The effects of Core-polarization on inelastic form factors of ^{10}B

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Abstract

Inelastic transverse and longitudinal form factors of same parity have been studied for ^{10}B nucleus in the frame work of the shell model for many particles, by using ^{4}He as an inert core and the remaining particles were distributed in $1p_{3/2},1p_{1/2}$ which form the model space. The calculations of the present work based on the harmonic oscillator potential with fixed size parameter (b). Here we use the first order correction for the perturbation theory and the interaction from Cohen-Kurath (CK). Adding the core-polarization effects to form factors calculations gave a good agreement with the experimental data. Calculations have been performed for the transverse excited states of: $(1^+,0)at(E=0.178MeV)$,. $(2^+,0)at(E=3.587MeV)$, $(3^+,0)at(E=4.774MeV)$, and longitudinal $(2^+,1)at(E=5.164MeV)$.

Key words

p-shell nuclei, Electron scattering form factor, first-order core-polarization effects.

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تأثير إستقطاب القلب على عوامل التشكل غير المرنة ^{10}B

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الخلاصة

لقد تمت دراسة عوامل التشكل غير المرنة المستعرضة والطولية والتي حُسبت للحالات متشابهة التناظر للبورون ^{10}B في إطار أنموذج القشرة للجسيمات المتعددة، حيث تم فرض القلب ^{4}He وباقي الجسيمات موزعة في المدارات $1p_{3/2}$ و $1p_{1/2}$ والتي تشكل فضاء الأنموذج.كما تم انجاز الحسابات في العمل الحالي اعتماد الدالة القطرية للمتذبذب التوافقي البسيط.وَ إختيار معامل الحجم (b; size parameter) بقيمة ثابتة. حيث تم تحليل عوامل التشكل في إطار استخدام التصحيح الاول لنظرية الاضطراب وبأعتماد تفاعل كوهين-كوراث(CK). وتم اضافة تأثير إستقطاب القلب الى الحسابات مما جعل النتائج المستحصلة اكثر توافقا مع النتائج العملية.كما اعتمدت كل من الإنتقالات المستعرضة للحالات المتهيجة التالية؛(1^+،0) للطاقة ($0.178MeV$)، (2^+،0) للطاقة ($3.587MeV$) و(3^+،0) للطاقة. هذا اضافة الى استخدام الإنتقال الطولي للحالة المتهيجة(2^+،1) للطاقة($5.164MeV$) .

Introduction

The framework of the nuclear shell model provides the main theoretical tool for understanding all properties of nuclei. It can be used in its simplest single particle form to provide qualitative understanding, but it also be used as a basis for much more complex and complete calculations. There appears to be limit within the near future to the expansion of its application [1]. Shell model within a restricted model space succeeded in describing static properties of nuclei. For p-shell nuclei, Cohen-Kurath [2] model explains well the low-energy properties of p-shell nuclei. However, at higher-momentum transfer, it fails to describe the form factors. Radhi [3, 4, 5, 6] have successfully proved that the inclusion of core polarization effects in the p-shell and sd-shell are very essential to improve the calculations of the form factors. The Coulomb form factors have been discussed for the stable sd-shell nuclei using sd-shell wave functions with phenomenological effective charges [7]. Comparisons between calculated and measured longitudinal and transverse electron scattering form factors have long been used as tests for models of nuclear structure [8, 9].

Restricted 1p-shell models were found to provide good predictions for the $({}^{10}B)$ natural parity level spectrum and transverse form factors [10]. However, in the same reference they were less successful for C2 form factors and give just 45% of the total observed C2 transition strength. Expand the shell model space to include 2ħω configurations in describing the form factors of ${}^{10}B$. Cichocki et al. [10] have found that only a 10% improvement was realized.

The aim of the present work is to study the form factors for ${}^{10}B$ by including higher-energy configurations as a first-order core polarization through a microscopic theory which combines shell model wave functions and highly excited states. Single-particle wave functions are used as a zero-th contribution and the effect of core polarization is included as a first-order perturbation theory with the modified surface delta interaction (MSDI) [11] as a residual interaction and a 2ħω for the energy denominator. The single-particle wave functions are those of the harmonic-oscillator (HO) potential with fixed size parameter (b).

