Elastic electron scattering from some 2*s*-1*d* shell nuclei

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Abstract

The charge density distributions (CDD) and the elastic electron scattering form factors F(q) of the ground state for some even mass nuclei in the $2s-1d$ shell (^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S) nuclei have been calculated based on the use of occupation numbers of the states and the single particle wave functions of the harmonic oscillator potential with size parameters chosen to reproduce the observed root mean square charge radii for all considered nuclei. It is found that introducing additional parameters, namely α, α_1 and α_2, which reflect the difference of the occupation numbers of the states from the prediction of the simple shell model leads to a remarkable agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r. The experimental electron scattering form factors for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei are in reasonable agreement with the present calculations throughout all values of momentum transfer q.

Key words
charge density distributions, elastic electron scattering form factors, Even- A $2s-1d$ shell nuclei, root mean square charge radii, occupation numbers of higher states for [^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei].

Article info
Received: Jun. 2012
Accepted: Dec. 2012
Published: Dec. 2012

الاستطاره الالكترونيه المرنه من بعض نوى القشرة النووية 2*s*-1*d*

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الخلاصة

حسب كل من توزيع كثافة الشحنة النووية و عوامل التشكل لاستطارة الالكترون المرنه للحاله الارضيه للنوى الزوجيه (^{20}Ne, ^{24}Mg, ^{28}Si و ^{32}S) الواقعة ضمن القشرة النووية $2s-1d$ بالاعتماد على كل من اعداد الأشغال للحالات النووية والدوال الموجية لجهد المتذبذب التوافقي ذي أعلومات حجمية اختيرت لكي تعيد انتاج مربع متوسط انصاف اقطارالشحنة لجميع النوى تحت الدراسة. لقد وجد ان ادخال الأعلومات الأضافية (α و α_1 و α_2)، والتي تعكس الفرق بين اعداد الأشغال للحالات النووية قيد الدراسة وبين تلك التي يتنبأ بها أنموذج القشرة البسيط ، يؤدي الى توافق ممتاز بين النتائج المحسوبة والنتائج العملية لتوزيع كثافة الشحنة النووية ولكل قيم المسافة r . أظهرت هذه الدراسة بأن عوامل التشكل لاستطارة الالكترون المرنه النظرية تتفق مع النتائج العملية للنوى (^{20}Ne, ^{24}Mg, ^{28}Si و ^{32}S).

Introduction

The charge density distributions (CDD) and form factors are some of the most important quantities in nuclear structure which were well studied experimentally over a wide range of nuclei. This interest in the CDD is related to the basic bulk nuclear characteristics such as the shape and size of nuclei, their binding energies, and other quantities which are connected with the CDD. Besides, the density distribution is an important object for experimental and theoretical investigations since it plays the role of a fundamental variable in nuclear theory. The CDD can be determined experimentally from the scattering of high-energy electrons by the nucleus. By measuring the elastic cross sections one obtains information about the distribution of the charges within the nucleus. Various theoretical methods [1, 2] are used for calculations of CDD, among them the theory of finite Fermi system, the Hartree-Fock method with skyrme effective interaction. A phenomenological method was applied that is based on the natural orbital representation to construct the ground state one body density matrix [1]. This method describes correctly both density and momentum distributions in closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca. Here, the parameters of the matrix are fixed by a best fit to the experimental density distribution and to the correlated nucleon momentum distribution. An analytical expression was derived for the one and two body terms in the cluster expansion of CDD and elastic form factors of $1s-1p$ and $2s-1d$ shell nuclei [3, 4]. This expression was used for the systematic study of the effect of the short-range correlations on the CDD and F(q). Their study depends on the harmonic oscillator size parameter (b) and the correlation parameter β, where these parameters were determined by fitting the theoretical charge form factors to the experimental ones. Many particles shell model wave functions were used to calculate the form factors of the even-even nuclei with mass number $A=20-36$ that have the positive parity states [5]. Many theoretical works (with various assumptions) concerning the calculations of CDD, elastic form factors and nuclear momentum distribution have been carried out [6-10] for even-A and odd-A of $2s-1d$ shell nuclei. The $CDDs$ of $2s-1d$ shell nuclei [11] was calculated with the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to $2-\alpha$ and $Z-10+\alpha$, respectively. Here, α represents the deviation of the shell charges from the prediction of the simple shell model and Z is the proton number (total charge of the nucleus). In general, the calculated CDD were in good agreement with those of experimental data for all considered $2s-1d$ shell nuclei.

