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Elastic electron scattering of palladium

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Abstract

Proton density distributions (PDD), their differences and the elastic electron- scattering form factor of the ground state for some shell nuclei, such as (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes have been calculated based on the use of occupation on the surface orbits of level 2p, 2s eroding shells and 1g, 1h gaining shells and the wave functions of the harmonic oscillator potential with size parameters chosen to reproduce the observed root mean square charge radii for all considered nuclei. It is found that introducing additional parameters, namely d_1 and d_2 which reflect the difference of the occupation numbers of the states from the prediction of the simple shell model SSM leads to a remarkable agreement between the calculated and experimental results of the proton density distributions (PDD) throughout the whole range of (r).

Key words

Difference of the PDD (Δρ), elastic electron scattering, form factor, Proton Density Distribution (PDD).

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الاستطارة الالكترونيه المرنه للبلاديوم

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الخلاصة

تم دراسة توزيعات كثافة البروتون للحالة الأرضية للنواة مثل نظائر (^{104}Pd، ^{106}Pd، ^{108}Pd، ^{110}Pd) على أساس نموذج القشرة المعدلة مع اعداد الاشغال للمستويات 2s، 2p (قشور معطيه) و 1g، 1h (القشور المكتسبه). تختلف احتمالات الاشغال لهذه المستويات بشكل ملحوظ عن تنبؤات نموذج القشرة البسيطة. يتم إجراء حسابات لعوامل التشكل F (q) لجميع النوى قيد الدراسة في الموجة المستويه لتقريب بورن (PWBA) وكانت في اتفاق جيد مع تلك البيانات التجريبية في جميع قيم نقل الزخم q. علاوة على ذلك ، تم حساب الفرق في توزيع كثافة البروتون بين (^{106}Pd - ^{104}Pd)، (^{108}Pd - ^{106}Pd)، (^{110}Pd - ^{108}Pd) لتوضيح تأثير النترونين الإضافيين على توزيع كثافة البروتون.

Introduction

Electron-Scattering is an interesting tool for considering the electromagnetic properties of nuclei, acquiring knowledge to the nuclear charge and current distribution. There is a few purposes behind using electron as test. To begin with, the electron interacts with the nucleus with the electromagnetic force, which is the best known connection, accurately described by quantum electrodynamics. The coupling consistent of the interaction is also adequately powerless to not essentially disturb the nuclear structure under investigation. What's more, the weakness of the interaction enables one to work in first order perturbation during the one photon exchange approximation. One more, in as opposed to the instance of real photons, one can vary the energy transfer and the momentum transfer independently, consequently mapping out the fourier change density [1]. Electron- Scattering studied experimentally and theoretically, it is considered a probe to study the internal structure of the nucleus [2]. Electron-Scattering has been previously considered by Antonov et al. for both light and heavy nuclei. For the He isotopes they found variations of the PDD and so likewise for the form factor, for ^{4}He and ^{6}He but not a significant change in the form factor between ^{6}He and ^{8}He. They also found that the proton density extends far with increasing neutron number [3]. The properties of the ground state of the atomic nucleus are calculated from the most important quantities of the comprehension nuclear physics has been verified atomic nucleus comprises of two kinds of nucleons are proton and neutron [4].

The number of protons in shell model is important for nuclei and is derived from the proton-removing reaction experiments [5] such as (d, ^{3}He) or (e, e'p) [6, 7]. Over the past half-century, electron- scattering experiments have been used as a powerful and precise instrument. They have revealed the distribution density of the charge, provided measurements on the radius of the charge of the proton and drawn detailed maps of elastic form factor (EFF) [8-10].

In the conventional ES model of nuclei, the PDD of the target is generally subrogated with approximately simple proton density models [11]. In other words, there are number of ways to connect the experimental to measure the form factor and PDD of including Fourier – Bessel (FB) see references [12, 13].

