

Chaotic features of energy spectrum in ^{68}Ge nucleus using the nuclear shell model

Hayder A. Abd ALabas¹ and Adel K. Hamoudi²

¹Ministry of Education, Muthana, Iraq

²Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

E-mail: Hydar375@gmail.com

Abstract

Chaotic properties of nuclear energy spectrum in ^{68}Ge are analyzed using the framework of the nuclear shell model. Nuclear energies of considered states are obtained via performing f5p shell model calculations using the OXBASH computer program together with the effective interaction of F5PVH. The ^{68}Ge nucleus is suggested to consist of a core ^{56}Ni with twelve particles (8 neutrons + 4 protons) in f5p-model space ($2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$). The density of energy level of studied classes have been found to have a form of Gaussian, which is in accord with the prediction of other theoretical studies. The distributions of the spacing $P(s)$ and Δ_3 statistic for the studied states are well described by the Gaussian orthogonal ensemble (GOE). Furthermore, these fluctuations are independent on the spin J.

Key words

Chaotic proprieties, random matrix theory, statistical fluctuations, nuclear f5p shell model.

Article info.

Received: Jan. 2020

Accepted: Feb. 2020

Published: Mar. 2020

الخواص الفوضوية لطيف طاقة النواة ^{68}Ge باستخدام نموذج القشرة النووي

حيدر علي عبد العباس¹ و عادل خلف حمودي²

¹وزارة التربية، المثنى، العراق

²قسم الفيزياء، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

تم استخدام نموذج القشرة النووي في دراسة الخواص الإحصائية لطيف طاقة النواة ^{68}Ge . أجريت حسابات نموذج القشرة النووي، بواسطة البرنامج الحاسوبي OXBASH سوية مع التفاعل المؤثر F5PVH، لغرض الحصول على الطيف الطاقوي للنواة أعلاه. تم افتراض النواة ^{68}Ge بأنها مؤلفه من القلب المغلق ^{56}Ni مع 12 جسيمة تتحرك ضمن أنموذج الفضاء النووي f5p المعرف بالمدارات $2p_{3/2}$ ، $1f_{5/2}$ و $2p_{1/2}$. أوضحت هذه الدراسة بأن كثافة مستويات الطاقة للحالات النووية، قيد الدراسة، لها شكل كاوسي (Gaussian shape). أن توزيعات الأحصاء $P(s)$ والأحصاء Δ_3 تتوافق تماما مع توزيع طاقم الحالات المتعامدة الكاوسي (GOE) للمصفوفات العشوائية. لقد وجدنا أيضا بأن التقلبات الإحصائية للتوزيعات أعلاه لا تعتمد على الزخم الزاوي الكلي J.

Introduction

Quantum chaos was explored hugely in the past 30 years [1]. Bohigas et al. [2] assumed a relationship amongst disorder in a classical system and the statistical fluctuations of nuclear spectrum of identical quantum system, where a

systematic evidence of Bohigas assumption is presented in [3]. At the moment, it is eminent that quantum similar of utmost classically disordered systems depict fluctuations in energies that come to an agreement with Random Matrix Theory (RMT) [4, 5] but quantum equivalents of classically

ordered systems depict variations in energies which come to an agreement with the Poisson limit. Under time reversal, the proper formula of RMT be the GOE. RMT had been, at first, operated toward illustrate the fluctuation features of nucleon resonance in the complex nucleus [6]. RMT developed into a typical scheme for probing of common chaotic features in disordered system [7 - 10].

Mean field approximation may be employed to explore the disordered manners of single particle dynamic in nuclei. However, the nuclear interactions mix various mean field which in sequence leads to change the fluctuations properties of the nuclear spectrum and wave functions. Actually, one can investigate these fluctuations through utilizing different models. The nuclear shell model provides an attractive context for such investigations, where effective two-body residual interactions are obtainable and the basis states are designated by exact quantum numbers of J (angular momentum), T (isospin) and π (parity) [11]. In [12-16], the context of the nuclear shell model was utilized to examine eigenvector component distributions. The basis vector amplitudes were found [14] to be in accordance with the Gaussian distribution (GOE prediction) in regions of large level density and diverged from Gaussian manners in further regions unless the computation employs degenerate single-particle energies. The investigation [16] as well recommended that computations by means of the degenerate single particle energies are disordered at lower excitation energy than that of realistic computations.

