Calculation of the longitudinal electron scattering form factors for the 2s-1d shell nuclei

***A. K. Hamoudi, R. A. Radhi and G. N. Flaiyh**
Department of Physics, College of Science University of Baghdad, Baghdad, Iraq
*e-mail: a.hamoudi@yahoo.com

Abstract

An Expression for the transition charge density is investigated where the deformation in nuclear collective modes is taken into consideration besides the shell model transition density. The inelastic longitudinal $C2$ and $C4$ form factors are calculated using this transition charge density for the ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. In this work, the core polarization transition density is evaluated by adopting the shape of Tassie model togther with the derived form of the ground state two-body charge density distributions (2BCDD's). It is noticed that the core polarization effects which represent the collective modes are essential in obtaining a remarkable agreement between the calculated inelastic longitudinal *F(q)*'s and those of experimental data.

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حساب عوامل التشكل للاستطارة الالكترونية الطولية لنوى القشرة 2s-1d

عادل خلف حمودي ، رعد عبد الكريم راضي و غيث نعمة فليح
قسم الفيزياء – كلية العلوم- جامعة بغداد- بغداد - العراق

الخلاصة

تم دراسة عوامل التشكل للاستطارة الطولية غير المرنة والمتضمنة انتقال كثافة الشحنة عندما نأخذ بنظر الاعتبار التشوه في الأنماط التجميعية النووية الى جانب كثافة الانتقال لأنموذج القشرة. حيث تم حساب عوامل التشكل للاستطارة الطولية $C2$ و $C4$ للنوى ^{20}Ne ^{24}Mg و ^{28}Si . أن تأثيرات استقطاب القلب لكثافة الانتقال حسبت بالاعتماد على شكل أنموذج Tassie الى جانب الصيغة الرياضية المشتقة لتوزيعات كثافة الشحنة النووية ذو صيغة الجسيمين في الحالة الارضية (2BCDD's) . لقد وجد بان تأثير استقطاب القلب الذي يمثل نمط تجميعي يكون جوهريا للحصول على توافق جيد بين حسابات الاستطارة الطولية غير المرنة (*F(q)*'s) و القيم العملية لجميع النوى قيد الدراسة.

Introduction

Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models for transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei [1]. Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. In spite of the success of the 1p-shell model on static properties of nuclei in this region , it fails to describe electron scattering data at high momentum transfer [1,2]. Extending the model space to include the $2\hbar\omega$ configurations improves the agreement with the transverse form factors in the beginning of the p-shell, but towards the end of the p-shell the situation deteriorates[2]. Calculations of form factors using the model space wave function alone is inadequate for reproducing the data of electron scattering [3]. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. These effects can be considered as a polarization of core protons by the valence protons and neutrons. Core polarization effects can be treated either by connecting the ground state to the J-multipole $n\hbar\omega$ giant resonances [3], where the shape of the transition densities for these excitations is given by Tassie model [4-6], or by using a microscopic theory [7,8] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Core polarization effects were incorporated within the p-shell wave function by Sato et al. [9], where the effects greatly improved the agreement with the experimental data. Coulomb form factors of $E4$ transitions in the sd-shell nuclei were discussed taking into account core polarization effects using self-consistent Hartree-Fock plus random phase approximation calculations, which gave a good agreement with experimental form factors [10].

In this study, we have derived an expression for the ground state two - body charge density distributions (2BCDD) of light nuclei, based on the use of the two - body wave functions of the harmonic oscillator and the full two–body correlation functions FC's (which include the tensor correlations (TC's) and short range correlations (SRC's)). This study is aimed to investigate the inelastic longitudinal electron scattering form factors, where the deformation in nuclear collective modes (which represent the core polarization effects) is taken into consideration besides the shell model space transition density. Core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state charge density distribution. Our investigation is devoted on $0^{+}0 \rightarrow 2^{+}0$ and $0^{+}0 \rightarrow 4^{+}0$ transitions in ^{20}Ne, ^{24}Mg, ^{28}Si and ^{34}S nuclei. It is found that the core polarization effects are essential for reproducing a remarkable agreement between the calculated and the observed inelastic longitudinal $C2$ and $C4$ form factors.

