# The nuclear level density parameter

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**Abstract**

The nuclear level density parameter $a$ in non Equi-Spacing Model (NON-ESM), Equi-Spacing Model (ESM) and the Backshifted Energy Dependent Fermi Gas model (BSEDFG) was determined for 106 nuclei; the results are tabulated and compared with the experimental works. It was found that there are no recognizable differences between our results and the experimental $a$-values. The calculated level density parameters have been used in computing the state density as a function of the excitation energies for $^{58}$Fe and $^{246}$Cm nuclei. The results are in a good agreement with the experimental results from earlier published work.

**Key words**

*Exciton model, level density parameter, state density.*

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## معلمة كثافة المستوي النووي

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**الخلاصة**

تم ايجاد معلمة كثافة المستوي النووي $a$ في نموذج المسافة الغير متساوية والمتساوية ونموذج غاز فيرمي المعتمد على الأزاحة المتأخرة للطاقة، لعدد 106 نوى. تم جدولة النتائج والمقارنة مع النتائج العملية وتبين عدم وجود اختلافات واضحة بين نتائجنا والنتائج العملية لحساب قيم $a$ . استخدم حساب معلمة كثافة المستوي النووي في حساب كثافة المستوي كدالة لطاقة التهيج لنظير $^{58}$Fe. وجد هنالك توافق جيد خلال المقارنة بين هذه النتائج والنتائج العملية للأخرين.

## Introduction

The levels of the nucleus can be divided into two energy regions, namely the low energy and high-energy excitations. The low-lying nuclear excited levels are small in the number, well separated, and rather simple in structure. With increasing excitation energy, the spacing between these levels is progressively reduced and the nature of the excitations becomes very complicated. The most relevant quantity describing the statistical nuclear properties is the level density of the system $\omega(E)$, expressed as a function of angular momentum, excitation energy, number of protons and neutrons or simply the mass number. Nuclear level densities are some of the most important quantities in nuclear physics, especially, for nuclear reaction models. The first major study of level densities was conducted by Hans Bethe in 1936 [1], then many experimental calculations for nuclear level density have been made [2-5]. These calculations are based on the formula introduced by Bethe[1]. The most important quantity that is required in calculating the nuclear level

density is the level density parameter $a$. The main challenge in this work is the determination of the level density parameter $a$ which affects the value of the nuclear level density $\omega(E)$.

**Fermi Gas Model**

The simplest system, which does not have equidistant levels, is the Fermi gas system, where the single-particle level density increases with the square root of the kinetic energy of the particles. In this model the nucleons are treated as freely moving non-interacting Fermions in a spherical potential well whose size corresponds to that of the nucleus, and its depth adjusted so that the Fermi energy raises the highest nucleons up to the observed binding energy usually near 8 MeV. In this model, the difference between neutron and proton single particle spacing is neglected, and the particles are supposed to occur with equal probability. The nucleus is considered as a free Fermi gas of neutrons and protons where both are move in a nuclear volume $V$.

The level density parameter ($a$) is related to the mass number $A$ and Fermi energy $E_F$ by the relation [6]:

$$a = \frac{\pi^2}{6} g \qquad (1)$$

where $g$ is the single particle level density and for equispacing model (ESM) it given by [7]:

$$g_0 = \frac{3A}{2E_F} \qquad (2)$$

For the non-equispacing model (NON-ESM) it is given by:

$$g(\varepsilon) = g_0 \sqrt{\frac{\varepsilon}{E_F}} \qquad (3)$$

where $\varepsilon$ is the single particle energy measured from the bottom of the potential well.

Therefore, the only difference between the equidistant and Fermi gas model expressions is the value of the level density parameter $a$.

There have been a number of ideas about the value of $a$, which affects the value of the nuclear level density. One of these ideas is the back shifted Fermi gas level density parameter.

