1}. Same behavior could be seen in the work of Booten [9], and Karataglidis et al. [10].

The inclusion of higher configurations supplies the necessary strength to reproduce the data. Fig.(2a) represents the results including the core-polarization effects (with e_p=1.35e and e_n=0.35e) with the exact value of c.m. correction (red-solid curve), and without the exact value of c.m. correction (blue-solid curve). Both calculations are close to each other, they give a good behavior for all momentum transfer regions, and fail to reproduce a second measured diffraction maximum.

The present results are compared with the (0+2)$\hbar\omega$ results of Booten [9] (dotted curve), (0+2+4)$\hbar\omega$ results of Karataglidis et al. [10] (blue dash-dotted curve),and with the (1p+CP) results of Radhi et al. [12] (cross symbol curve). This comparison is shown in Fig.(2b). Good agreement can be noted between the experimental data and both present results and Radhi et al. [12] results.

The present results are very close to that of Radhi et al. [12](cross symbol curve) for momentum transfer up to q >3.0 fm^{-1}. They reproduce very well the measured first maximum value ($\sim 8\times10^{-3}$) at q ~ 1.2 fm^{-1}, and they are slightly different from that of Booten [9] (dotted curve) and Karataglidis et al. [10] (blue dash-dotted curve). The first maximum was underpredicted by an order of magnitude in both results of Booten [9], and Karataglidis et al. [10].

The present results and that of the above three models fail to reproduced the second diffraction maximum. The predicted value of B(C2) with the exact value of c.m. correction (5.49 $e^2.fm^4$) gives a reasonable agreement with that of Refs. [9, 12] and less than the observed value ($8.3\ \pm 0.5$ $e^2.fm^4$) of Ref.[17], as shown in table (1).

Table (1): The calculations of the reduced transition probabilities B(C2↑) (in unit of $e^2.fm^4$) in comparison with experimental values and other theoretical calculations. The effective charge for all transition is e_p=1.35e, e_n=0.35e.

Nucleus	$J_f^\pi T_f$	Energy (MeV)	Present work (1P+corr.*)		Experiment values	Other theoretical results		
			Without exact c.m. value	With exact c.m. value		Ref.[9]	Ref.[10]	Ref.[12]
^{7}Li	1/2$^-$ 1/2	0.478	6.78	5.49	8.3 ± 0.5^a	5.75	7.23	6.507
^{7}Li	7/2$^-$ 1/2	4.63	11.10	8.99	3.5^b 7.5 ± 0.8^c	10.48	3.32	10.57

[a] Ref. [17], [b] Ref. [18] [c], Ref. [7]

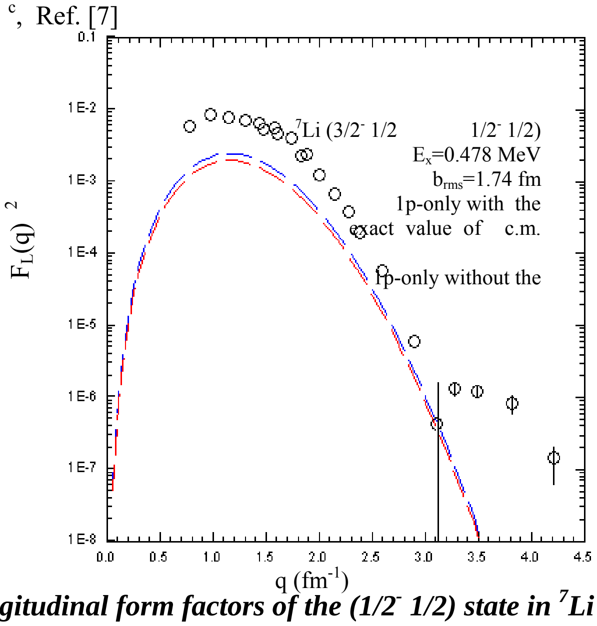

Fig (1): The longitudinal form factors of the (1/2$^-$ 1/2) state in ^{7}Li calculated in 1p-shell model space only. The red dashed curve represents the results with exact value of c.m. correction, and the

blue dashed curve represents the results without the exact value of c.m. correction. The experimental data are taken from Ref. [6] (circles).