Theory

The many particle reduced matrix elements of the longitudinal operator, consists of two parts; one is for the model space and the other is for core polarization matrix element[5,6]:

$$\left\langle f \left\| \hat{T}_J^L(\tau_Z, q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{T}_J^L}(\tau_Z, q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{T}_J^L}(\tau_Z, q) \right\| i \right\rangle \quad (1)$$

The model space matrix element has the form [6]:

$$\left\langle f \left\| \overset{ms}{\hat{T}_J^L}(\tau_Z, q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{ms}{\rho}_{J,\tau_Z}(i, f, r) \quad (2)$$

where $\overset{ms}{\rho}_{J,\tau_z}(i, f, r)$ is the transition charge density of model space given by [3]:

$$\overset{ms}{\rho}_{J,\tau_Z}(i, f, r) = \sum_{jj'}^{ms} OBDM(i, f, J, j, j', \tau_z) \left\langle j \left\| Y_J \right\| j' \right\rangle R_{nl}(r)\, R_{n'l'}(r) \quad (3)$$

where $OBDM$ is the One Body Density Matrix. The core- polarization matrix element is given by[٣]:

$$\left\langle f \left\| \hat{T}_J^{L\,core}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{core}{\rho}_{J,\tau_Z}(i,f,r)$$

(4)

where $\overset{core}{\rho}_{J,\tau_Z}$ is the core-polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J,\tau_Z}(i,f,r) = \overset{ms}{\rho}_{J,\tau_Z}(i,f,r) + \overset{core}{\rho}_{J,\tau_Z}(i,f,r)$$

(5)

where $\overset{core}{\rho}_{J,\tau_Z}$ is assumed to have the form of Tassie shape and given by [٤].

$$\rho_{Jt_z}^{core}(i,f,r) = N\frac{1}{2}(1+\tau_z)\, r^{J-1}\frac{d\rho(i,f,r)}{dr}$$

(6)

where N is a proportionality constant. It is determined by adjusting the reduced transition probability *B(CJ)* and given by [1١]:

$$N = \frac{\int_0^\infty dr\, r^{J+2}\, \overset{ms}{\rho}_{J\tau_Z}(i,f,\mathrm{r}) - \sqrt{(2J_i+1)\,B(CJ)}}{(2J+1)\int_0^\infty dr\, r^{2J}\rho(i,f,\mathrm{r})}$$

(7)

Here, $\rho(i,f,r)$ is the ground state two-body charge density distribution derived as follow; we have produced an effective two-body charge density operator by folding the two-body charge density operator with the two-body correlation functions $\tilde{f}_{ij}$ as[1٢]:

$$\hat{\rho}_{eff}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\tilde{f}_{ij}\left\{\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}-\vec{r}_{ij}\right]+\delta\left[\sqrt{2}\,\vec{r}-\vec{R}_{ij}+\vec{r}_{ij}\right]\right\}\tilde{f}_{ij}$$

(8)

where $\vec{\mathrm{r}}_{ij}$ and $\vec{R}_{ij}$ are relative and center of mass coordinates and the form of $\tilde{f}_{ij}$ is given by [13]:

$$\tilde{f}_{ij} = f(r_{ij})\Delta_1 + f(r_{ij})\{1+\alpha(A)S_{ij}\}\Delta_2$$

(9)

It is clear that eq. (9) contains two types of correlations:

1. The two body short range correlations (SRC) presented in the first term of eq. (9) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body functions with the exception of $^3S_1$ and $^1D_3$ states. It should be noted that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances,where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of $f(r_{ij})$ is given as [13]:

$$f(r_{ij}) = \begin{Bmatrix} 0 & for\ \ r_{ij}\le r_c \\ 1-\exp\left\{-\mu(r_{ij}-r_c)^2\right\} & for\ \ r_{ij} > r_c \end{Bmatrix}$$

(10)

where r_c (in fm) is the radius of a suitable hard core and $\mu = 25\ fm^{-2}$ [13] is a correlation parameter.

2. The two-body tensor correlations (TC) presented in the second term of eq.(9) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto $^1S_3$ and $^1D_3$ states only. S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2} (\vec{\sigma}_i . \vec{r}_{ij}) (\vec{\sigma}_j . \vec{r}_{ij}) - \vec{\sigma}_i . \vec{\sigma}_j \tag{11}$$

The parameter $\alpha(A)$ is the strength of tensor correlations and it is non zero only in the ${}^1S_3 - {}^1D_3$ channels.

