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The difference in the charge density distribution of ^{90}Zr and ^{92}Mo nuclei from elastic electron scattering

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Abstract

The calculation of the nuclear charge density distributions ρ(r) and root mean square radius (RMS) by elastic electron scattering of medium mass nuclei such as (^{90}Zr, ^{92}Mo) based on the model of the modified shell and the use of the probability of occupation on the surface orbits of level 2p, 2s eroding shells and 1g gaining shell. The occupation probabilities of these states differ noticeably from the predictions of the SSM. We have found an improvement in the determination of ground charge density and this improvement allow more precise identification of (CDD) between (^{92}Mo-^{90}Zr) to illustrate the influence of the extra two protons on the charge density distributions and was agree with those of experimental data and Hartree.– Fock. (H.F) wave functions.

Key words
Difference, of the CDD (Δρ),elastic, electron, scattering. EES and charge density distribution.

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الفرق في توزيع كثافة الشحنة للنوى^{90}Zr و^{92}Mo من استطارة الإلكترونات المرنة

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الخلاصة

حساب توزيعات كثافة الشحنة النووية ρ (r) ونصف قطر الجذر التربيعي المتوسط (RMS) عن طريق الاستطارة الالكترونيه المرنه لنواة الكتلة المتوسطة مثل (^{90}Zr، ^{92}Mo) بناءً على نموذج القشره المعدل واستخدام احتمالية الاشغال على مدارات السطح من المستوى 2p، 2s (القشورالمعطيه) و 1g (القشور المكتسبه). تختلف احتمالات اشغال هذه الحالات بشكل ملحوظ عن توقعات نموذج القشرة البسيطة. لقد وجدنا تحسنا في تحديد كثافة الشحنة الأرضية وهذا التحسن يسمح بتحديد أكثر دقة لـ (CDD) بين (^{92}Mo - ^{90}Zr) لتوضيح تأثير البروتونين الإضافيين على توزيعات كثافة الشحنة وكان متفقا مع تلك الخاصة بالبيانات التجريبية ووظائف دالة هارتري - فوك (H.F).

Introduction

In 1953, Hofstadter and his partners were the first to utilize high– energy electron beams given by the Stanford electron direct quickening to observe electron scattering. They observed a clear deviation in the angular distribution from that for a point charged particle the Mott cross-section, which was attributed to a finite spatial spread of the charge of those nuclei. A series of (EES) tests for various nuclei decided the gross properties of the nuclear charge distributions and the estimations for the proton confirmed that the proton has limited size. These discoveries won the Nobel Prize in 1961 [1].

Electron scattering has been previously considered by Antonov et al. for both light and heavy nuclei for the He isotopes they found variations of the charge densities and so likewise for the form factors for ^{4}He and ^{6}He but not a significant change in the form factor between ^{6}He and ^{8}He [2]. The properties of the ground state of the atomic nucleus are calculated from the most important quantities of the understanding of nuclear physics has been verified atomic nucleus consists of two types of nucleons are protons and neutrons [3]. The number of the occupation and the natural orbits of the nucleus are obtained theoretically from the natural orbital method [4, 5] and the coherent density fluctuation model, the formalisms occupation numbers which were discussed by Antonov, hodgsonand petkov [6]. Depending on the situation in which the charge density is distributed correctly the numbers of the occupation can be determined [7].

Shell model is a theoretical model to portray the atomicnucleus. The nuclear shell model was proposed by Dmitry Ivanenko in 1932 and further developed independently by several physicists such as Maria Goeppert-Mayer and Eugene Paul Wigner et al. in 1949. It must be noticed this model depends on the pauliexclusion principle to portray the structure of the nucleus in terms of energy levels shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. Nucleons are added to shells which increment with energy that orbit around a central potential.

The two-body powerful interaction is a key ingredient for the success of the nuclear shell model, which determines the accuracy of the shell - model calculations that assume an appropriate core to be inert and a limited space [8].

In modern literature many of the theoretical works are taught to contrast form factors along isotopes and isotonic chains of medium and heavy mass nuclei. From the theoretical side the difference between the distribution of protons and neutrons can be obtained in the framework of Hatree - Fock (HF) method [9, 10].

Astrategy method, which is somewhat analogous, is the insertion ofthe short –range correlation (SRC) inthe Slater determinant. Various efforts were made in this trend relating for the most part light closed shell nuclei in the context of the Born approximation [11]. The variety of the charge FF along isotonic chains of medium and heavy mass nuclei. It has been discovered that when the quantity of (protons) in these isotonic chains increases, the squared modulus of the charge FF and the situation of its minima appear, respectively, an upward pattern and a significant internal moving inthe momentum transfer [12].