The general calculation method of nuclear distributions (radius and form factor) of the harmonic- oscillator shell model is based on the analytical expressions derived from this model where they are modified using the probability of occupation of surface orbits, which are distributed in orbits of (ℓ- levels), the method is finally applied in the study of nuclei by comparing them with empirical data [14].

The number of the occupation and the natural orbits of the nucleus are obtained theoretically from the natural orbital method [15, 16].

We conducted calculation of the PDD and EESFF of some 2s-1d shell nuclei [17, 18] on the basis of a modified shell model (MSM) with fractional occupation numbers of the states 2s, 2p and illustrated that the inclusion of the higher 1f-2p shell in the calculations lead to produced a good results in comparison with those of the experiment data, the same procedures done for some 1f-2p shell nuclei [19-21] but in [21] the modified shell model (MSM) with occupation numbers of the state 2s,2p and 1g.

The article is organized in the following way. Above section I and section II is devoted to the theoretical formalism. The numerical results and discussions of calculations of charge densities of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes and come into the possession of the proton occupancies of the surface shells of these nuclei which fit the experimental data by electron scattering. A similar analysis allows us to obtain new information regarding the shell structure of these nuclei different from a simple shell model. The proton occupancies of these nuclei were determined theoretically by comparison with the experimental charge densities and were found to be different from 0 to 1, while the derived form of the PDD is employed in determining the EESFF for these nuclei are presented in section III. Finally, our conclusions of this study is laid in section IV.

Theory

In short, this section describes the derivation of nuclear distributions such as proton density distribution (PDD), root mean square radius (RMS) and EESFF of the ground state for some even mass nuclei in the 2p-1f shell for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes. By a harmonic oscillator can be evaluated by means of the radial part of the wave functions $R_{nl}(r)$ [18].

$$\rho(r)\frac{1}{4\pi}\sum_{nl}\xi_{nl}2(2l+1)|R_{nl}(r)|^2 \quad (1)$$

$\rho(r)$ is the PDD of nuclei, ξ_{nl} is the proton occupation probability of the state nl $\xi_{nl}=0$ or 1 for closed shell nuclei and $0<\xi_{nl}<1$ for open shell nuclei.

Well-ordered to improve the description of the proton density we presumed model of the shell of the nucleus, which has the occupation numbers of each of the model in which the core filled consists of (1s, 1p, 1d) is filled and are re-distribution of protons (2s, 1f, 2p, 1g, 1h), So that (2s, 1f, 2p) is eroding shells and (1g, 1h) is gaining shells and redistribution as follows. For (^{104}Pd, ^{106} Pd, ^{108}Pd, ^{110} Pd) isotopes. The (d_1, d_2) Protons of shells (2s, 2p) respectively were transferred the proton occupation numbers in (2s, 1f, 2p, 1g and 1h) shells are equal to ($2-d_1$), 14, ($6-d_2$), (Z-40) and (d_1+d_2), respectively instead of to 2, 14, 6, (Z-40) and 0 as in simple shell model., we will obtain form for the ground state (PDD) in 1g – 1h shell (^{104}Pd, ^{106} Pd, ^{108}Pd, ^{110} Pd) isotopes for as:

$$\rho(r)=\frac{1}{4\pi}\{2|R_{10}(r)|^2+6|R_{11}(r)|^2+10|R_{12}(r)|^2+(2-d_1)|R_{20}(r)|^2+14|R_{13}(r)|^2+(6-d_2)|R_{21}(r)|^2+(z-40)|R_{14}(r)|^2+(d_1+d_2)|R_{15}(r)|^2\} \quad (2)$$

$$\rho(r)=\frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{\left(5-\frac{3}{2}d_1\right)+\left(10+2d_1-\frac{5}{3}d_2\right)\left(\frac{r}{b}\right)^2+\left(-4-\frac{2}{3}d_1+\frac{4}{3}d_2\right)\left(\frac{r}{b}\right)^4+\left(\frac{8}{3}-\frac{4}{15}d_2\right)\left(\frac{r}{b}\right)^6+\left(\frac{16}{945}(z-40)\left(\frac{r}{b}\right)^8\right)+\frac{32}{10395}(d_1+d_2)\left(\frac{r}{b}\right)^{10}\right\} \quad (3)$$