The ground state two body charge density distribution $\rho_{ch}(r)$ is given by the expectation value of the effective two-body charge density operator of eq(8) and written as

$$\rho_{ch}(r) = \langle \psi | \hat{\rho}_{eff}^{(2)}(\vec{r}) | \psi \rangle = \sum_{i<j} \langle ij | \hat{\rho}_{eff}^{(2)}(\vec{r}) | [| ij \rangle - | ji \rangle] \quad , \tag{12}$$

where the two particale wave function is given by [14]

$$|ij\rangle = \sum_{JM_J} \sum_{TM_T} \langle j_i m_i \, j_j m_j \mid JM_J \rangle \langle t_i m_{t_i} \, t_j m_{t_j} \mid TM_T \rangle \mid (j_i j_j) JM_J \rangle \mid (t_i t_j) TM_T \rangle \tag{13}$$

Wher J and M_J denote the total angular momentum and it's projection of a pair of particles formed by coupling j_i and j_j while T and M_T denote their total isospin and isospin projection formed by coupling t_i and t_j.

It is important to indicate that our effective two body charge density operator of eq(8) is constructed in terms of relative and centre of mass coordinates, therefore the space-spin part $\mid (j_i j_j) JM_J \rangle$ of the two particale wave function constructed in jj-coupling scheme must be transformed in terms of relative and centre of mass coordinates. This transformation can be achieved as follow:

1. Switching from jj to λS coupling schemes as [15]

$$|(j_i j_j) JM_J\rangle \equiv |(\ell_i \tfrac{1}{2}) j_i, (\ell_j \tfrac{1}{2}) j_j; JM_J\rangle = \sum_{\lambda S} \hat{j}_i \hat{j}_j \hat{\lambda} \hat{S} \begin{Bmatrix} \ell_i & \ell_j & \lambda \\ \tfrac{1}{2} & \tfrac{1}{2} & S \\ j_i & j_j & J \end{Bmatrix} |(\ell_i \ell_j) \lambda (\tfrac{1}{2} \tfrac{1}{2}) S; JM_J\rangle \tag{14}$$

where the notation $\hat{A} = (2A+1)^{1/2}$ and the bracket $\begin{Bmatrix} .... \\ .... \\ .... \end{Bmatrix}$ is the 9-j symbol.

2. We next use the Brody–Moshinsky transformation brackets [15] to transform the spatial part of the two-body wave function $|(\ell_i \ell_j)\lambda\rangle$ in terms of relative and center of mass coordinates.

$$|(\ell_i \ell_j)\lambda\rangle \equiv |n_i \ell_i \, n_j \ell_j ; \lambda\rangle = \sum_{n \ell N L} \langle n\ell, NL; \lambda | n_i \ell_i, n_j \ell_j ; \lambda \rangle | n\ell, NL; \lambda \rangle \tag{15}$$

where the coefficient $\langle n\ell, NL; \lambda | n_i \ell_i, n_j \ell_j ; \lambda \rangle$ is an overlap integral and called a transformation bracket. For the purpose of extending the calculation to open shell nuclei we replace the factors $\hat{j}_i$ and $\hat{j}_j$ in eq.(14) as

$$(2j_i + 1)^{1/2} \Rightarrow \{\eta_{n_i \ell_i j_i} (2j_i + 1)\}^{1/2}$$
$$(2j_j + 1)^{1/2} \Rightarrow \{\eta_{n_j \ell_j j_j} (2j_j + 1)\}^{1/2} \tag{16}$$

where $\eta_{n_i \ell_i j_i}$ and $\eta_{n_j \ell_j j_j}$ are the occupation probabilities of the states $n_i \ell_i j_i$ and $n_j \ell_j j_j$, respectively. These parameters equal to (zero or 1) for closed shell nuclei while for open shell nuclei they are larger than zero or less than one (i.e. $0 < \eta_{n_i \ell_i j_i} < 1$) and ($0 < \eta_{n_j \ell_j j_j} < 1$).