Theory

1. General Theory

In the plane-wave Born approximation (PWBA), the differential cross-section for the scattering of an electron from a nucleus of charge (Ze) and mass (M) into a solid angle (dΩ) is given by [12]

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{rec} \sum_J \left|F_J(q,\theta)\right|^2 \qquad (1)$$

where

$$F_J^2(q,\theta) = \left(\frac{q_\mu}{q}\right)^4 \left|F_J^L(q)\right|^2 + \left[\frac{q_\mu^2}{2q^2} + \tan^2(\theta/2)\right] \left|F_J^T(q)\right|^2 \quad (2)$$

and

$$\left|F_J^T(q)\right|^2 = \left|F_J^{El}(q)\right|^2 + \left|F_J^{mag}(q)\right|^2 \qquad (3)$$

The form factor of multi polarity (J) as a function of momentum transfer is written in terms of the reduced matrix elements of the transition operator as:

$$\left|F_J^\eta(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left|\left\langle J_f \left\| T_J^\eta(q) \right\| J_i \right\rangle\right|^2 \quad (4)$$

The nuclear states have a well-defined isospin. So using the Wigner-Eckart theorem in isospin space, the form factor can be written in terms of the matrix element reduced both in total angular momentum (spin) (J) and isospin (T) (triple-bar matrix elements). Also, in the realistic calculation of the form factor, it is necessary to take into account the effects of finite size, center of mass motion and Coulomb distortion of electron waves. Thus, the form factor for a given multipolarity (J) can be written in terms of the matrix elements reduced both in spin and isospin spaces as:

$$\left|F_J^\eta(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left|\sum_{T=0,1} (-1)^{T_f - T_{zf}} \begin{pmatrix} T_f & T & T_i \\ -T_{Zf} & M_T & T_{Zi} \end{pmatrix} \left\langle \Gamma_f \left\| T_{J,T}^\eta(q) \right\| \Gamma_i \right\rangle\right|^2 \times \left|F_{c.m}(q)\right|^2 \times \left|F_{f.s}(q)\right|^2 \quad (5)$$

The multipolarity (J) in the last equation is restericted by angular momentum selection rule:

$\left|J_i - J_f\right| \le J \le J_i + J_f$ or $\Delta(J_i, J_f, J)$

and

$$\left|T_i - T_f\right| \le T \le T_i + T_f \qquad (6)$$

The parity selection rules:

$$\Delta\pi^{El} = (-1)^J \qquad (7)$$
$$\Delta\pi^{Mag} = (-1)^{J+1}$$

2. Core-Polarization Effects (CP)

The core polarization effect on the form factor (the effects from outside the 1p-shell model space) is based on a microscopic theory that combines shell-model wave functions and configurations with higher energy as first order perturbations.

The reduced matrix elements of the electron scattering operator consist of two parts, one is the "Model space" matrix elements and the other is the "Core-polarization" matrix elements

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP} \quad (8)$$

$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS}$ is the model-space matrix elements

$\left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP}$ is the core-polarization matrix elements

$\left|\Gamma_i\right\rangle$ and $\left|\Gamma_f\right\rangle$ are described by the model-space wave functions.

The model-space matrix elements are expressed as the sum of the product of the one-body density matrix elements (OBDM) times the single-particle matrix elements which are given by:

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} = \sum_{\alpha,\beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{MS} \qquad (9)$$

The core-polarization matrix element can be written as follows

$$\left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{cp} = \sum_{\alpha,\beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \left\langle \alpha \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{cp} \qquad (10)$$

According to the first – order perturbation theory, the reduced single – particle matrix element of the one body operator is expressed as the sum of three terms, a model space matrix element and two core – polarization matrix elements

$$(\alpha \| \hat{T}^{\eta}_{\Lambda} \| \beta) = \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{MS} + \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \frac{Q}{E-H^{(0)}} V_{res} \right\| \beta \right\rangle_{CP} + \left\langle \alpha \left\| V_{res} \frac{Q}{E-H^{(0)}} \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{CP} \qquad (11)$$

The first term is the zero- order contribution. The second and third terms are the first–order contribution which give the higher–energy configuration.