The article is organized in the following way. Above section I and section II is devoted to the theoretical formalism. The numerical results and discussions of calculations of charge densities of the (^{90}Zr, ^{92}Mo) nuclei and come into the possession of the proton occupancies of the surface shells of these nuclei which fit the experimental data by electron scattering. A similar analysis allows us to obtain new information regarding the shell structure of these nuclei different from a simple shell model. The proton occupancies of these nuclei were determined theoretically by comparison with the experimental charge densities and were found to be different from to 1 in section III. Finally, our conclusions of this study is laid in section IV.

Theory

In short, this section describes the derivation of nuclear distributions such as proton density distribution (PDD) and root mean square radius (RMS) of the ground state for even mass nuclei in the 2p-1f shell for (^{90}Zr, ^{92}Mo). By a harmonic oscillator can be evaluated by means of the radial part wave function.$R_{nl}(r)$ [13]

$$\rho_c(r) = \frac{1}{4\pi}\sum_{nl} \xi_{nl} 2(2l+1)|R_{nl}(r)|^2 \quad (1)$$

where $\rho_c(r)$ is the PDD of nuclei, ξ_{nl} is the proton occupation probability of the state nl (ξ_{nl}= 0 or 1 for closed shell nuclei and 0 < ξ_{nl}< 1 for open shell nuclei) and $R_{nl}(r)$ is the radial. Part of the single-particle harmonic oscillator wave function. To derive an explicit form for the PDD of consider nuclei, it is supposed that there is a core of filled 1s and 1p and 1d shells and the proton occupation numbers in 2s, 1f, 2p, 1g shells are equal to (2-d_1), 14, (6-d_2) and $(Z-40+d_1+d_2)$ for ^{90}Zr, ^{92}Mo but not to 2, 14, 6 and (Z-40) as in SSM. Using this assumption in Eq.(1), an analytical form for the ground state PDD of the (^{90}Zr, ^{92}Mo) nuclei is expressed as

$$\rho(r) = \frac{1}{4\pi}\{2|R_{10}(r)|^2 + 6|R_{11}(r)|^2 + 10|R_{12}(r)|^2 + (2d_1)|R_{20}(r)|^2 + 14|R_{13}(r)|^2 + (6-d_2)|R_{21}(r)|^2 + (z-40+d_1+d_2)|R_{14}(r)|^2\} \quad (2)$$

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{\begin{matrix}\left(5-\frac{3}{2}d_1\right) + \left(10+2d_1-\frac{5}{3}d_2\right)\left(\frac{r}{b}\right)^2 + \left(-4-\frac{2}{3}d_1+\frac{4}{3}d_2\right)\left(\frac{r}{b}\right)^4 + \\ \left(\frac{8}{3}-\frac{4}{15}d_2\right)\left(\frac{r}{b}\right)^6 + \frac{16}{945}(z-40+d_1+d_2)\left(\frac{r}{b}\right)^8\end{matrix}\right\} \quad (3)$$

where Z is the atomic number of nuclei, the parameters (d_1, d_2) characterize the deviation of the proton occupation numbers from the prediction of the SSM and b is the harmonic oscillator size parameter where the normalization condition of the $\rho_c(r)$ is given by [14]

$$Z = 4\pi\int_0^\infty \rho_c(r) r^2 dr\ , \quad (4)$$

The central $\rho_c(r=0)$ is obtained from Eq.(3) as

$$\rho_c(0) = \frac{1}{\pi^{3/2}b^3}\left\{5-\frac{3}{2}d_1\right\}, \quad (5)$$

then d_1 is obtained from the central PDD of Eq.(5) as

$$d_1 = \frac{2}{3}\left\{5-\pi^{3/2}b^3\rho_c(0)\right\} \quad (6)$$

The mean square charge radius (MSR) can be determined according to the following equation [14].