The longitudinal form factor is related to the charge density distribution through the matrix elements of multipole operators $\hat{T}_J^L(q)$[3].

$$\left| F_J^L(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle \right|^2 \left| F_{cm}(q) \right|^2 \left| F_{fs}(q) \right|^2 \qquad (17)$$

where Z is the proton number in the nucleus and $F_{cm}(q)$ is the center of mass correction, which removes the spurious state arising from the motion of the center of mass when shell model wave function is used, and given by [11]:

$$F_{cm}(q) = e^{q^2 b^2 / 4A} \qquad (18)$$

where A is the nuclear mass number and b is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the finite size correction, considered as a free nucleon form factor and assumed to be the same for protons and neutrons, and it takes the form[11]:

$$F_{fs}(q) = e^{-0.43 q^2 / 4} \qquad (19)$$

Results, Discussions and Conclusions

The inelastic longitudinal $C2$ and $C4$ form factors of ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei are presented in figures (1) and (2), respectively. The model space transition density is obtained by eq.(3), where the $(OBDM)$ elements of above nuclei are calculated by OXBASH code [16] using the USDB interaction [17]. The *B(C2)* and *B(C4)* values displayed in table (1) and needed for the calculation of the proportionality constant N are calculated from the microscopic theory. This theory allows a particle-hole excitation from the core orbits and also from the model space orbits, with $2\hbar w$ excitation [7], using the relastic Michigan three Yukawa (M3Y) interaction [18]. So, the constant N is determined theoretically and not as adjustable parameter. For considering the collective modes of the nuclei, the core polarization transition density of eq.(6) is evaluated by adopting the Tassie model [4] together with the calculated ground state 2BCDD of eq.(12). All parameters required in the calculations of 2BCDD's such as the values of the harmonic oscillator spacing parameter $\hbar\omega$, the occupation probabilities η's of the states and the values of $\alpha(A)$ are presented in table (2). The calculated inelastic longitudinal $C2$ and $C4$ form factors for the transitions $J_i^\pi T_i = 0_1^+ 0$ to $J_f^\pi T_f = 2_1^+ 0$ and $4_1^+ 0$ of considered nuclei are displayed in Fig. 1 and Fig. 2, respectively. The dash-dotted curves represent the contribution of the model space where the configuration mixing is taken into account, the dashed curves represent the core polarization contribution where the collective modes are considered and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects. The experimental data are represented by solid circles. Core polarization effects enhance the $C2$ form factors at the first and second maximum and bring the calculated values very close to the experimental data. The locations of the diffraction minimum are slightly displaced in comparison to those of the *sd*-shell model (dashe-dotted lines). For higher q values, $q \geq 2.2\, fm^{-1}$, the core-polarization results are shifted towards lower values of q , bringing the theortical results very close to the experimental data. The modification of the form factors due to core-polarization effects are also reflected in $C4$ form factors. There is a significant improvement in the form factors over the model space results, as shown in Fig. 2. For momentum transfer region $q > 2.\, fm^{-1}$, the calculated form factors with including the core polarization effects are slightly shifted towards the lower values of q, bringing the calculated results of the solid distributions very close to the experimental data. Therefore core polarization effects show a strong q dependence modification to the form factors as seen in the solid distributions of Fig. 2. For ^{32}S, no $C4$ electron scattering data have been available for analyzing. The experimental form factor for ^{32}S doubled of the first 4^+ and second 2^+ states has been analyzed in Ref.[22]. Then energies of these two states are very close (4.29 MeV for the second 2^+ and 4.46 MeV for the first 4^+). We present in Fig. 3 the form factors of these two states, where the solid line represents the sum of the form factors of the two states. The upper panel represents the calculations with *sd*- shell model wave function (without core polarization), while the lower panel represents those, which include core polarization. An excellent overall agreement is obtained with the data. The form factors for q beyond 1.5 fm^{-1} are almost totally predicted by $C4$ excitation.

It is concluded that the core polarization effects, which represent the collective modes, are essential in obtaining a remarkable agreement between the calculated and experimental $C2$ and $C4$ longitudinal form factors of the stable even-even, $N = Z,\ 2s-1d$ shell nuclei .

Table(1):Theortical calculations of the reduced transition probabilities B(C2) (in units $e^2\ fm^4$) and B(C4) (in units of $e^4\ fm^8 \times 10^3$) in comparison with experimental values.