Results and Discussion

In the present work, the shell-model technique was used to deal with form factors which have been measured experimentally, from the electron scattering for the momentum transfer range $0 \le q \le 4.0\ fm^{-1}$. The interaction between the electron and the nucleus is treated by first-order perturbation theory; this is thought to be a good approximation because the estimate of the higher-order (especially the second order) interaction is small compared with the first order effect [11-13]. So, we use the plane-wave solutions for the incident and scattered electrons. If the distortion of the electron wave function by the Coulomb field of the nucleus is quite small, relativistic effects of the recoiling nucleus or nucleons will be ignored; this is a good approximation as long as the electron energy is small compared with the rest energy of the nucleon [12].

The core polarization effects are calculated with the MSDI as a residual interaction. The parameters of the MSDI are denoted by A_T, B and C [12], where T indicates the isospin and takes values 0 or 1. These parameters are taken to be $A_0 = A_1 = B = 25/A$ and C = 0, where A is the mass number. In Figs.1, 3 and 5, the dashed, dashed-dotted and dotted lines are the multi-polarities of the transition whereas the solid line is the total form factors. While the dashed lines in Figs. 2, 4, 6, and 7 are the results obtained using the 1p-shell wave functions based on Cohen-Kurath interaction (CK) [2]. The results of CP effects are shown as the crossed symbols while the solid line gives the total contribution of both the model space and core-polarization effects (1p+CP). The size parameter (b) is taken to be 1.611 fm [14] to get the single-particle wave functions of the harmonic-oscillator potential. The ground state of ^{10}B is $(J^{\pi}, T = 3^{+}, 0)$.

1.Inelastic transverse form factors for ^{10}B to $(1^+,0)$ state of (E = 0.718MeV)

The calculations for the E2 and M3 isoscalar transition from the ground state $((J^{\pi}, T = 3^{+}, 0))$ to the exited state $((J^{\pi}, T = 1^{+}, 0))$ at Ex = 0.718MeV are shown in Fig.1. the multi-pole decomposition is displayed as indicated by E2 and M3. The total form factor is shown by the solid curve, where the data are reasonably described in all the momentum transfer regions between $(1 \le q \le 2\ fm^{-1})$.

The CP effects enhance the form factor appreciably over the 1p-shell calculation. As shown in Fig.2, this enhancement brings the total form factor (solid curve) very close to the experimental data. Similar results are obtained in Ref.[10].

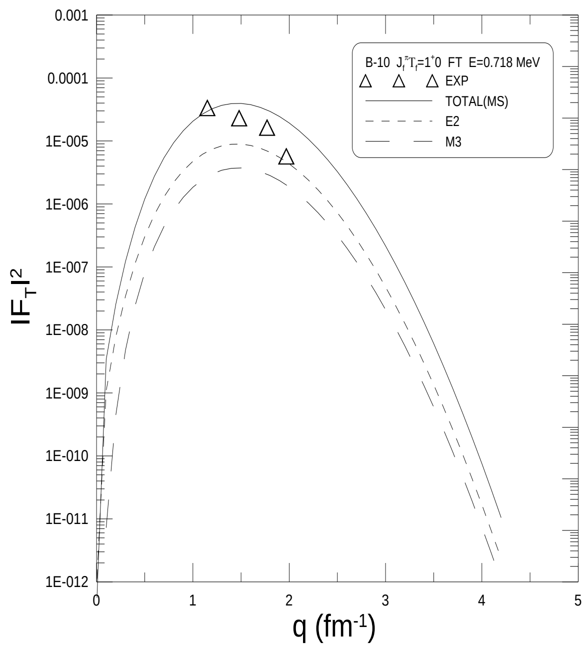

Fig.1:** The inelastic transverse (E2+M3)form factors for the isoscalar* $(J^{\pi}, T = 1^{+}, 0)$ ***(E=0.718 MeV) transition in** ^{10}B **compared with the experimental data which are taken from Ref. [10].