b≡The harmonic- oscillator size parameters, Z ≡ The atomic number of nuclei, d_1, d_2≡The occupation number to the deviation of proton from the prediction of the simple shell model d=0 proton configurations of these nuclei are the central PDD ρ(r=0) is obtained from Eq.(3) as:

$$\rho(0)=\frac{1}{\pi^{3/2}b^3}\left[5-\frac{3d1}{2}\right] \quad (4)$$

Also, the parameter d_1 which we can obtain from the central PDD of equation (4) as:

$$d_1=\frac{2}{3}\left\{5-\pi^{\frac{3}{2}}b^3\rho(0)\right\} \quad (5)$$

d_2 calculated in Ref. [18] as:
for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes:

$$d_2=\frac{Z}{b^2}\langle r^2\rangle+35-\frac{3}{2}d_1+\frac{11}{4}z \quad (6)$$

The normalization condition of the $\rho(r)$can be expressed as [18].

$$Z=4\pi\int_0^\infty\rho(r)r^2dr \quad (7)$$

The mean square charge radius for 1f-2p shell nuclei according to the following equation

$$\langle r^2\rangle=\frac{4\pi}{Z}\int_0^\infty\rho(r)r^4dr \quad (8)$$

MSR for (^{104}Pd, ^{106} Pd, ^{108}Pd, ^{110}Pd) isotopes:

$$\left\langle r^2\right\rangle=\frac{b^2}{Z}\{-70+3d_1+2d_2+\frac{11}{2}Z\} \quad (9)$$

The elastic monopole charge form factors $F_{C0}(q)$, of the target nucleus is expressed in form [18]

$$F_{C0}^{g}(q)=\frac{1}{Z}\int_0^\infty F\ (q,x)dx \quad (10)$$

where the form factor of uniform charge density distribution is given by

$$F\ (q,x)=\frac{3Z}{(qx)^2}\left[\frac{\sin(qx)}{(qx)}-\cos(qx)\right] \quad (11)$$

Inclusion of the correction due to the finite nucleon size $f_{fs}(q)$ and the center of mass correction $f_{cm}(q)$ in the calculations requires multiplying the form factor of Eq. (10) by these correction. Here, $f_{fs}(q)$ is considered as free nucleon form factor which is assumed to be the same for protons and neutrons [18].

$$f_{fs}(q)=e^{\left(\frac{-0.43q^2}{4}\right)} \quad (12)$$

The correction $f_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and is given by [18].

$$f_{cm}(q)=e^{\left(\frac{b^2q^2}{4A}\right)} \quad (13)$$

Multiplying eq. (10) by these corrections, yields:

$$F_{C0}(q)=\frac{1}{Z}\int_0^\infty F\ (q,x)dx\ f_{fs}(q)\,f_{cm}(q), \quad (14)$$

Results and discussion

1-Proton density distribution

The size and shape of nucleus cannot be uniquely determined from the RMS radius only. Proton distribution is necessary to measure the study of the internal structure [2]. In the present work we take simple analytical expressions of nuclear charge in the context of the wave performance of the harmonic-oscillator of the shell model, the expressions are modified assuming the probability of occupying the surface orbits. Re-distribution protons of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes d_1 and d_2 protons shells 2s, 2p go to 1h a new composition of the proton appears for these nuclei as follows: The proton distribution on shells for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes when d1=0, d2=0 in the simple shell model (SSM) is given by ($1s_2$, lp_6 , $1d_{10}$, $2s_2$, $1f_{14}$, $2p_6$, $1g_6$, $1h_0$) can be expressed by

$$\rho(r)=\frac{\exp(-x^2)}{\pi^{3/2}b^3}\sum_{m-0}^{4}\xi_m x^{2m}.$$

From the Eq. (3) the proton configuration of these nuclei ξ_m are equal to $\xi_0=5,\ \xi_1=10, \xi_2=-4,\ \xi_3=\frac{8}{3},\ \xi_4=32/315,\ \xi_5=0.$