Electromagnetic probabilities in nuclei are observables which are related to the wave function. The examination of their fluctuations would enhance the universal spectral

investigation as well as help as an extra sign of disorder in the quantum system. In the previous investigations [17-22] we adopted the context of the RMT together with the nuclear shell model to explore the physical characteristics of chaos in nuclear spectra, electromagnetic probabilities, and moments for various nuclei located in different shell model spaces. As a whole, the results were very good depicted by the GOE limit.

In [23] the statistical fluctuations of energy spectra in ^{32}A (^{32}S , ^{32}P and ^{32}Si) nuclei was investigated using the empirical interaction of Wildenthal [24]. The statistical fluctuations were in very well accordance with the GOE. Furthermore, they demonstrated independency on spins J and isospin T . In [25], the work was repeated as in [22] but this time we considered the higher model space of N82 and chose the nucleus ^{138}Ba as case study, where the interaction of N82K was adopted in the calculations [26].

In this research, the spectral variations in ^{68}Ge nucleus are explored using an effective two body residual interaction of f5pvh for twelve particles (4 protons + 8 neutrons) in the f5p-space. The nuclear level density of considered states is noticed have a Gaussian shape, which is consistent with the prediction of other theoretical studies. The statistical properties of $P(s)$ and Δ_3 distributions are well described by the Gaussian orthogonal ensemble of random matrices. Furthermore, these fluctuations are independent on the spin J .

Theory

The effective shell model Hamiltonian of many particle systems can be expressed by [14].

$$H = H_0 + H' \quad (1)$$

where H_0 and H' are the independent particle (one body) Hamiltonian and the residual two-body interaction of H .

The one-body (unperturbed) Hamiltonian

$$H_0 = \sum_{\lambda} e_{\lambda} a_{\lambda}^{\dagger} a_{\lambda} \quad (2)$$

characterizes non-interacting fermions in the mean field of the appropriate spherical core. The single-particle orbitals $|\lambda\rangle$ have quantum numbers $\lambda = (l j m \tau)$ of orbital (l) and total angular momentum (j), projection $j_z = m$ and isospin projection τ . The antisymmetrized two-body interaction H' of the valence particles is written as

$$H' = \frac{1}{4} \sum_{\lambda\mu;\nu\rho} V_{\lambda\mu;\nu\rho} a_{\lambda}^{\dagger} a_{\mu}^{\dagger} a_{\nu} a_{\rho} \quad (3)$$

The many-body wave functions with good spin J and isospin T quantum numbers are constructed via the m -scheme determinants which have, for given J and T , the maximum spin and isospin projection [14],

$$|M = J, T = T_3; m\rangle \quad (4)$$

where m span the m -scheme subspace of states with $M = J$ and $T_3 = T$.

The many-body hamiltonian

$$H_{kk'}^{JT} = \sum_k \langle JT; k | H | JT; k' \rangle \quad (5)$$

is eventually diagonalized to obtain the eigenvalues E_{α} and the eigenvectors

$$|JT; \alpha\rangle = \sum_k C_k^{\alpha} |JT; k\rangle \quad (6)$$

here, the eigenvalues E_{α} are considered as the main object of the present investigation.

The statistical fluctuation of nuclear energy spectrum are gotten by the nuclear level density and the statistical measures of $P(s)$ and Δ_3 statistics [4, 27]. The staircase function $N(E)$ of the nuclear shell model spectrum is firstly build, where $N(E)$ is the number of levels with excitation energies $\leq E$. Here, a smooth fitting to $N(E)$ has been achieved adopting the polynomial of 8th degree. The unfolded energy spectrum is in sequence expressed by the relationship [12].

$$\tilde{E}_i = \tilde{N}(E_i) \quad (7)$$

The real spacings reveal strong fluctuations whereas the unfolded levels \tilde{E}_i have a constant average spacing.

Level density is denoted as the level number per energy at a given excitation energy, locates at the range E to $E + dE$, and expressed as

$$\rho(E) = \frac{dN}{dE} \quad (8)$$

The distribution of $P(s)$ is described as the probability of the adjacent energy levels are far apart. The i^{th} spacing s_i is found via $s_i = \tilde{E}_{i+1} - \tilde{E}_i$. An ordered system is predicted to behave with the Poisson limit

$$P(s) = \exp(-s) \quad (9)$$

If the system is classically chaotic, one expects to obtain the Wigner distribution or Wigner surmise

$$P(s) = \frac{\pi}{2} s \exp\left(\frac{-\pi s^2}{4}\right) \quad (10)$$

which is in agreement with GOE distribution.