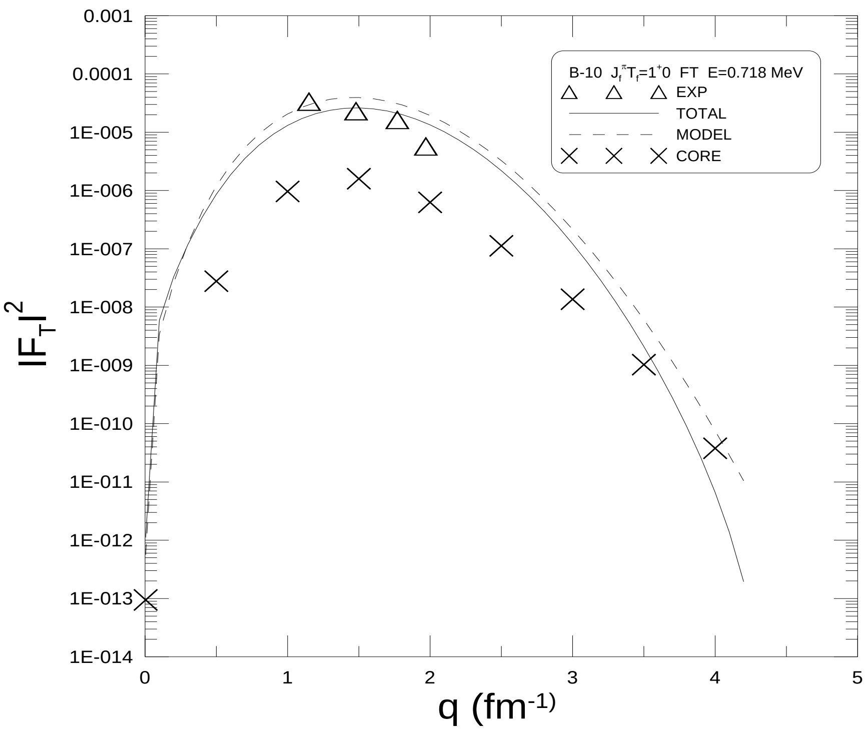

Fig. 2: Core polarization effect on the $(J^{\pi},T=1^{+},0)$ ***(E=0.718 MeV) transition in*** ^{10}B **.**

2.Inelastic transverse form factors for ^{10}B **to (2⁺,0) state of (E = 3.587MeV)**

Fig.3 displays calculations of the total form factors as a solid line which consists of the (M1+E2+M3) multi-polarities without CP effects. The 1p-shell model (the dashed line) calculation underestimates the experimental data and the inclusion of the CP (the cross-symbols)in Fig. 4 enhances the calculations and brings the total form factors(1p+cp)as a solid line to the experimental values especially in the momentum transfer region lies between $(1.4 \le q \le 2\ fm^{-1})$.