Either at the proton distribution on shells for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes when d1≠0, d2≠0 in the modified shell model (MSM) and can be expressed by:

$$\rho(r)=\frac{\exp(-x^2)}{\pi^{3/2}b^2}\sum_{m-0}^{5}\xi'_m x^{2m}.$$

A new proton density distribution (PDD) taking into account the core, eroding and gaining shells can be written in the general analytic form the Eq.(3), where the new coefficients $\xi'_m$for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes are equal to

$$\xi'_0=5-\frac{3}{2}d_1,\xi'_1=10+2d_1-\frac{5}{3}d_2,\xi'_2=-4-\frac{2}{3}d_1+\frac{4}{3}d_2,\xi'_3=\frac{8}{3}-\frac{4}{15}d_2,\xi'_4=\frac{32}{315},\xi'_5=\frac{32}{10395}(d_1+d_2)$$

where x= r/b ;b≡ the harmonic oscillator size parameter, which can be chosen so as to imitate the experimental root mean square (RMS) radii of nuclei. The coefficients ξ_m, ξ'_m for (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes which shown above.

We display the values parameters and the experimental values of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes and as well as the values for the FB, the properties the charge density and (RMS) of the nucleus are shown in the Table 1 where we compare (RMS) our values with the experimental paper data note that it coincide with each other where it depend (RMS) on the occupation number of the state of the proton and significantly. it was observed in (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes that the increase of two neutrons caused a slight increase in the (RMS) radius, we also found the possibility of occupation proton of the shells 2s, 2p, 1g and 1h of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes shown in the Table 2. The distribution of the charge density of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes is calculated on the basis of the modified shell model MSM as shown in Fig.1 (a, b, c, d) (solid curves) the occupation was determined by the case where the charge density is correct gave a good description of PDD more model of shell (dashed curves), a small variation between theoretical and experimental is clearly associated with the tail region. In the Fig. 1(a) ^{104}Pd, (b)^{106} Pd, (c) ^{108}Pd, (d) ^{110} Pd nucleus. The computations of the dashed curves shape discord with the experimental data particularly at the region when ($r \leq$ 4.2 fm) and it slightly discord at the region ($2.8 \leq r \leq 3.7$fm) the computations of the solid curves a little discord with the experimental data (solid circles) in the region of r.

Table 1: Parameters employed in the present calculations for charge densities.

Nucleus	Z	Type of CDD[22]	ρ_{exp} (0) (fm^{-3}) [22] p.w.Eq.(4)	b(fm)		$< r^2 >^{\frac{1}{2}}_{exp}$ (fm)[22]	$< r2 >^{1/2}_{cal}$ (fm)		d_1 Obtained from Eq.(5)	d_2 Obtained from Eq.(6)
				ssm	msm		Obtained from Eq.(8)	Obtained from Eq.(9)		
^{104}Pd	46	FB	E-027.337517	2.233	2.171	4.437(10)	4.4361	4.42071	5.4448E-01	3.7530
^{106}Pd	46	FB	E-027.430921	2.246	2.185	4.467(11)	4.4611	4.44877	4.5265E-01	3.9206
^{108}Pd	46	FB	6.798767E-02	2.276	2.215	4.524(10)	4.5154	4.50165	5.8893E-01	3.5623
^{110}Pd	46	FB	E-026.73444	2.285	2.221	4.541(10)	4.5318	4.51737	5.9275E-01	3.7574

Table 2: The occupation probabilities of the shell 2s, 2p, 1g and 1h of the nuclei.