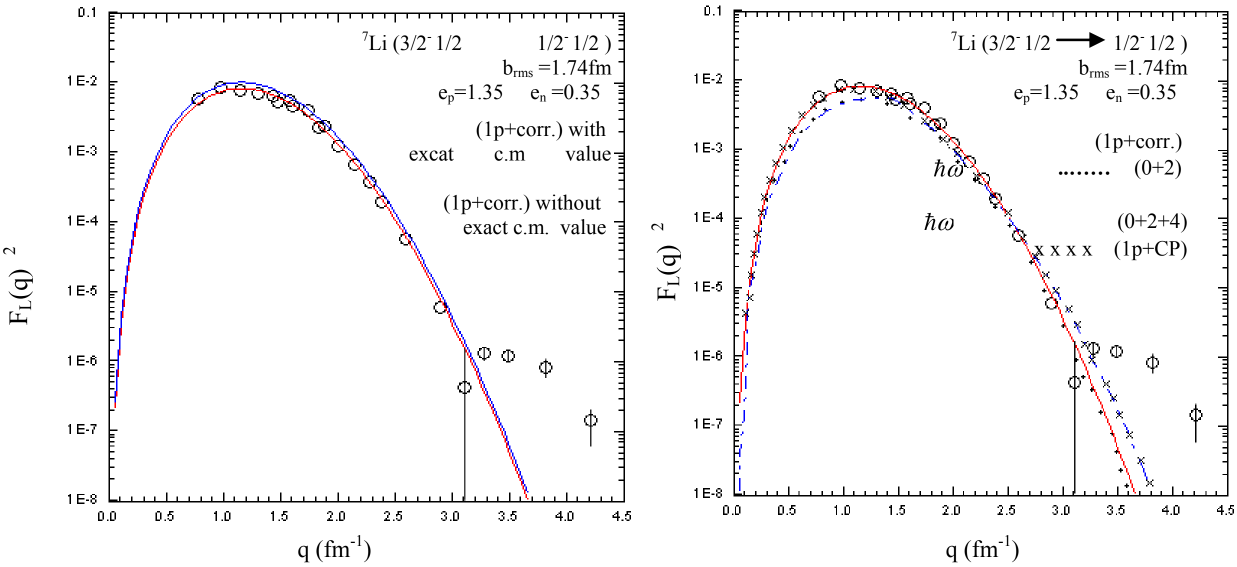

Fig (2) The longitudinal form factors of the (1/2⁻ 1/2) state in ^{7}Li calculated with (1p+corr). (a) The present results with and without exact value of c.m. correction (red and blue solid curves respectively). (b) The present results (red solid curve) are compared to that of Ref. [12] (cross symble curve), Ref. [10] (blue dash-dotted curve) and to the results of Ref. [9] (dotted curve).

2.<u>The 4.63 MeV (7/2⁻ 1/2) State</u>

The transition to the $J^{\pi}T=\frac{7}{2}^{-}\frac{1}{2}$ (4.63 MeV), state is associated with C2 and C4 multipoles. The C4 multipole is absent in the 1p-shell model space, since the largest multipolarity for any transition involving one-body operator is L=3.The calculated form factors with size parameter $b_{r.m.s}$= 1.74 fm [2] and with free charges are shown in Fig.(3).The form factor is entirely dominated by C2 multipole.

The 1p-shell results with and without exact value of c.m. correction (red and blue dashed curves respectively) fail to match the magnitude of the experimental data of Lichtenstadt et al. [7] (circles), over all range of momentum transfers.

The addition of higher energy admixtures into the model space acts as the core-polarization corrections normally associated with 1p-shell calculations.

For the calculations with the exact value of the c.m. correction, the inclusion of the core-polarization effects with effective charges (e_p=1.35e and e_n= 0.35e) provides a very good agreement with the first measured peak, but the results fail to reproduce the second diffraction maximum as shown in Fig.(4a) (red solid curve). The results without exact value of the c.m. correction (blue solid curve) show almost same behavior. The comparison between the present results and that of Booten [9] (dotted curve), Karataglidis et al. [10] (blue dash-dotted curve) and (1p+CP) Radhi et al. [12] (cross symbol curve) is presented in Fig.(4b). The present results and that of Radhi et al. [12] (cross symbol curve) reproduce very well the measured first maximum value ($\sim 13\times10^{-3}$) at q ~ 1.1 fm^{-1}. They are close to each other for momentum transfer up to q > 2.8 fm^{-1}, and slightly different from that of Booten [9] (dotted curve), and Karataglidis et al. [10] (blue dash- dotted curve). The second diffraction maximum is not observed in the present results as well as in all above three models.