The Δ_3 statistic, which are employed to determine the stiffness of energy spectrum, defined by [18]

$$\Delta_3(\alpha, L) = \min_{A,B} \frac{1}{L} \int_{\alpha}^{\alpha+L} [N(\tilde{E}) - (A\tilde{E} + B)]^2 d\tilde{E} \quad (11)$$

It determines the deviancy of the $N(E)$ from a conventional line. It is well-known that rigid (soft) spectra have small (large) values of Δ_3 . For the purpose of getting a smooth distribution for the $\overline{\Delta_3}(L)$, the $\Delta_3(L)$ is averaged over a number n_{α} of intervals $(\alpha, \alpha + L)$

$$\overline{\Delta_3}(L) = \frac{1}{n_{\alpha}} \sum_{\alpha} \Delta_3(\alpha, L) \quad (12)$$

The successive intervals are taken to overlap by $L/2$. In the Poisson limit, $\Delta_3(L) = L/15$. In the GOE limit, $\Delta_3 \approx L/15$ for small L , while, $\Delta_3 \approx \pi^{-2} \ln L$ for large L .

Results and discussion

The computations of the shell-model (for the ^{68}Ge) have been carried out via the program of OXBASH [28]. The ^{68}Ge nucleus is investigated using an effective two body residual interaction of F5PHV [29] for 12 nucleons (4 protons + 8 neutrons) in the f5p model space with $T = 2$.

Table 1 display the dimensions of all considered $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ with $T=2$ for 12 valence particles in the f5p space.

Table 1: Dimensions of all considered $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ states for 12 particles in the f5p space.

J^π	Dimension (N)
0^+	874
1^+	2319
5^+	3059
9^+	393
10^+	160

Fig.1 presents the computed level density $\rho(E)$ (histograms) for the J^π (0^+ , 1^+ , 5^+ , 9^+ , and 10^+) classes of states. The dashed line, which describes the Gaussian fit, is also presented for comparison. The level density $\rho(E)$ of ^{68}Ge ($T = 2$) is plotted as a function of the excitation energy

E_x (in MeV). In this figure evident that the distributions of $\rho(E)$ (histograms) have a bell-shaped curve that are symmetric about the mean energy E_0 where these histograms are fitted by the Gaussian shape [30] (the dashed line). In the cases of taking a finite Hilbert space, $\rho(E)$ vanishes at lower and upper boundaries of the spectrum while in the middle of the spectrum it reaches to maximum. It is clear that with moving from $J^\pi = 0^+$ towards $J^\pi = 10^+$, the level density becomes narrow in width (shrink) due to the lack of the matrix dimension that exists in a Table 1. Anyway, level density (histograms) $\rho(E)$ has a Gaussian shape that is in agreement with Gaussian orthogonal ensemble (GOE). The values of parameters E_0 (the mean energy) and σ (the standard deviation) for each J^π class of states used in the Gaussian fit, are displayed in Table 2. It is obvious that there is no significant change in E_0 for all considered values of J^π . It is also seen no a clear change in the parameter σ for lower value of J^π but found an obvious change for higher values of J^π as a result of the lack of dimensions in the higher J^π (look at Table 1).