**The level density parameter in BSEDFG model**

The level density parameter in BSEDFG can be represented as [8]:

$$a(E,z,N) = \breve{a}\left[1 + \frac{S(z,N) - \Delta}{E - E_2} f(E - E_2)\right] \qquad (4)$$

where $\breve{a}$ is the asymptotic value of $a$, $S(z,N)$ is the shell correction and it defined as [8]:

$$S(z,N) = M_{\exp} - M_{LD} \qquad (5)$$

$M_{\exp}$ is the experimental mass taken from ref. [9] and $M_{LD}$ is the liquid drop mass estimated using the liquid drop formula:

$$M_{LD}(N,z) = NM_n + zM_p - \frac{E_b}{c^2}$$

where $M_p$ is the proton mass, $M_n$ is the neutron mass and $E_b$ is the binding energy[8]:

$$E_b = -\left\{-15.65 \times A + 17.63A^{2/3} + \frac{3e^2}{5r_0} z^2 A^{-1/3} + A\left(27.72 - 15.6A^{-1/3}\right)\left(\frac{N-z}{A}\right)^2\right\} \qquad (6)$$

where $N$, $z$ and $A$ are the neutron number, proton number and the mass number respectively, $e$ is the electron charge and $r_0 = 1.233\ fm$.

The pairing term was not included in Eq.(6), therefore, it was subtracted from the shell correction, Eq. (4), and it has deferent values depending on whether the nuclei is even-even, odd-odd or odd mass, it was calculated using[8]:

$$\Delta = \begin{cases} +0.5P_d \\ 0 \\ -0.5P_d \end{cases} \quad (7)$$

The function $f(E-E_2)$ can be calculated as:

$$f(E-E_2) = 1-e^{-\gamma(E-E_2)} \quad (8)$$

where $E_2$ is the back shift energy and $\gamma = 0.06\ MeV^{-1}$ [8].

The back shifted energy $E_2$ is defined as [8]:

$$E_2 = \begin{cases} P_1 - 0.5P_d + P_4 \dfrac{dS(z,N)}{dA} & \text{even - even} \\ P_2 - 0.5P_d + P_4 \dfrac{dS(z,N)}{dA} & \text{odd mass} \\ P_3 - 0.5P_d + P_4 \dfrac{dS(z,N)}{dA} & \text{odd - odd} \end{cases} \quad (9)$$

With

$P_1 = -0.477, P_2 = -0.577, P_3 = -0.231$ and $P_4 = 0.442$

The asymptotic value of the level density parameter was calculated using:

$$\frac{\breve{a}}{A} = P_1 + P_3 A \quad (10)$$

with $P_1 = 0.127$ and $P_3 = -9.05\times10^{-5}$ and the deuteron pairing $P_d$ is given by [8]:

$$P_d = \frac{1}{4}(-1)^{Z+1}\left[S_d(A+2,z+1) - 2S_d(A,z) + S_d(A-2,z-1)\right] \quad (11)$$

$S_d$ is the deuteron separation energy.

The level density parameter $a$, resulting from the fitting procedure done in this work, for the experimental results [10], in case of the effective moment of inertia $I_{eff}=0.5\ I_{rig}$, where $I_{rig}$ is the rigid body moment of inertia for different nuclei, is given by :

$$a = 0.32\times Z^5 + 0.75\times Z^4 - 0.67\times Z^3 - 2.9\times Z^2 + 3.9\times Z + 15 \quad (12)$$

where:

$$Z = \frac{A - 1.4\times10^2}{55} \quad (13)$$

**Results, Discussions and Conclusions**

As shown in Eq.(12), an empirical formula for the level density parameter has been formulated and tabulated in Table (1) for even-even, odd-odd and odd mass nuclei starting from z=18 to z=96 for about 106 different nuclei, except for the 209-230 mass nuclei due unavailable experimental data. Table (1) includes the values of the level density parameter taken from the experimental calculations [10], polynomial value of the parameter-*a* which refer to it as $a_{polynomial}$ is calculated in this work. The ESM parameter-*a*, the energy dependent level density parameter (where the single-particle energy value was taken to be 100 MeV and the Fermi energy is 38 MeV) and Finally, BSEDFG level density parameter, are tabulated in Table (1) and sketched in Fig.1.