Nucleus	J_i^π	T_i	J_f^π	T_f	$(E_x\ \ MeV)$	*sd+cp*	Exp.[Ref.]
^{20}Ne	0^+ 0^+	0 0	2^+ 4^+	0 0	1.630 4.248	278.3 32.5	292.07 ± 37.72 [19] 38 ± 8 [20]
^{24}Mg	0^+ 0^+	0 0	2^+ 4^+	0 0	1.370 6.01	404.7 36	428.9 ± 8.74 [19] 43 ± 6 [20]
^{28}Si	0^+ 0^+	0 0	2^+ 4^+	0 0	1.780 4.617	415 27.7	327.24 ± 9.47 [19] 27.5 ± 5 [21]
^{32}S	0^+ 0^+	0 0	2^+ 4^+	0 0	2.237 4.459	235 50.7	300.33 ± 11.9 [19] 49.9 [3]

Table(2): Parameters to the ground state 2BCDD's for some open shell nuclei.

Nucleus	$\hbar w$ (MeV)	$\eta_{1S_{1/2}}$	$\eta_{1P_{3/2}}$	$\eta_{1P_{1/2}}$	$\eta_{1d_{5/2}}$	$\eta_{2s_{1/2}}$	$\eta_{1d_{3/2}}$	$\alpha(A)$
^{20}Ne	11.6	1	1	1	0.1917	0.425	–	0.085
^{24}Mg	11.53	1	1	1	0.5	0.5	–	0.084
^{28}Si	11.5	1	1	1	0.783	0.65	–	0.081
^{32}S	10.9	1	1	1	1	0.7	0.15	0.075