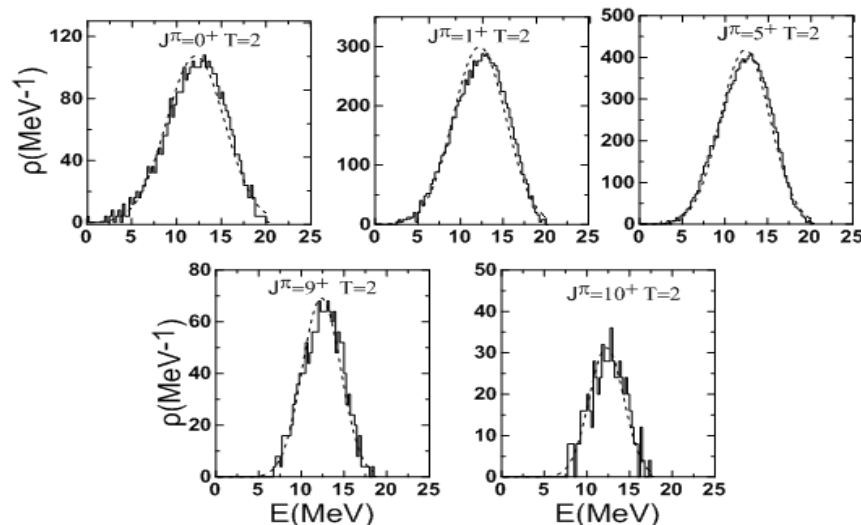


Fig. 1: The level density (histograms) is compared with the Gaussian fit (dash curves) in ^{68}Ge for $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ states.

Table 2: Values of E_0 and σ to ^{68}Ge for various $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ states for 12 particles move in the f_{5p} -shell model space.

J^π	$E_0(\text{MeV})$	$\sigma (\text{MeV})$
0^+	12.119730	3.246461
1^+	12.224740	3.084587
5^+	12.374110	2.918783
9^+	12.356870	2.266879
10^+	12.293750	2.042502

Fig. 2 shows the level spacing $P(s)$, for different unfolded J^π ($0^+, 1^+, 5^+, 9^+$ and 10^+) classes of states with $T = 2$ (^{68}Ge). The solid line describes the GOE distribution, and characterizes chaotic systems. The dashed line describes the Poisson distribution, and depicts regular systems. The histograms denote the computed $P(s)$ distribution and demonstrate a chaotic presentation, which in sequence reveal

a good accordance with GOE limit. The repulsion of levels at small spacing (a distinctive property of chaotic level distribution), formed due to the mixing via the off-diagonal interactions, is noticeably found in the computed histograms. Inspection of Fig.2 provides the conclusion that the nearest neighbor level spacing distributions are independent of the spin J^π .

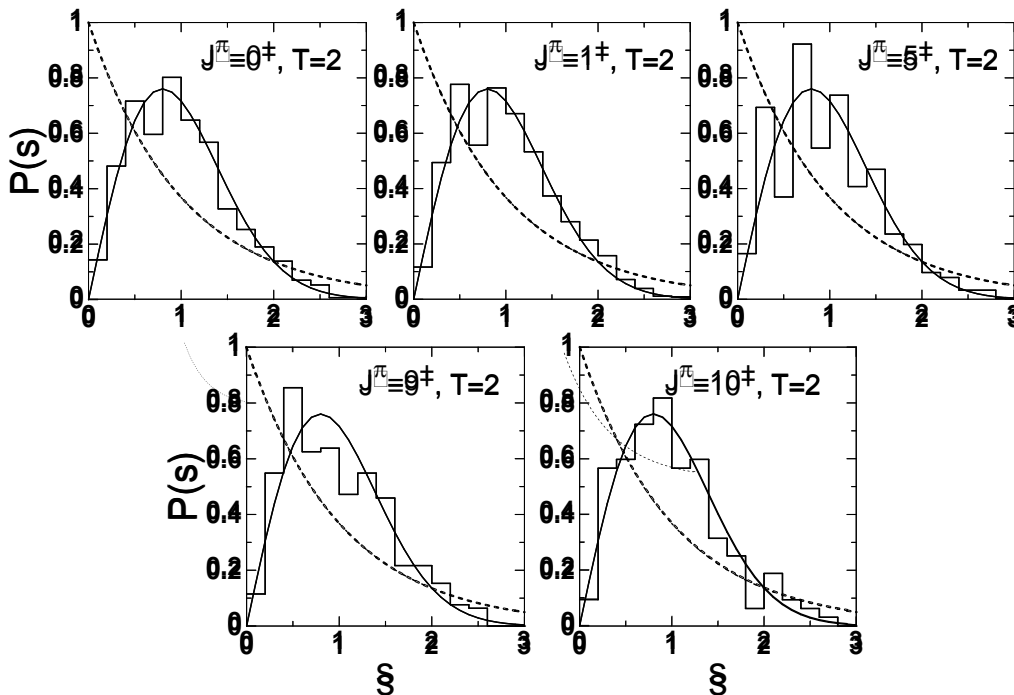


Fig. 2: The nearest neighbor level spacing distributions $P(s)$ in ^{68}Ge for various $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ states. The GOE (solid line) and Poisson distributions (dashed line).