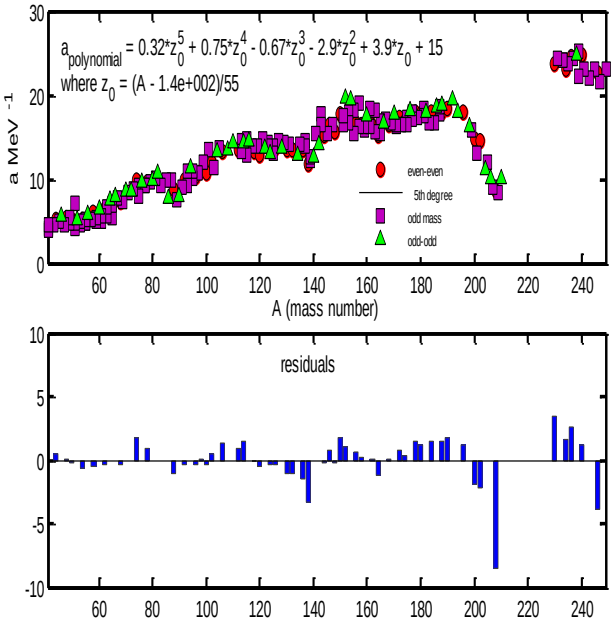


***Fig.1: The level density parameter a extracted from the data tabulated in ref.[10] with the polynomial fitting of the data.***

To give a preliminary survey Fig. 2 shows the level density parameter $a$ as a function of the mass number for even-even, odd-odd and odd-mass nuclei, where the results of the present empirical evaluations are shown as triangles, squares and circles, the full line shows the fitting of these experimental results. The dashed line represents the energy dependent level density parameter, the dashed dotted line gives the results for the case of energy independent level density parameter, and finally the back shifted energy dependent level density parameter.

Referring to this figure one can pointed out no recognizable differences between the experimental and BSEDFG *a*-values for even-even, odd-mass and odd-odd nuclei, due to the pairing correction $P_d$ that was added to the calculations. Also, we will find no recognizable differences in the *a*-values for even, odd-mass and odd nuclei; Even the slight decrease of the *a*-values at masses between 150 and 185. Fig. 2 is expected as the result of the decreased single particle level density with increasing deformation. An error may be noticed in the figure come from the contribution of strong p-levels to the number of observed resonances even at low neutron energy in those mass ranges where the p-wave strength considerably exceeds the s-wave strength $(A \cong 90, A \cong 140 and A \cong 240)$ the level density parameter calculated using Eqs. (2 and 3) give a linear dependence on the mass number.

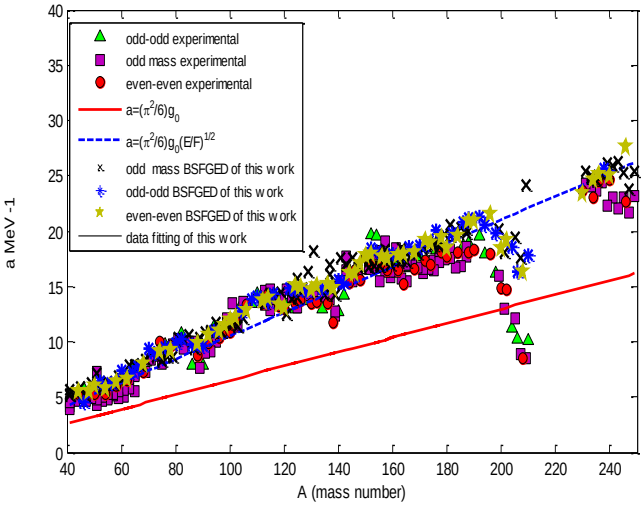


***Fig.2: The level density parameter as a function of the mass number for even-even, odd-odd and odd mass nuclei taken from the experimental results of ref.[10] compared with the energy dependent level density parameter with $\varepsilon = 100$ MeV, the energy independent level density parameter and the BSFG ED level density parameter calculated in this work and taken from Table 1.***