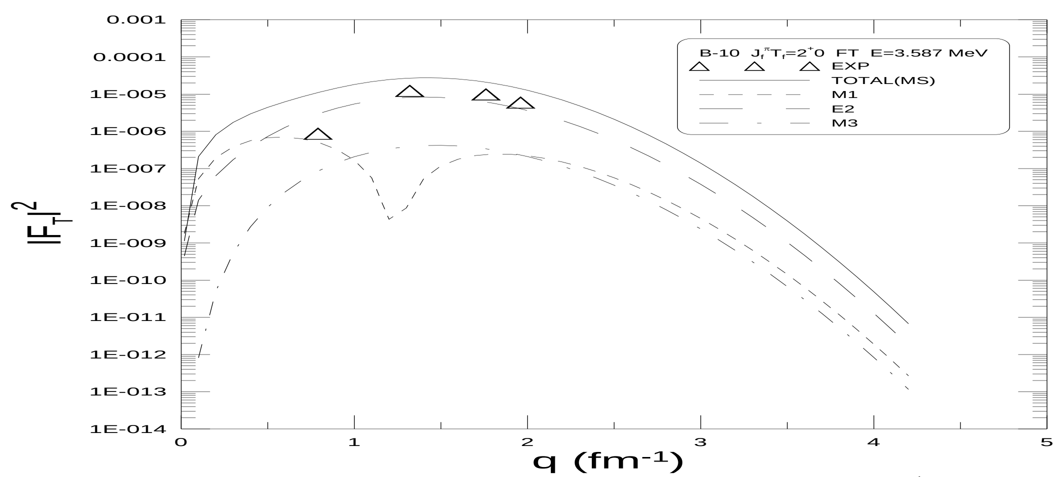

Fig.3: The inelastic transverse (M1+E2+M3) form factors for the isoscalar$(2^+,0)$ (E=3.587MeV) transition in ^{10}B ***compared with the experimental data which are taken from Ref. [10]***

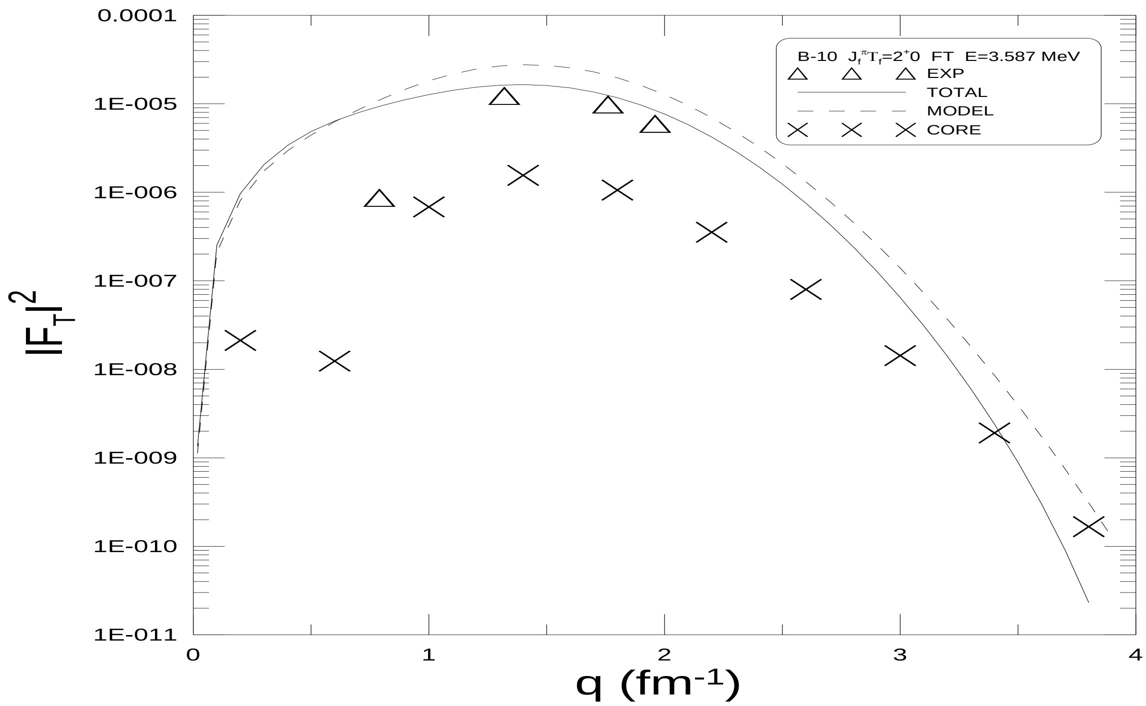

Fig. 4: The core polarization effect on the (2^+,0) (E=3.587MeV) transition in ^{10}B .

3.Inelastic transverse form factors for ^{10}B to (3^+,0) state of (E=40774MeV)

The transverse form factor for the (3^+,0) at Ex=4.774MeV is shown in Fig.5, the 1p-shell model calculations without CP effects describes the experimental data very well at the momentum transfer (1.0≤ q ≤ 1.6 fm^{-1})and beyond this limit the calculations start to deviate from the experimental data. The inclusion of the CP effect overestimates the total form factors (1p+cp) up to (q ~2.0 fm^{-1}) as shown in the Fig.6.