In the present work, the method of Gul'karov et al. [11] is followed but with including some higher shells into consideration with the aim of driving an analytical expressions for the CDD and F(q), based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states, applicable through out the whole region of $2s-1d$ shell nuclei. The derived expressions are then employed for determination of CDD and the elastic form factors for even-A of $2s-1d$ shell nuclei ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S. The calculated CDD and the elastic form factors demonstrate a remarkable agreement with those of experimental results.

Theory

In the simple shell model, the CDD is evaluated in terms of the radial part of the

harmonic oscillator wave functions $R_{nl}(r)$ as

$$\rho(r)=\frac{1}{4\pi}\sum_{nl}2(2l+1)\left|R_{nl}(r)\right|^2 \qquad (1)$$

where the form of $R_{nl}(r)$ is very well known [11, 12] while n and l are the single particle principal and orbital angular momentum quantum numbers, respectively. In the simple shell model, the $2s-1d$ shell nuclei are considered as an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to 2 and $Z-10$, respectively. According to the assumption of the simple shell model of Eq.1, an analytical expression for the *CDD* of $2s-1d$ shell nuclei is obtained as [11]:

$$\rho(r)=\frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{5+\left(\frac{4Z}{15}-\frac{4}{3}\right)\left(\frac{r}{b}\right)^4\right\} \qquad (2)$$

and the corresponding mean square charge radii (*MSR*) can be determined by [13]:

$$\langle r^2\rangle=\frac{4\pi}{Z}\int_0^\infty\rho(r)r^4dr \qquad (3)$$

where the normalization condition of the *CDD* is given by [2]:

$$Z=4\pi\int_0^\infty\rho(r)r^2dr \qquad (4)$$

Introducing Eq.2 into Eq. 3 and integrating, the *MSR* of $2s-1d$ shell nuclei is obtained as:

$$\langle r^2\rangle=b^2\left(\frac{7}{2}-\frac{10}{Z}\right) \qquad (5)$$

In this study, the higher shells are included in the calculations of *CDD* by using the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in the shells $2s, 1d$ and $2p$ are equal to, respectively, $2-\alpha$, $Z-10+\alpha_1$ and α_2 and not to 2, $Z-10$ and 0 as in the simple shell model of Eq. 2. The parameters α, α_1 and α_2 (with $\alpha=\alpha_1+\alpha_2$) represent the deviation of the shell charges $2s, 1d$ and $2p$, respectively, from the prediction of the simple shell model. Using this assumption with the help of Eq.1, an analytical expression of the *CDD* for the $2s-1d$ shell nuclei is obtained as:

$$\rho(r)=\frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{\begin{array}{l}5-\frac{3\alpha}{2}+\left[\frac{11\alpha}{3}-\frac{5\alpha_1}{3}\right]\left(\frac{r}{b}\right)^2+\\ \left(\frac{4Z}{15}-\frac{4}{3}-2\alpha+\frac{8\alpha_1}{5}\right)\left(\frac{r}{b}\right)^4+\frac{4\alpha_2}{15}\left(\frac{r}{b}\right)^6\end{array}\right\} \qquad (6)$$

and the corresponding *MSR* is

$$\langle r^2\rangle=b^2\left(\frac{7}{2}-\frac{10}{Z}+\frac{\alpha_2}{Z}\right) \qquad (7)$$

The central *CDD*, i.e. at $r=0$, is determined from Eq. 6 as:

$$\rho(0)=\frac{1}{\pi^{3/2}b^3}\left\{5-\frac{3\alpha}{3}\right\} \qquad (8)$$

The elastic electron scattering form factors from spin zero nuclei is determined by the ground state CDD and defined as [14]:

$$F(q)=\frac{4\pi}{Z}\int_0^\infty j_0\rho(r)r^2dr \qquad (9)$$

where

$$j_0(qr)=\frac{\sin(qr)}{qr} \qquad (10)$$

is the zeroth order spherical Bessel function, q is the momentum transfer from the incident electron to the target nucleus and the $\rho(r)$ is the CDD of the ground state. In the limit $q\to 0$, the target will be represented as a point particle, and from eq.(9) with the help of Eq. 4, the form factor of this target is equal to unity, i.e. F(q = 0) =1.