Nucleus	P_{2s}	P_{2p}	P_{1g}	P_{1h}
^{104}Pd	0.7278	0.3745	0.3333	0.1953
^{106}Pd	0.7737	0.3465	0.3333	0.1987
^{108}Pd	0.7055	0.4062	0.3333	0.1885
^{110}Pd	0.7036	0.3737	0.3333	0.1977

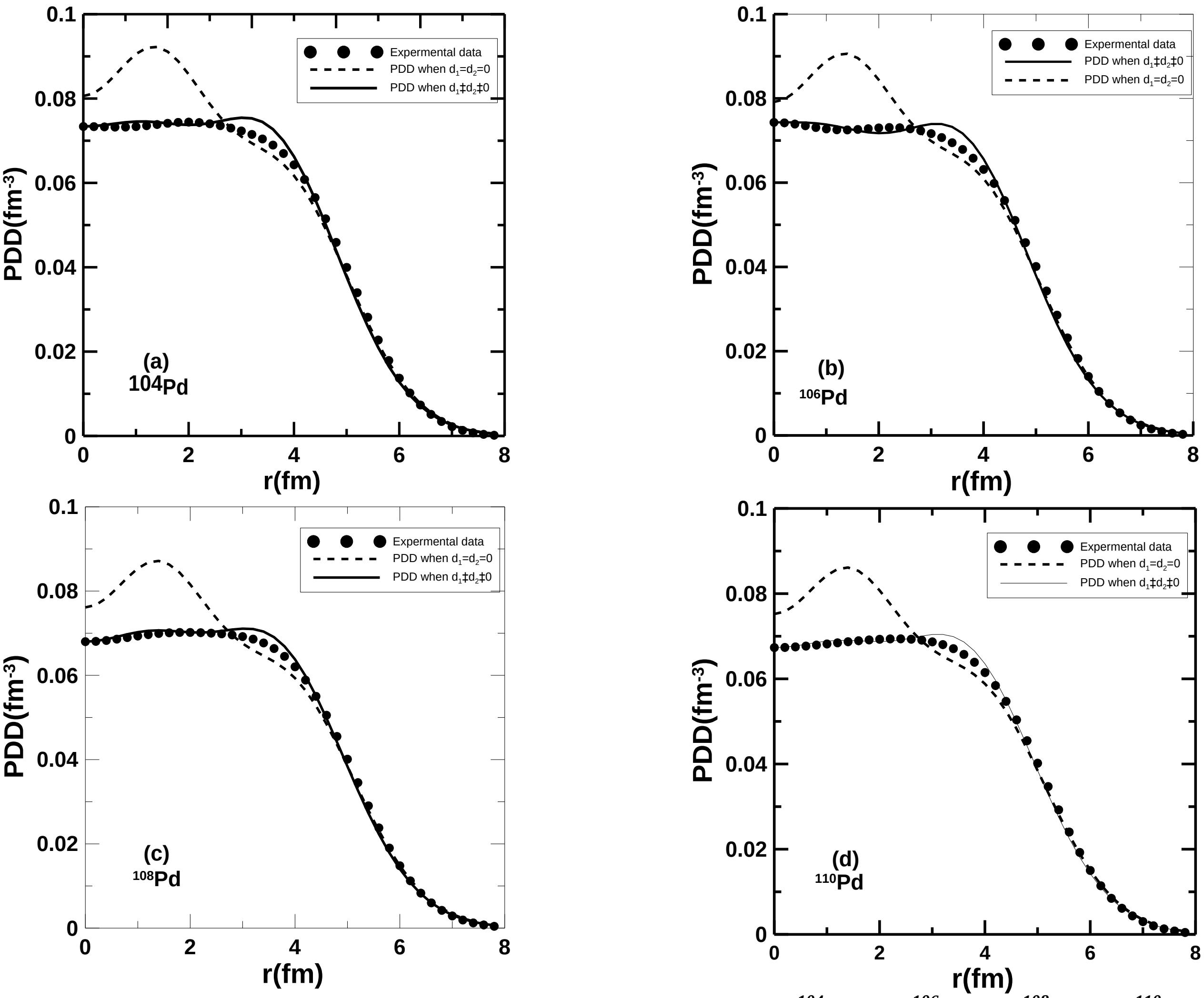

***Fig.1:** Dependence of the PDD (fm-3) on r (fm) for (a) ^{104}Pd, (b) ^{106}Pd, (c) ^{108}Pd, (d) ^{110}Pd nucleus. The dashed and solid curves are the calculated PDD of Eq. (3), when (d1, d2=0) and (d1, d2≠0), respectively. The dotted symbols are the experimental data of (FB) PDD of Ref. [23].*

2. Elastic electron scattering form factor

In Fig. 2 the computed squared of the EESFF of even mass (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd), we present elastic electron scattering form factors that are calculated by (PWBA) on the momentum transfer (q) (in fm^{-1}) for ^{104}pd (Fig. 2(a)), ^{106}pd (Fig. 2(b)), ^{108}pd (Fig. 2(c)) and ^{110}pd (Fig. 2(d)) nuclei. The dotted symbols indicate the experimental data. It is clear from Fig. 2(a-d) that both the dashed and solid curves agree well with the experimental data at the region $q \le 0.75$ fm^{-1} while at higher region $q > 0.75$ fm^{-1}, the data is under predicted noticeably by the dashed curve and slightly by the solid curve. the observed first minimum is very well described by both the solid and dashed curves at $(q \approx 0.859\ \text{fm}^{-1})$ and $(q \approx 0.85\ \text{fm}^{-1})$, respectively, while that of second minimum is located at the correct place by the curve solid and diverged obviously by the dashed curve at $(q \approx 1.6\ \text{fm}^{-1})$ and $(q \approx 1.4\ \text{fm}^{-1})$ respectively. Generally, considering the higher orbitals (the solid curve) in our calculations leads to enhance the computed form factors at the region $q > 0.75$ fm^{-1} which in sequence tends to improve the computed results and make them to be closer to the experimental data. We observe in all regions the momentum transfer along (q) of calculated form factors of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes and both behavior and the magnitudes are reasonable agreement with agreement with the empirical data.