$$\langle r^2\rangle = \frac{4\pi}{Z}\int_0^\infty \rho_c(r) r^4 dr, \quad (7)$$

The corresponding MSR and d_1 for (^{90}Zr, ^{92}Mo) is

$$\langle r^2\rangle = \frac{b^2}{Z}\{-70+2d_1+d_2+\frac{11}{2}Z\}, \quad (8)$$

$$d_2 = \frac{Z}{b^2}\langle r^2\rangle + 70 - 2d_1 - \frac{11}{2}Z, \quad (9)$$

Results and discussion

1-Proton density distribution

In SSM (when d_1=0, d_2=0), the CDD of ^{90}Zr, ^{92}Mo nuclei obtained from theoretical consideration of the Eq.(3). In this case, these equations are simplified to the form $\rho(r) = \frac{\exp(-x^2)}{\pi^{3/2}b^2}\sum_{m-0}^{4}\xi_m x^{2m}$. *In MSM* (when $d_1 \neq$, $d_2 \neq 0$), the general form of the Eq.(3) can be expressed by $\rho(r) = \frac{\exp(-x^2)}{\pi^{3/2}b^2}\sum_{m-0}^{5}\xi'_m x^{2m}$. Here,

$x = r/b$; b is the harmonic oscillator size parameter, which can be chosen so as to imitate the experimental root mean square (RMS) charge radii of considered nuclei. The coefficients ξ_m, ξ'_m for the considered nuclei are shown in Table 1 and the proton configuration of these nuclei is shown in Table 2. The values of parameters d_1 obtained from Eq.(6) and d_2 are evaluated by the Eq.(8) for (^{90}Zr, ^{92}Mo). The value of $< r^2 >^{1/2}_{cal}$ obtained from the equation (8) for (^{90}Zr, ^{92}Mo,). In Table 3, we display the values for the parameters b, d_1, d_2 and the experimental and calculated values of $\rho_{exp}(0)$, $<r^2>^{1/2}_{Cal}$ and $<r^2>^{1/2}_{exp}$ as well as the values for the FB which are utilized in this study for ^{90}Zr, ^{92}Mo nuclei and the experimental and calculated values to generate the densities of fitted FB. In Table 4, the proton occupation probabilities P_{2s}, P_{2p}, P_{1g}, and P_{1h} with MSM case are displayed for ^{90}Zr, ^{92}Mo nuclei.

The dependence of the CDD (in fm^{-3}) on r (in fm) of the studied nuclei are shown in figure ^{90}Ze [Fig. 1(a)], ^{92}Mo [Fig. 1(b)]. The dashed curves and the solid curves in figures ^{90}Ze [Fig. 1(a)], ^{92}Mo [Fig. 1(b)], are the calculated CDD using Eq.(3) for (^{90}Zr, ^{92}Mo) nuclei and figure ^{90}Ze [Fig. 1(a)], ^{92}Mo [Fig.1(b)] with (d_1, $d_2 = 0$) and (d1, d2 $\neq$ 0) respectively. The experimental data of the (FB) Fermi Bessel and designated by the dotted symbols, are also displayed in this figure for comparison. It is obvious that the dashed distributions are in poor accordance with the experimental data, especially for small (r).

The introduction of the fractional occupation numbers of the shells 2s and 2p (eroding shells) and 1g (gaining shells) tends to improve the (CDD) (the solid curves) which sequential leads the results to be in accordance with the experimental data. It is obvious the figure ^{90}Ze [Fig. 1(a)], ^{92}Mo [Fig. 1(b)] the computations of the dashed curves constitute a large disagreement with the experimental data (solid circles) in the central especially at the region (r≤4.5 fm). Besides, the computations of the solid curves a little discord with the experimental data (solid circles) in the region of r. i.e., for ^{90}Zr, ^{92}Mo nucleus (2.8 ≤ r≤ 3.2).

Table1: Coefficients of the charge density of nuclei in the simple (ξ_m) and modified (ξ'_m) shell models.

Coefficient Nucleus	ξ_0	ξ'_0	ξ_1	ξ'_1	ξ_2	ξ'_2
^{90}Zr	5	$5-\frac{3}{2}d_1$	10	$10+2d_1-\frac{5}{3}d_2$	-4	$-4-\frac{2}{3}d_1+\frac{4}{3}d_2$
^{92}Mo	5	$5-\frac{5}{2}d_1$	10	$10+2d_1-\frac{5}{3}d_2$	-4	$-4-\frac{2}{3}d_1+\frac{4}{3}d_2$

Coefficient Nucleus	ξ_3	ξ'_3	ξ_4	ξ'_4	ξ_5	ξ'_5
^{90}Zr	$\frac{8}{3}$	$\frac{8}{3}-\frac{4}{15}d_2$	0	$\frac{16}{945}(d_1+d_2)$	0	0
^{92}Mo	$\frac{8}{3}$	$\frac{8}{3}-\frac{4}{15}d_2$	$\frac{32}{945}$	$\frac{16}{945}(2+d_1+d_2)$	0	0

Table 2: Proton configuration of the nuclei SSM, simple shell model; MSM, modified shell model.