An analytical expression for elastic electron scattering form factor F(q) can be obtained by introducing the form of the CDD of Eq. 6 into Eq.9, and performing the integration i.e.

$$F(q)=\frac{e^{-q^2b^2/4}}{Z}[Z+(\frac{5-Z}{3}-\frac{\alpha_2}{6})q^2b^2+$$
$$(\frac{3\alpha_2-\alpha}{40}+\frac{Z-5}{60})q^4b^4-\frac{\alpha_2}{240}q^6b^6] \qquad (11)$$

The form factor given in Eq.11 must be corrected to convert it into a representation appropriate for comparison with the experimental form factors. Therefore, Eq. 11 must be multiplied by the correction due to the finite nucleon size $F_{fs}(q)$ and the center of mass correction $F_{cm}(q)$ [14]:

$$F_{fs}(q) = \exp[-\frac{0.43q^2}{4}] \qquad (12)$$

$$F_{cm}(q) = \exp[\frac{q^2b^2}{4A}] \qquad (13)$$

where A is the mass number of the nucleus. In this study, we compare the calculated *CDD* of considered even-A nuclei with those of three parameter Fermi model $3PF$ [13], which are extracted from the analysis of elastic electron-nuclei scattering experiments, and are given by [13]

$$\rho_{3PF}(r) = \rho_0\left(1+\frac{wr^2}{c^2}\right)\Big/\left(1+e^{\frac{r-c}{z}}\right) \qquad (14)$$

where the constant ρ_0 is obtained from the normalization condition of the charge density distribution of Eq.4.

Results and Discussion

Our analytical expression of Eq.6 has been used for the study of *CDD* for the odd-A of $2s-1d$ shell nuclei ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. In Eq.6, the harmonic oscillator size parameter b is chosen in such a way to reproduce the experimental root mean square charge radii $\langle r^2\rangle^{1/2}_{\exp}$ of the considered nuclei, the parameter α is determined by introducing the experimental $\rho_{\exp}(r=0)$ into Eq.8, the parameter α_2 is determined by introducing the experimental *MSR* into Eq.7 and the parameter α_1 is determined from the relation $\alpha_1 = \alpha - \alpha_2$. It is important to remark that when $\alpha = \alpha_1 = \alpha_2 = 0$, Eqs.6 and 7 coincide with those of eqs. (2) and (5), respectively. The calculated *CDDs* of considered even-A nuclei are compared with those of $3PF$ [13]. In Table (1), we present the parameters ω, c and z required by the experimental *CDD* of $3PF$ together with their root mean square charge radii $\langle r^2\rangle^{1/2}_{\exp}$ and central charge densities $\rho_{\exp}(r=0)$ for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. Table (2) displays all parameters needed for calculating $\rho(r)$ of Eq.6, such as the harmonic oscillator size parameter b and the calculated parameters of α, α_1 and α_2 for considered nuclei. Table (3) demonstrates the calculated occupation numbers for $2s, 1d$ and $2p$ shells and the calculated root mean square charge radii $\langle r^2\rangle^{1/2}_{cal}$ obtained by using Eqs.5 and 7.

The dependence of the *CDD* (in units of fm^{-3}) on r (in unit of fm) is shown in Fig. 1 for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. The solid circles (●) are the experimental data of $3PF$ [13]. The dashed symbol distributions are the calculated *CDD* obtained either from Eq.2 or from eq.6 using the values $\alpha = \alpha_1 = \alpha_2 = 0$. The solid distributions are the calculated *CDD* when the higher shells are included in the calculations and obtained by Eq. 6 using the values of α, α_1 and α_2 given in Table (2). It is obvious that the form of the *CDD* represented by Eq.2 or 6 behaves as an exponentially decreasing function as seen by the plus symbol distributions or solid distributions for all considered nuclei of Fig. 1. This figure shows that the probability of finding a proton near the central region ($0 \le r \le 2$ *fm*) of $\rho_{ch}(r)$ is larger than the tail region ($r > 2$ *fm*). presented in Table (2), into Eq. 6 leads to reducing significantly the central region of $\rho_{ch}(r)$ and increasing slightly the tail region of $\rho_{ch}(r)$ as seen by the solid distributions. This means that the

effect of inclusion of higher shells in our calculations tends to increase the probability of transferring the protons from the central region of the nucleus towards its surface and tends to increase the root mean square charge radius $< r^2 >^{1/2}$ of the nucleus (see Table (3)) and then makes the nucleus to be less rigid than the case when there is no this effect. Fig. 1 also illustrates that the dashed symbol distributions in all considered nuclei are not in good accordance with those of $3PF$, especially at the central region of $\rho_{ch}(r)$, but once the higher shells are considered in
the calculations due to the introduction the calculated values of α, α_1 and α_2 given in Table (2) into Eq. 6, the results for the CDD become in astonishing accordance with those of $3PF$ throughout the whole values of r.