For the case of including the single-particle energy, the increasing of the particle energy increases the level density parameter, while the energy independent level density parameter gives a constant value for each nucleus, which may give unacceptable results. The level density parameter that calculated in this work which depends on the incident particle energy gives accurate results and close to the experimental results, this is clear and can be seen when applying the level density parameter value calculated in this work in the estimation of the state density as seen later in Fig. 3.

For the purpose of making sure the present calculations are matching with the experimental works of others, Fig. 3 shows the detail of these comparisons with experimental results for the state density, taken from ref. [8], and that estimated by using the level density parameter values in Table 1. The following form of the total state density was adopted [10]:

$$\omega(E) \cong \frac{1}{12\sqrt{2}} \frac{\exp\left[2\left[a(E-\Delta)^{1/2}\right]\right]}{\sigma a^{1/4}(E-\Delta+t)^{5/4}} \qquad (15)$$

With:

$$\sigma^2 = \frac{I_{rigid} t}{\hbar^2} \cong 0.0150 A^{5/3} t \qquad (16)$$

The values of $\Delta$ and t are 0.66 [12], and 1.0114 [13] respectively for $^{58}$Fe and equal 0.2 and 0.5721 for $^{246}$Cm. A close agreement is seen between our results and the experimental results of the state density as a function of the excitation energy for energies up to 25 MeV. Using the energy independent level density parameter gives results very far from the practical results, that is why it is preferable not to use the energy independent level density parameter in the calculation of the state density in order to get the correct results.

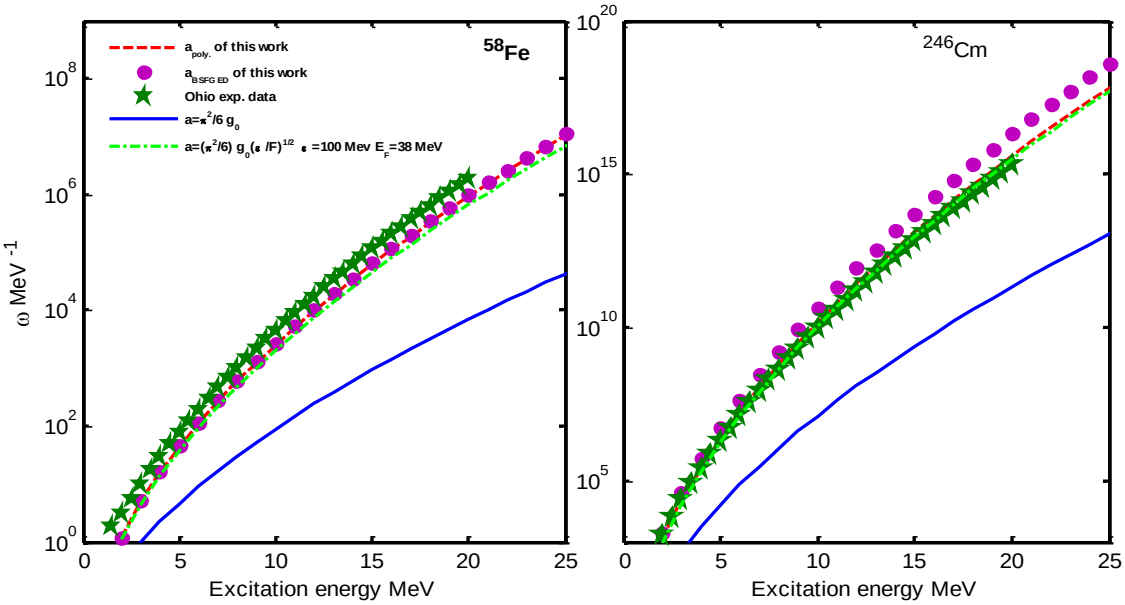


***Fig.3: The state density calculated for $^{58}Fe$ and $^{246}Cm$ as a function of excitation energy with different values of the level density parameter taken from Table 1. All the results are compared with the experimental data of the state density [11].***