Fig.3 presents the rigidity of the nuclear spectrum (Δ_3 distribution), for the $T=2$ (^{68}Ge) nucleus. The computed $\Delta_3(L)$, depicted by open circles, is plotted against L for the unfolded J^π ($0^+, 1^+, 5^+, 9^+$ and 10^+) classes. For the aim of comparison the Poisson and GOE limits are also

presented and described by the dashed and solid lines, respectively. The computed $\Delta_3(L)$ statistics of all considered states, which have the feature of chaotic systems, are in astonishing accordance with GOE limit. The calculated distribution of Δ_3 statistics for $J^\pi = 9^+$ and 10^+ states

reveal a slight oscillations about the GOE limit. These oscillations are caused by the number of intervals n_α , which is related to the dimension of the J^π classes (see Table 1). In addition, they exhibit independency on the spin

J . It is apparent that the results that we have obtained in Fig. 3 for the spectral rigidity $[\Delta_3(\alpha, L)$ statistic] are consistent with those obtained in Fig. 2 for the nearest neighbor level spacing distributions $P(s)$.

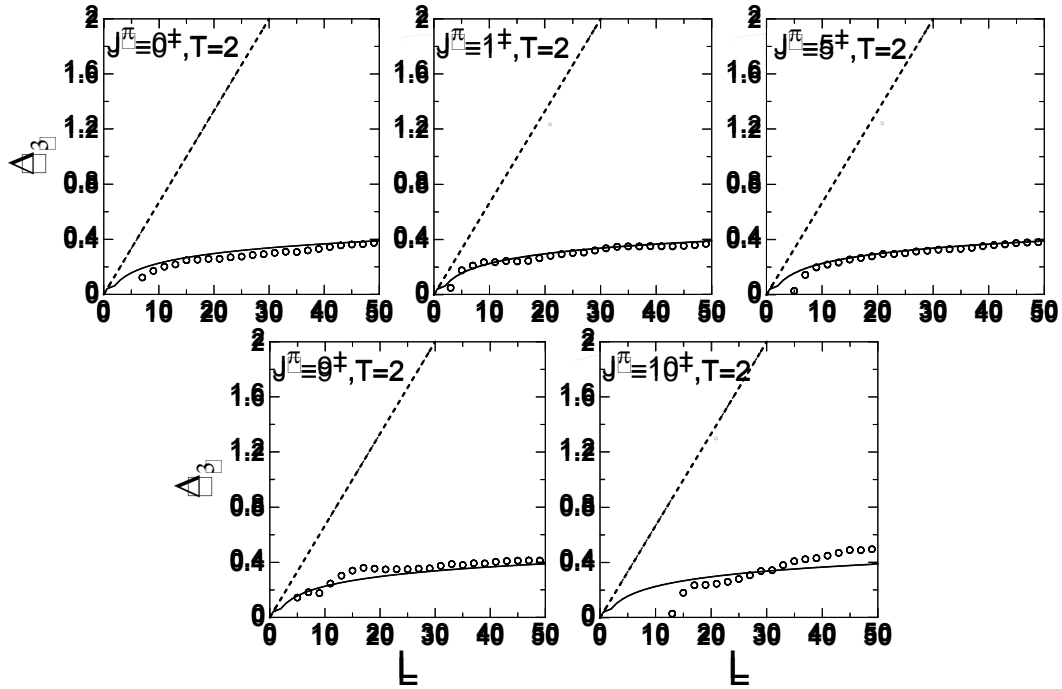


Fig. 3: The average Δ_3 statistics in ^{68}Ge for various $J^\pi = (0^+, 1^+, 5^+, 9^+, \text{ and } 10^+)$ states. The open circles are the calculated results with full Hamiltonian. The solid line is GOE distribution and dash line is the Poisson distribution.

Conclusions

It is found that the nuclear level density has a Gaussian shape, which is in agreement with Gaussian orthogonal ensemble (GOE). The nearest neighbor level spacing distribution $P(s)$ and the Δ_3 statistics are found to be in accordance with the Gaussian orthogonal ensemble (GOE) of random matrices. Besides, the distributions of $P(s)$ and Δ_3 are independent of the spin J .

Acknowledgment

We would like to express our appreciations to Professor B. A. Brown of Michigan State University (the National Superconducting Cyclotron Laboratory) for providing the code OXBASH code.