In Fig.(2), we present our results for elastic form factors as a function of the momentum-transfer q for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei, respectively. The circles in this figure are the experimental results of Ref.[15] , the solid and dashed curves are our calculated results. The experimental data of ^{24}Mg are very well described by the calculated ones up to q = 3 fm^{-1} whereas the remainder nuclei ^{20}Ne, ^{28}Si and ^{32}S exhibit a remarkable agreement between the calculated result and experimental ones.

***Table (1): Values of various parameters required by the* CDD *of* 3PF**

Nucleus	Type of CDD [13]	ω	c	z	$\rho_{\exp}(0)$ [13] (fm^{-3})	$< r^2 >_{\exp}^{1/2}$ [13] (fm)
^{20}Ne	$3PF$	-0.168	2.791	0.698	0.08807012	2.992
^{24}Mg	$3PF$	-0.163	3.108	0.607	0.08433872	3.075
^{28}Si	$3PF$	-0.233	3.340	0.580	0.09019782	3.086
^{32}S	$3PF$	0.160	2.540	2.191	0.08946637	3.239

***Table (2): Calculated parameters used in eq. (6) for the calculations of the* CDD**

Nucleus	Z	b	α	α_1	α_2
^{20}Ne	10	1.885	1.14922	0.9014770	0.2477427
^{24}Mg	12	1.880	1.251738	1.1479850	0.1037537
^{28}Si	14	1.846	1.225740	1.1005480	0.1251923
^{32}S	16	1.901	1.050355	0.6012179	0.4491367

Table (3): Calculated occupation numbers of* 2s, 1d *and* 2p *shells and the calculated $\langle \mathbf{r}^2 \rangle_{\mathbf{cal}}^{1/2}$

Nucleus	Z	Occupation No. of $2s$	Occupation No. of $1d$	Occupation No. of $2p$	$\langle r^2 \rangle_{cal}^{1/2}$ obtained eq.(5)	$\langle r^2 \rangle_{cal}^{1/2}$ obtained eq.(7)
^{20}Ne	10	0.850780	0.901477	0.247742	2.97728	2.99200
^{24}Mg	12	0.748262	3.147985	0.103753	3.07003	3.07500
^{28}Si	14	0.774260	5.100548	0.125192	3.08106	3.08600
^{32}S	16	0.949645	6.601217	0.449136	3.22330	3.23900