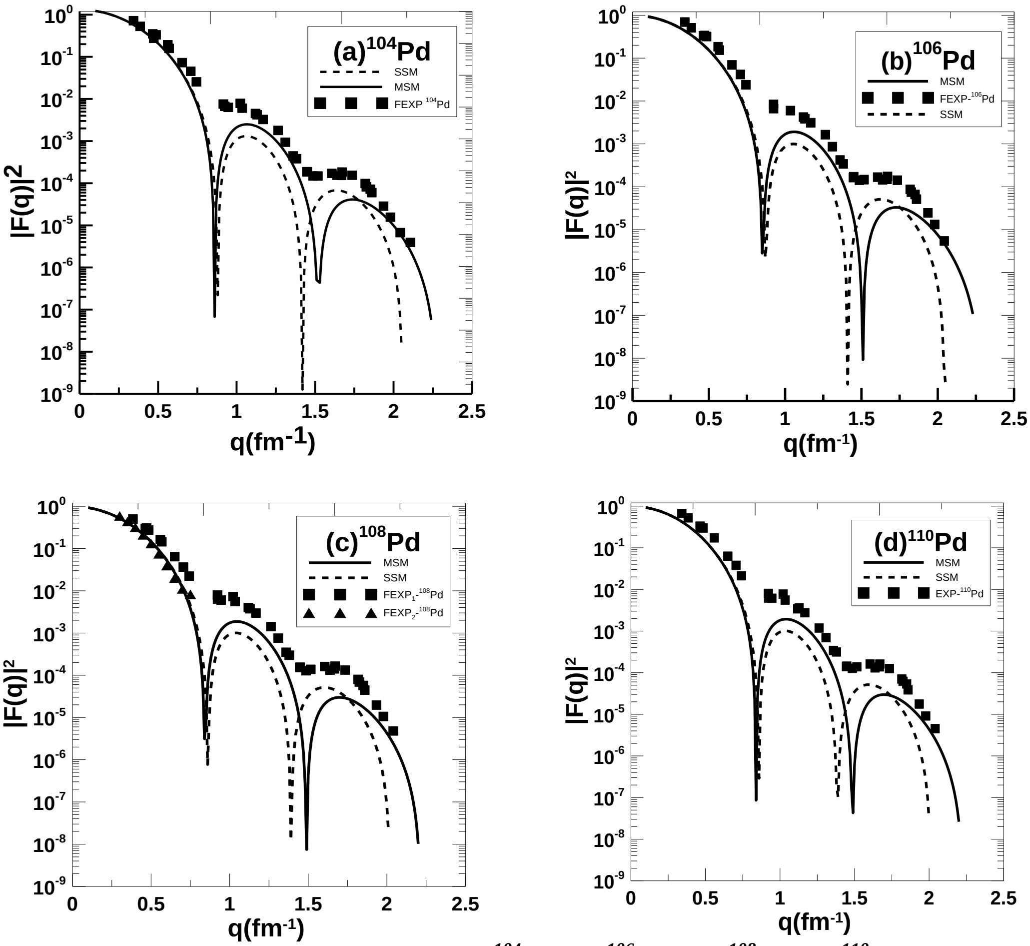

Fig.2: Elastic charge form factor of (a)^{104}Pd, (b)^{106} Pd, (c)^{108}Pd, (d)^{110} Pd nucleus using SSM (dashed curves) and MSM (solid curves) are the calculated charge form factors calculated using methods through Eq.(13) in comparison with the experimental data [23].

3-The difference of proton density distributions

By searching in the proton density distribution in the nuclei, as shown in Fig.1 (a, b, c) clarifying that the addition to the neutrons to the nuclei (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) respectively, leading to a simple change in the distribution of protons because shells the nuclear reactions that will occur between these added neutrons and protons. These interactions with some dwindle in PDD particularly in the central regions of these nuclei by the added neutrons of these isotopes. The difference between (^{106}Pd- ^{104}Pd), (^{108}Pd- ^{106}Pd) and (^{110}Pd- ^{108}Pd) were recalculated to explain that the addition of two neutrons affects the distribution of the proton density as shown in the forms in Fig. 3. We calculated the proton density distribution for the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) with experimental data as shown in Fig. 3 (a, b, c) respectively. This is an important step to calculate the PDD difference between these isotopes. The solid curve represents the difference in the PDD with the dashed curve of the empirical data taken from J.B. Vanderlaan [23].