There is some doubt on the measured B(C2) for this transition. From the q noted γ-decay rate [18] this is 3.5 e^2.fm^4, while the value obtained from an analysis of the longitudinal inelastic electron scattering form factor is 7.5 $\pm$0.8 e^2.fm^4 [7].The calculated B(C2) value (8.99 e^2.fm^4) is in a reasonable agreement with the experimental value (7.5 $\pm$ 0.8 $e^2.fm^4$) [7] and with that of Ref.[9] and Ref. [12] as shown in table (1). Radhi et. al. [19] presented a calculation of the form factor for the two mentioned stats of ^7Li, using Woods-Saxon potential for the radial part of the single-particle wave functions. A second diffraction maximum was obtained in this case, and explained the diffractive structure in these two states. So, the high q-data depend strongly on the radial part of the single-particle wave functions.

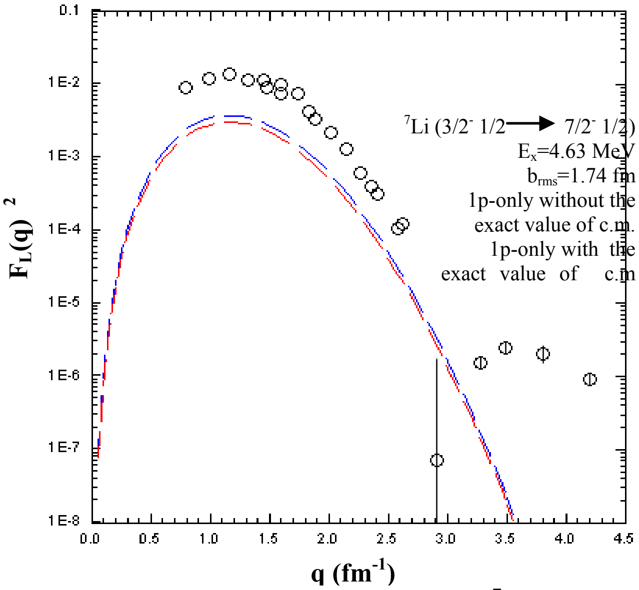

Fig (3) The longitudinal form factors of the (1/2$^-$ 1/2) state in ^7Li calculated in 1p-shell model space only. The red dashed curve represents the results with exact value of c.m. correction, and the blue dashed curve represents the results without the exact value of c.m. correction. The experimental data are taken from Ref. [7] (circles).