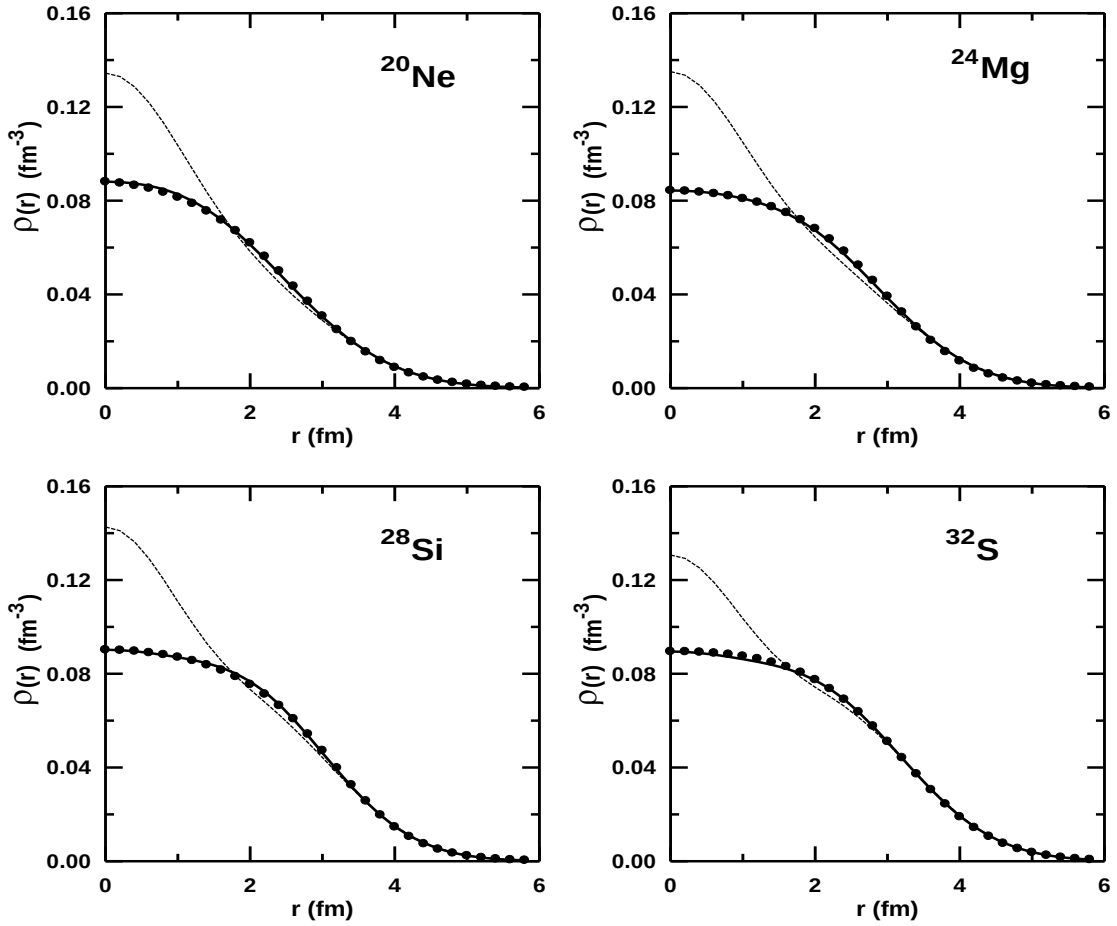

Fig.1: The dependence of the charge density distributions $\rho_{ch}(r)$ (in fm^{-3}) on r (in fm) for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. The solid circles (●) are the experimental data [13] of $2PF$ (for ^{20}Ne and ^{28}Si nuclei) and $3PF$ (for ^{24}Mg and ^{32}S nuclei). The dashed curves are the calculated CDD obtained either by eq. (2) or by eq. (6) using the values $\alpha = \alpha_1 = \alpha_2 = 0$. The solid distributions are the calculated CDD obtained by eq. (6) using the values of α, α_1 and α_2 given in Table (2).

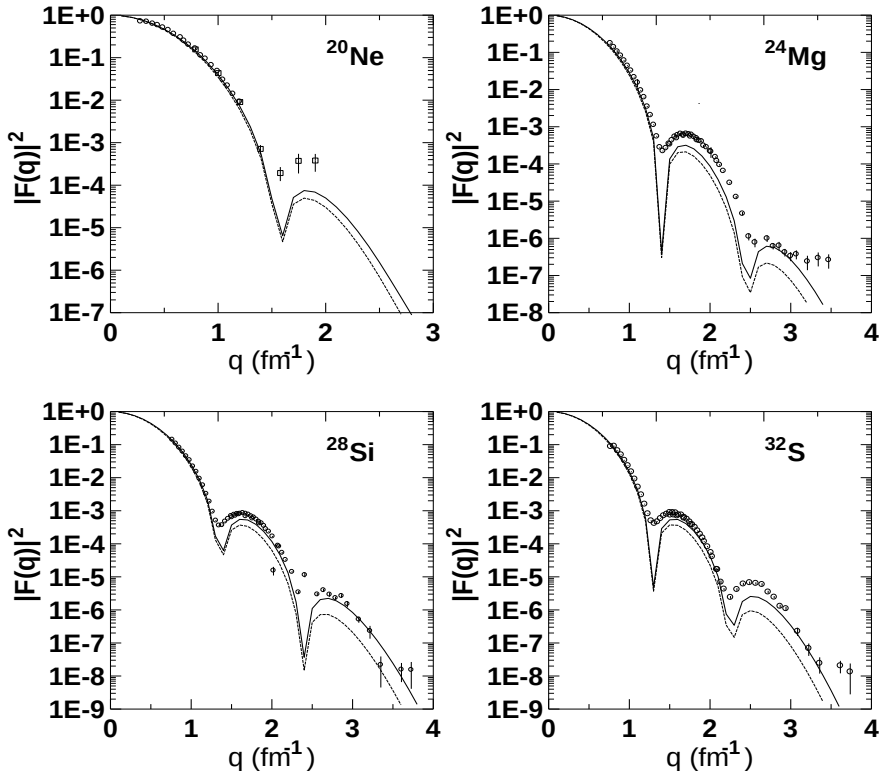

Fig.2: The dependence of the form factor of $F(q)$ on the momentum transfer q (in fm^{-1}) for ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei. The open circles (●) are the experimental data [15] the solid and dashed curves are the calculated result of eq. (11) without and with including the effect of the correction due to $F_{fs}(q)$ and the $F_{cm}(q)$ obtained, respectively.

Conclusions

This study leads to the conclusion that the introduction of additional parameters α, α_1 and α_2, that reflect the difference of the occupation numbers of the states from the prediction of the simple shell model gives very good agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r. The experimental elastic electron scattering form factors from ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S nuclei are in reasonable agreement with the present calculations throughout all values of q.

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