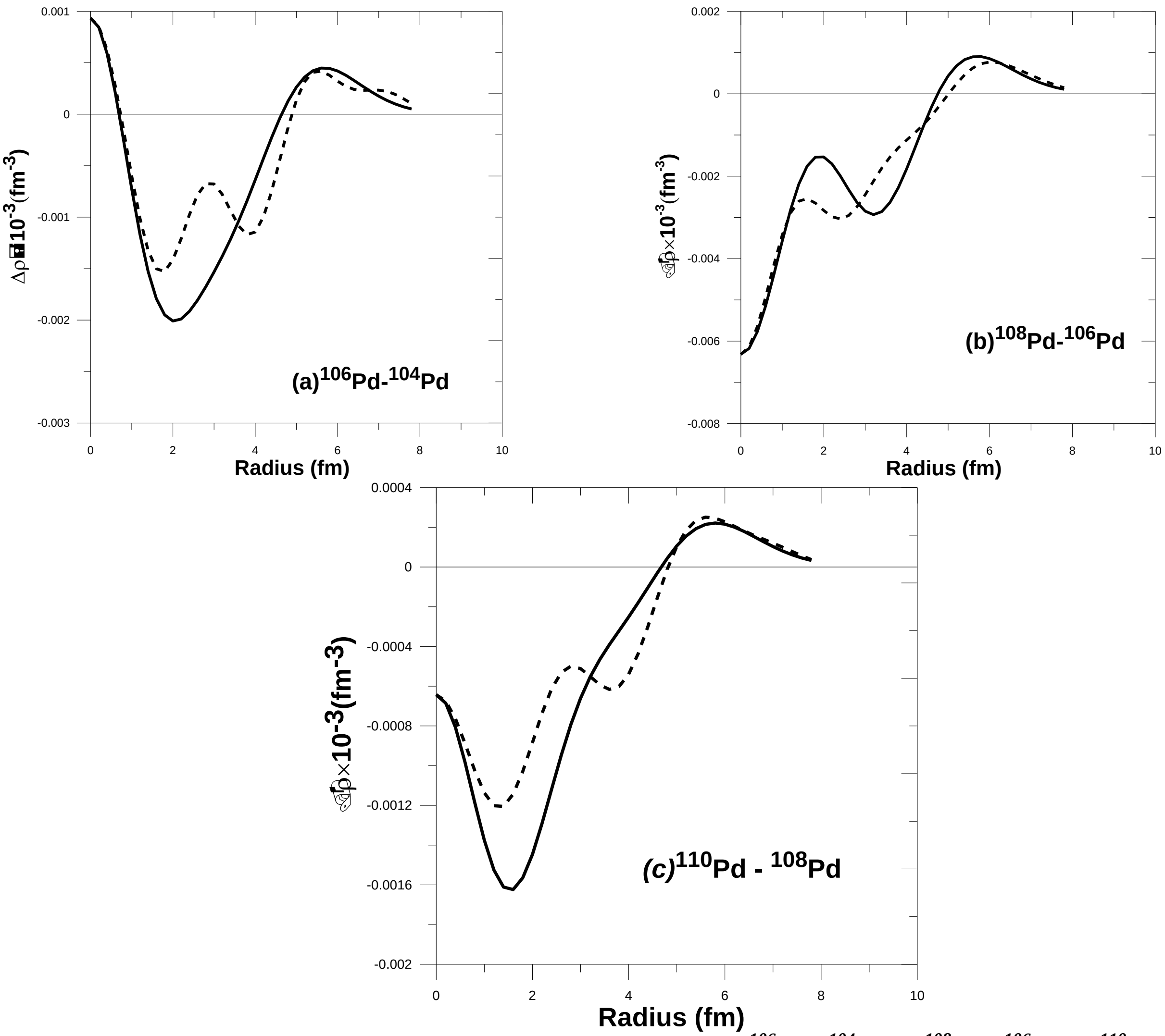

Fig.3: Dependence of the difference of the PDD of (^{106}Pd- ^{104}Pd), (^{108}Pd- ^{106}Pd), (^{110}Pd- ^{108}Pd) isotopes$\Delta\rho(r)$on (r). The solid curves represents the calculated difference of the PDD with (d_1, $d_2 \neq 0$), and the dashed curves are the fitted to the experimental FB data, taken from ref. [23].

Conclusions

The basic results of this paper can be formulated as follows.

a- The proton density distribution of the (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes was calculated on the basis MSM for the probability of occupation of the state. The nuclei have the core filled and eroding shells (2s, 2p), the gaining shell (1g, 1h)) and results for the probability of occupation differed from the expectations of the SSM and agree with the experimental charge density.

b- The calculated elastic electron scattering form factors from (^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd) isotopes are agreement with the fitted to the experimental data.

c- If two neutrons were added to the nucleus, it may be explained by the proton redistribution due to the nuclear interaction between those additional neutrons and the protons.

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