4.Inelastic longitudinal form factors for ^{10}B to (2^+,1) state of (E = 5.164MeV)

Fig.7 shows the comparison of the isovector longitudinal (C2) form factors from the ground state (3^+,0) to the excited state (2^+,1) at Ex = 5.164MeV. The 1p-shell model calculations under estimates slightly the experimental data at the region of momentum transfer 0.4≤ q ≤ 1.0 fm^{-1}, where the other regions of q have no experimental data, and the inclusion of the CP effects make the calculations more worse and bring it lower than 1p calculation.

Conclusion

In Figs. 2 and 4 the inclusion of CP effects enhances the 1p-shell model calculations and consequently gives a remarkably good agreement with the experimental data. In Fig.6 the CP effects does not clearly enhance the 1p-shell model results. In Fig. 7, the inclusion of the CP effects brings the result lower than the 1p calculation and making the result far away from the experimental data. All calculations presented in this work have been performed by employing the MSDI as residual interaction and core polarization as a first order correction on the form factor. Therefore, using of modern effective interaction may give a better description for the form factors of this nucleus.

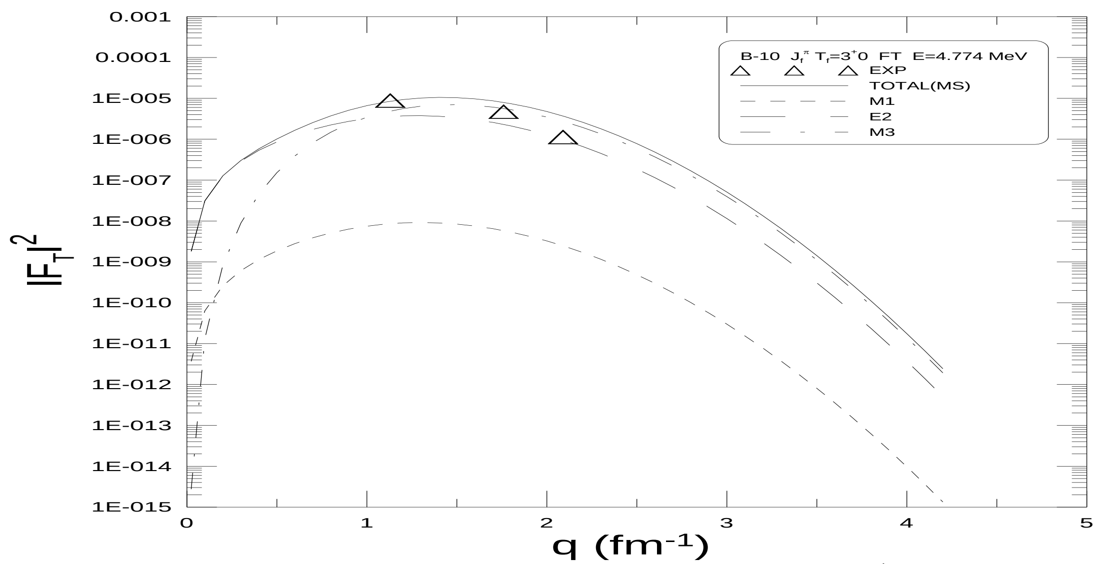

Fig.5: The inelastic transverse M1+E2+M3 form factor for the isoscalar(3^+,0) (E=4.774 MeV) transition in ^{10}B compared with the experimental data which are taken from Ref. [10].