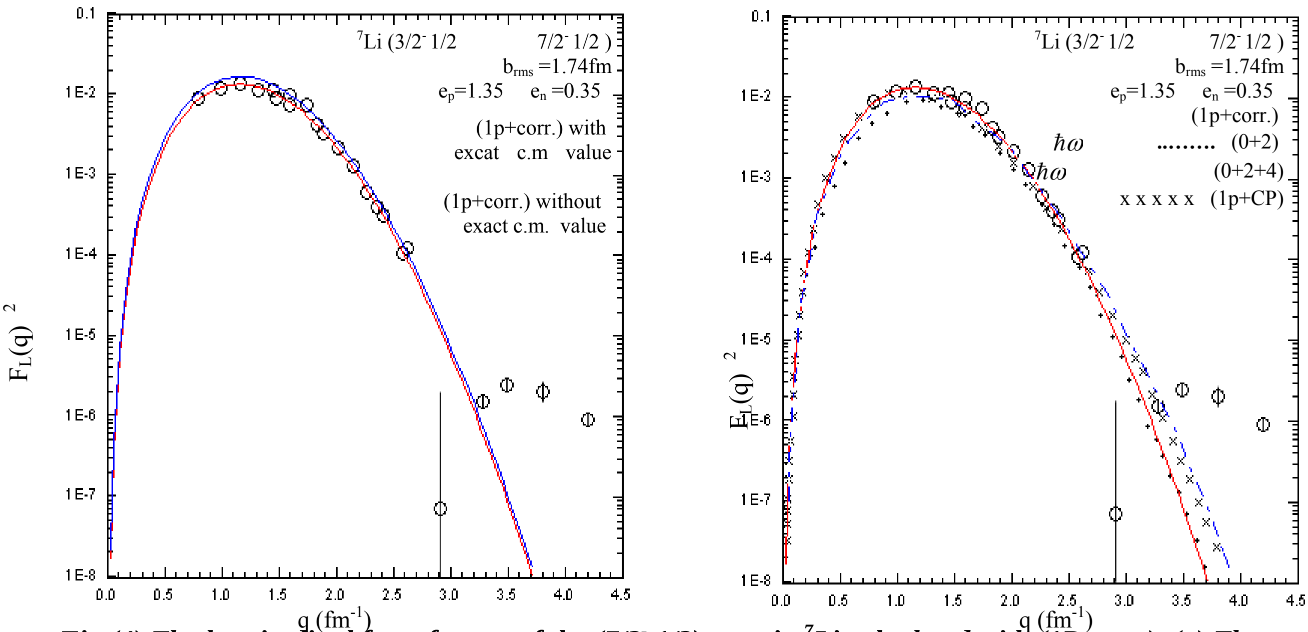

Fig (4) The longitudinal form factors of the (7/2$^-$ 1/2) state in ^7Li calculated with (1P+corr). (a) The present results with and without exact value of c.m. correction (red and blue solid curves respectively). (b) The present results (red solid curve) are compared to that of Ref.[12] (cross symble curve), Ref.[10] (blue dash-dotted curve) and to the results of Ref. [9] (dotted curve).

Conclusions

The most important conclusions of the present work can be briefly summarized by the following words: the longitudinal inelastic electron scattering form factors are fairly well predicted with the CP effects. For both C2 transitions, the inclusion of effective charges (e_p=1.35e and e_n= 0.35e) are adequate to obtain a good agreement between the predicted and measured form factors. The inclusion of the exact value of c.m. correction has a remarkable role on the B(C2) value and minor role on q dependence form factors. The predicted values of B(C2) are reduced with the exact value of c.m. correction.

References

[1] T. De Forest and J. D. Walecka; Adv. Phys., 15, 1 (**1966**).

[2] J. D. Walecka; Nucl. Phys., A574, 271 C (**1994**).

[3] J. D. Walecka; Overview of the CEBAF scientific program (**1992**).

[4] F. A. Bumiller, F. R. Buskirk and J. N. Dyer, Phys. Rev., C5, No.2, 391, (**1972**).

[5] J. Lichtenstadt, J. Alster, M.A.Moinester, J. Dubach, R. S. Hicks, G. A. Peterson and S. Kowalski, Phys. Lett., B121, No.6, 377 (**1983**).

[6] S. Cohen and P. Kurath; Nucl. Phys., 73, 1 (**1965**).

[7] A. Weller, Phys. Rev. Lett. 55, No. 5, 480 (**1985**)

[8] J. Lichtenstadt, J. Alster, M. A. Moinester, J.Dubach, R.S.Hicks, G.A.Peterson and S.Kowalski;Phys. Lett., B219, 394 (**1989**).

[9] J. Lichtenstadt, J. Alster, M. A. Moinester, J.Dubach, R.S.Hicks, G.A.Peterson and S.Kowalski; Phys. Lett., B244, No.2, 173 (**1990**).

[10] A. A. Wolters, A.G. M. Van Hess and P. W. M. Glaudmans; Phys. Rev. C42, 2053 (1990); C42, 2062 (**1990**).

[11] J. G. L. Booten, Ph. D. Thesis, Netherlands (**1992**).

[12] S. Karataglidis, B. A. Brown, K. Amos and P. J. Dortmans; Phys. Rev., C55, 2826 (**1997**).

[13] Bogdan Mihaila and Jochen H. Heisenberg; Phys. Rev., C60, 054303,(**1999**).

[14] R. A. Radhi, A. A. Abdulla, Z. A. Dakhil and N. M. Adeeb; Nucl. Phys., A696, 442 (**2001**).

[15] R. A. Radhi, N. M. Adeeb and A. K. Hashim; J. Phys G: Nucl. Part. Phys. 36 105102 (**2009**).

[16] L. J. Tassie and F. C. Barker; Phys.Rev., 111, 940 (**1958**).

[17] H. Chandra and G. Sauer; Phys. Rev., C13, 245 (1976).

[18] B. A. Brown, B. H. Wildenthal, C. F. Williamson, F. N. Rad, Sikowalski, H. Crannel and J. I. O Brien; Phys. Rev., C32, No.4, 1127 (**1985**).

[19] R. Yen, L. S. Cardman, D. Kalinsky, J. R. Legg, C.K. Bockelman, Nucl. Phys. A235, 135 (**1974**).

[20] F. Ajzenberg-Selove; Nucl. Phys. A490, 1 (**1988**).

[21] R. A. Radhi, A. K. Hashim and K. S.Jassim; Indian J.phys. 81, 695 (**2007**).