Nucleus	Core Shell			Eroding Shell						Gaining Shell			
	1S	1P	1d	2s		1f		2p		1g		1h	
				SSM	MSM	SMM	MSM	SSM	MSM	SSM	MSM	SSM	MSM
^{90}Zr	2	6	10	2	$2\text{-}d_1$	14	14	6	$6\text{-}d_2$	0	(d_1+d_2)	0	0
^{92}Mo	2	6	10	2	$2\text{-}d_1$	14	14	6	$6\text{-}d_2$	2	$(2+d_1+d_2)$	0	0

Table 3: The Values of various parameters employed in the present calculations together with $\rho_{\exp}(0)$ and $<r^2>^{1/2}_{\exp}$.

Nucleus	Type of CDD [16]	$\rho_{\exp}(0)$ (fm^{-3}) [16] P.W.Eq.(4)	$<r^2>^{1/2}_{\exp}$ (fm) [16]	b (fm)		$<r^2>^{1/2}_{Cal}$ (fm)		d_1 obtained from Eq.(6)	d_2 obtained from Eq.(9)
				SSM	MSM	obtained from Eq.(7)	obtained from Eq.(8)		
^{90}Zr	FB	6.891573E-2	4.258(8)	2.204	2.161	4.2572	4.2494	7.471E-1	3.687
^{92}Mo	FB	6.882336E-2	4.294(16)	2.199	2.159	4.2949	4.2847	7.603E-1	3.603

Table 4: Proton occupation probabilities of the shell 2s, 2p, 1g, 1h of the nuclei.

Nucleus	P_{2s}	P_{2p}	P_{1g}	P_{1h}
^{90}Zr	0.6264	0.3854	0.2463	0
^{92}Mo	0.6198	0.3994	0.3535	0

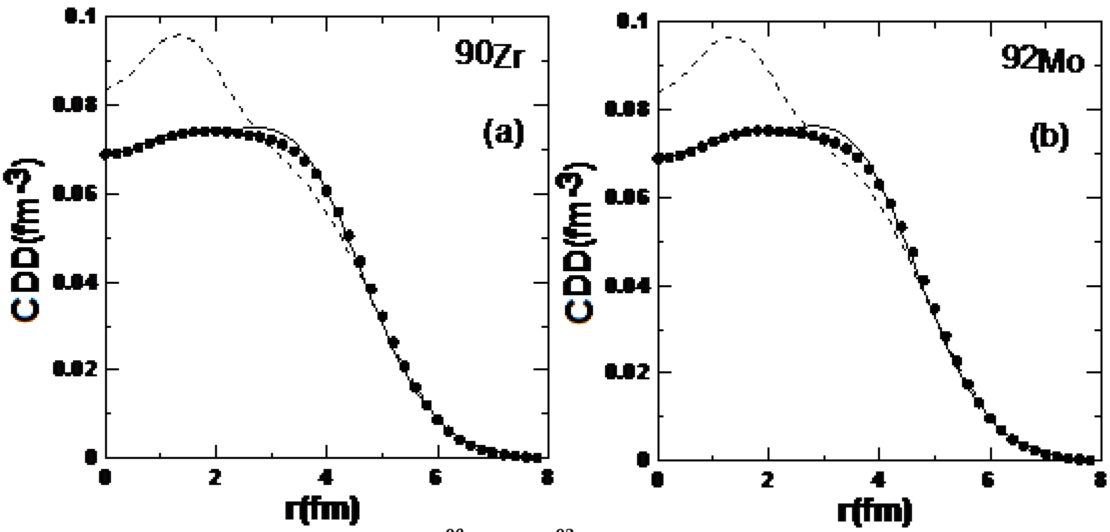

Fig.1: Dependence of CDD on r for ^{90}Zr and ^{92}Mo nuclei. The dashed and solid curves are the calculated CDD of equation (2) when $d_1, d_2 = 0$ and $d_1, d_2 \neq 0$, respectively. The dotted symbols are the best fitted to the experimental data for FB [17, 18].