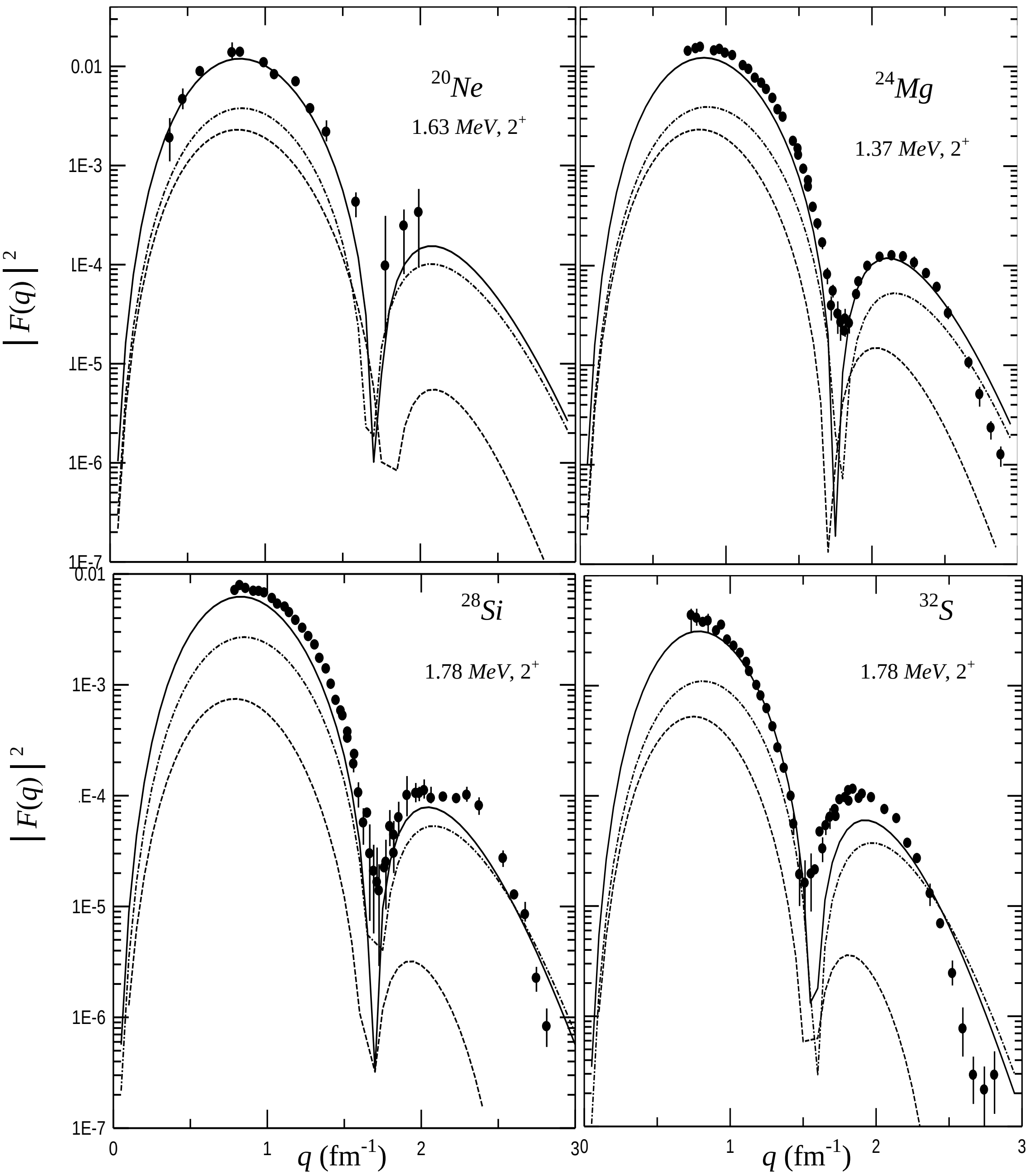

Fig. 1: Inelastic longitudinal $C2$ form factors for the transitions to the 2_1^+ in ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. The dash-dotted curves represent the contribution of the model space, the dashed curves represent the core polarization contribution and the solid curves represent the total form factors obtained by the sum of model space and core polarization contributions. The experimental data, which are represented by solid circles, are taken from Ref. [20] for ^{20}Ne and from Ref. [21] for ^{24}Mg, ^{28}Si and ^{32}S.