References

[1] F. Haak, "Quantum Signature of Chaos", 2nd enlarged ed. Springer Verlag, Berlin, (2001).
 [2] O. Bohigas, M. J. Giannoni, C. Schmit, Phys. Rev. Lett., 52 (1986) 1-4.
 [3] S. Heusler, S. Muller, A. Altland, P. Braun, F. Haak, Phys. Rev.Lett., 98, 4 (2007) 1-6.
 [4] M. L. Mehta, "Random Matrices", 3rd ed. Academic, New York, (2004).
 [5] T. Papenbrock and H. A. Weidenmuller, Rev. Mod. Phys., 79, 3 (2007) 997-1013.
 [6] C. E. Porter, "Statistical Theories of Spectra", Fluctuations Academic, New York, (1965).
 [7] T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, S. S. M.

- Wong, Rev. Mod. Phys., 53 (1981) 385-479.
- [8] T. Guhr, A. Müller-Groeling, H. A. Weidenmüller, Phys. Rep., 299 (1998), 189-425.
- [9] Y. Alhassid, Rev. Mod. Phys., 72 (2000) 895-968.
- [10] M. C. Gutzwiller, "Chaos in Classical and Quantum Mechanics", Springer-Verlag, New York, (1990).
- [11] V. Zelevinsky, B. A. Brown, N. frazier, M. Horoi, Phys. Rep., 276 (1996) 85-176.
- [12] R. R. Whitehead, A. Watt, D. Kelvin, A. Conkie, Phys. Lett., B 76 (1978) 149-152.
- [13] J. J. M Verbaarschot and P. J. Brussaard, Phys. Lett., B 87 (1979) 155-158.
- [14] B. A. Brown and G. Bertsch, Phys. Lett., B 148 (1984) 5-7.
- [15] H. Dias, M. S. Hussein, N. A. de Oliveira, B. H. Wildenthal, J. Phys. G, 15, 5 (1989) L79.
- [16] V. Zelevinsky, M. Horoi, B. A. Brown, Phys. Lett., B 350, 2 (1995) 141-146.
- [17] Y. Alhassid, A. Novoselsky, N. Whelan, Phys. Rev. Lett., 65 (1990) 2971-2974.
- [18] Y. Alhassid and A. Novoselsky, Phys. Rev., C 45 (1992) 1677-1687.
- [19] Y. Alhassid and D. Vretenar, Phys. Rev., C 46 (1992) 1334-1338.
- [20] J. J. M. Verbaarschot and P. J. Brussaard, Phys. Lett., B 87 (1979) 155-158.
- [21] A. Hamoudi, R. G. Nazmidinov, E. Shahaliev, Y. Alhassid, Phys. Rev., C 65 (2002) 1-5.
- [22] A. Hamoudi, Nucl. Phys., A 849 (2011) 27-34.
- [23] A. K. Hamoudi and T. A. Abdul Hussein, Am. J. Phys. Appl., 5 (2017) 35-40.
- [24] B. H. Wildenthal, Prog. Part. Nucl. Phys., 11 (1984) 5-51.
- [25] A. K. Hamoudi and S. F. Murad, Iraqi J. of Science, 59, 4c (2018) 2225-2233.
- [26] A. Hamoudi and A. A. Majeed Al-Rahmani, Nucl. Phys., A 892 (2012) 21-33.
- [27] F. S. Stephens, M. A. Deleplanque, I. Y. Lee, A. O. Macchiavelli, D. Ward, P. Fallon, M. Cromaz, R. M. Clark, M. Descovich, R. M. Diamond, E. Rodriguez-Vieitez, Phys. Rev. Lett., 94 (2005) 1-18.
- [28] B. A. Brown, A. Etchegoyen, N. S. Godwin, W. D. M. Rae, W. A. Richter, W. E. Ormand, E. K. Warburton, J. S. Winfield, L. Zhao, C. H. Zimmerman, MSU-NSCL Report, No. 1289 (2004).
- [29] P.W.M. Glaudemans, P.J. Brussaard, R. H. Wildenthal, Nucl. Phys., A 102 (1967) 593-601.
- [30] M. I. Ribeiro, Instituto Superior Tecnico, Av. Rovisco Pais, 1, 1049-0.001 Lisboa, Portugal,