2- The difference of charge density distribution of (^{92}Mo-^{90}Zr)

In Fig.2, the difference in the ground state charge densities between the magic nucleus for an isotonic include this pair (^{92}Mo - ^{90}Zr) the solid curve represent the calculated difference of the PDD with (d_1,$d_2 \neq 0$), and the dotted curve is the calculated differenceof the PDD by Hartree – Fock (H.F) method with taken from Ref. [15], and shaded area represent the experimental data with its error bar taken from Ref. [15].At the region of $r \leq 11.5$ fm, the performance of our calculated result (the solid curve) is in accord with the experimental result (the shaded area), while the magnitude at the region overestimates clearly the experimental data. For $r > 11.5$ fm both performanal and magnitude are in very well accord with these of the experimental result. It is obvious that the dashed curve under predicts slightly the data at the region $r \leq 7.5$ fm but its behavior agree the data at this region. In addition, both behavior and magnitude of this curve are in very well accordance with data at the region $r > 7.5$. The isotone pair ^{92}Mo-^{90}Zr is chosen as a typical case and analysis in terms of the difference charge distribution for ($\rho_{Mo} - \rho_{Zr}$).

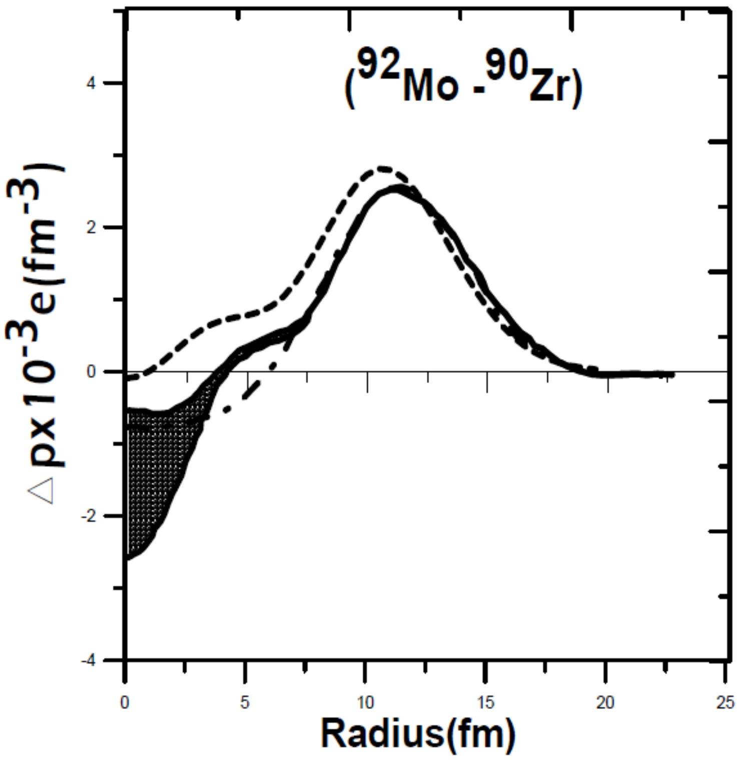

Fig.2: Dependence of the difference of the CDD of (^{92}Mo, ^{90}Zr)$\Delta\rho(r)$ on (r). The solid curve is the calculated difference of the CDD with (d_1, $d_2 \neq 0$), the dotted curve is the calculated difference of the CDD by Hartree - Fock (H.F) method with taken from Ref. [15], and the shaded area represents the calculated difference of the CDD with (d1,d2)=0 with the error bar.

Conclusions

The basic results of this paper can be formulated as follows

a-The distribution of the charge density of the nuclei (^{90}Zr, ^{92}Mo) was calculated on the basis of a MSM for the probability of occupation of the state. The nuclei have the core filled and eroding shells (2s, 2p), the gaining shell (1g) and results for the probability of occupation differed from the expectations of the simple shell model and more in agreement with theexperimental proton density.

b- Dependence on the harmonic oscillator parameter b and the root mean square radius of nuclei (^{90}Zr,^{92}Mo) This indicates that quantities can be described smoothly and on the basis of Eq.(3).

c- In the paper there is a clear argument that the correlations between nucleons are important to obtain the correct description for the distribution of the density of the charge. The modified shell model (MSM) allows us to obtain the numbers of the occupation of the paralysis cases from the experimental data for the electron scattering.

d- The difference of the charge density distribution between the magic nucleus for an isotonic pair (^{92}Mo - ^{90}Zr) are in very well from where behavior and magnitude of this curve together with experimental data.

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