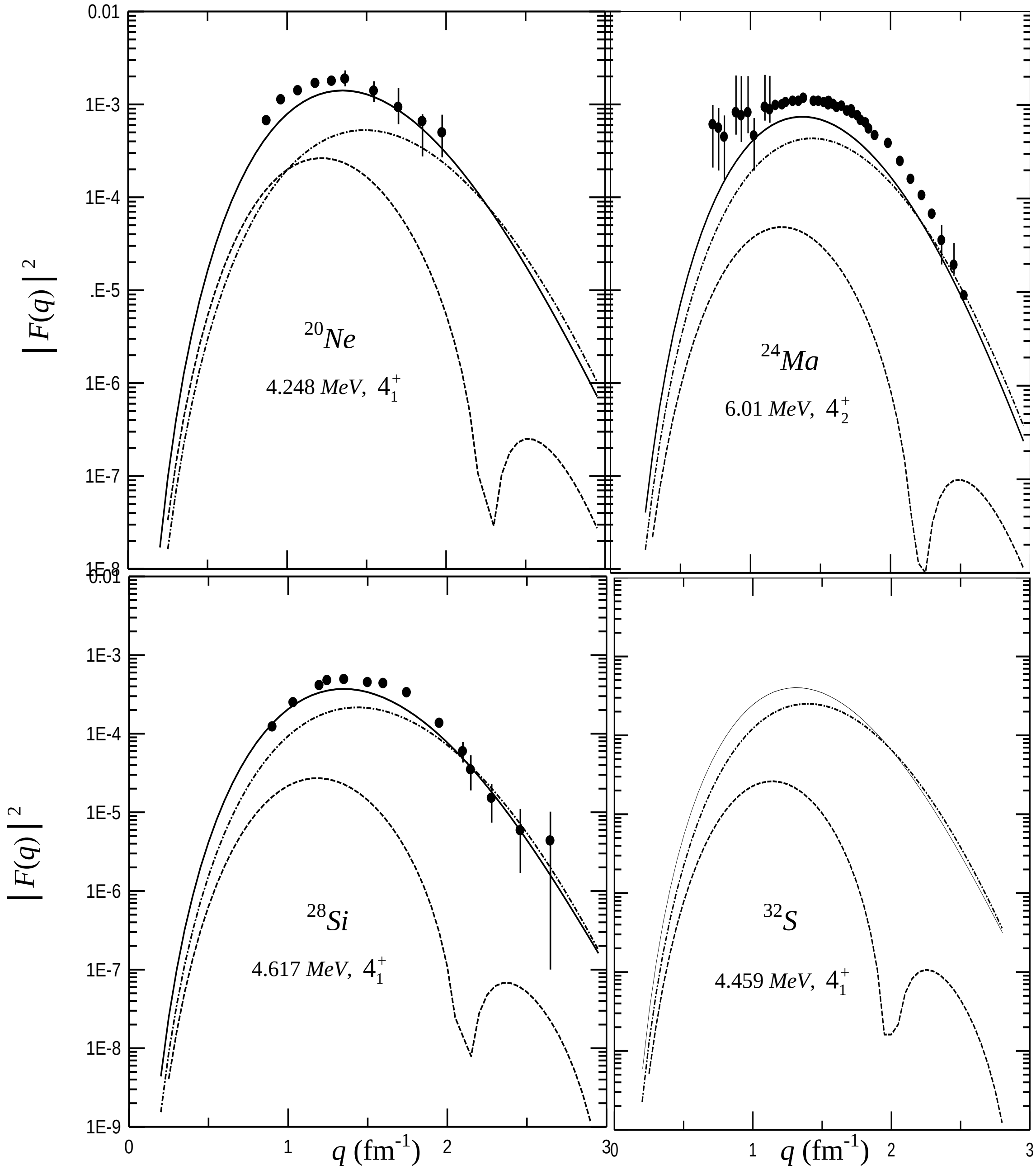

Fig. 2: Inelastic longitudinal $C4$ form factors for the transitions to the 4_1^+ state in ^{20}Ne, ^{28}Si and ^{32}S and , the 4_2^+ state in ^{24}Mg. The dash-dotted curves represent the contribution of the model space, the dashed curves represent the core polarization contribution and the solid curves represent the total form factors obtained by the sum of model space and core polarization contributions. The experimental data, which are represented by solid circles, are taken from Ref. [21].

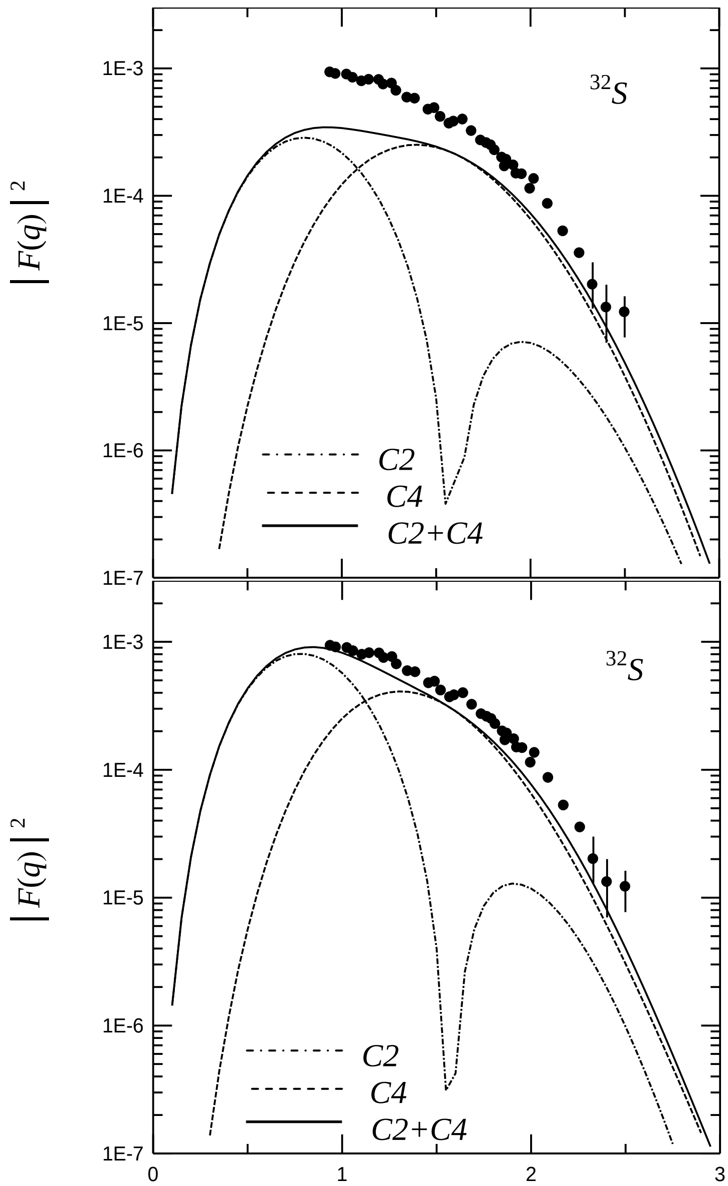

Fig. 3: Inelastic longitudinal form factors for the first 4_1^+ *+ second* 2_2^+ *doublet in* ^{32}S *. The upper panal represents the calculation with sd-shell model only. The lower panel represent the calculation which include core polarization. The experimental data, which are represented by solid circles, are taken from Ref.[22]*

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