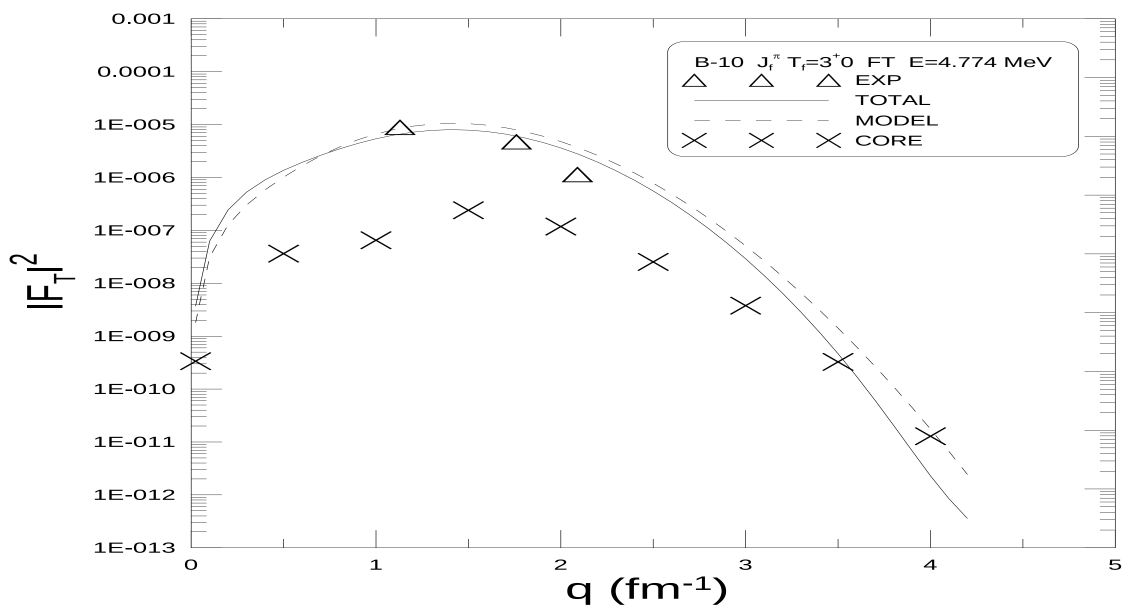

Fig.6: The core polarization effect on the(3^+,0) (E=4.774 MeV) transition in ^{10}B .

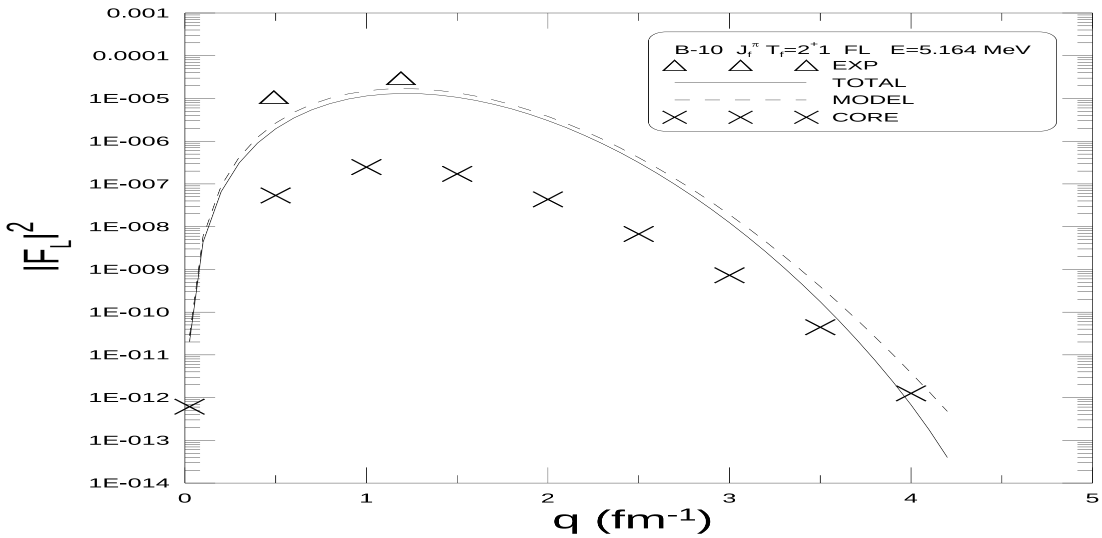

Fig.7: The core polarization effect on the (2^+,1) (E=5.164MeV) transition in ^{10}B compared with the experimental data which are taken from Ref. [10]

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