***Table 1: The level density parameter-a calculated in this work for various 106 nuclei using different methods.***

| Even-even nuclei | A | $a_{\exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}$ (present work) |
|---|---|---|---|---|---|---|
| $_{20}$Ca | 44 | 5.29 | 4.69747832 | 2.85409474 | 4.629953043 | 5.50148736 |
| $_{22}$Ti | 48 | 5.37 | 5.17900673 | 3.11355789 | 5.050857865 | 5.52146137 |
| | 50 | 5.26 | 5.4116378 | 3.24328947 | 5.261310276 | 6.02975355 |
| $_{24}$Cr | 54 | 5.28 | 5.86514761 | 3.50275263 | 5.682215098 | 5.76140283 |
| $_{26}$Fe | 58 | 5.98 | 6.30810296 | 3.76221579 | 6.10311992 | 6.3415782 |
| $_{28}$Ni | 62 | 6.48 | 6.74561987 | 4.02167895 | 6.524024742 | 6.65503324 |
| $_{30}$Zn | 68 | 7.25 | 7.40020635 | 4.41087368 | 7.155381975 | 8.00875581 |
| $_{32}$Ge | 74 | 9.96 | 8.0606976 | 4.80006842 | 7.786739208 | 9.09099973 |
| $_{34}$Se | 78 | 9.66 | 8.50683111 | 5.05953158 | 8.20764403 | 9.254572 |
| $_{38}$Sr | 88 | 8.75 | 9.6442271 | 5.70818947 | 9.259906085 | 9.83765017 |
| $_{40}$Zr | 92 | 9.92 | 10.1060055 | 5.96765263 | 9.680810907 | 10.6224629 |

| Even-even nuclei | A | $a_{\exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}$ (present work) |
|---|---|---|---|---|---|---|
| $_{42}$Mo | 96 | 10.37 | 10.5693824 | 6.22711579 | 10.10171573 | 11.0340466 |
| | 98 | 11.05 | 10.8010097 | 6.35684737 | 10.31216814 | 11.4501553 |
| $_{44}$Ru | 100 | 10.85 | 11.0321965 | 6.48657895 | 10.52262055 | 12.0379756 |
| | 102 | 12.02 | 11.2626176 | 6.61631053 | 10.73307296 | 12.2459708 |
| $_{46}$Pd | 106 | 13.35 | 11.7197779 | 6.87577368 | 11.15397778 | 12.8785042 |
| $_{48}$Cd | 112 | 13.5 | 12.3907793 | 7.26496842 | 11.78533502 | 13.6744365 |
| | 114 | 14.36 | 12.6089769 | 7.3947 | 11.99578743 | 14.0274351 |
| $_{50}$Sn | 118 | 13.28 | 13.0348032 | 7.65416316 | 12.41669225 | 13.3750416 |
| | 120 | 12.96 | 13.2416427 | 7.78389474 | 12.62714466 | 13.1627797 |
| $_{52}$Te | 124 | 13.53 | 13.6412326 | 8.04335789 | 13.04804948 | 15.0849397 |
| | 126 | 13.77 | 13.8332288 | 8.17308947 | 13.25850189 | 14.9903118 |
| $_{54}$Xe | 130 | 13.46 | 14.1998244 | 8.43255263 | 13.67940672 | 14.7373784 |
| | 132 | 13.59 | 14.3737486 | 8.56228421 | 13.88985913 | 15.0480592 |
| $_{56}$Ba | 136 | 13.54 | 14.7013029 | 8.82174737 | 14.31076395 | 15.1558631 |
| | 138 | 11.76 | 14.8543806 | 8.95147895 | 14.52121636 | 14.8659909 |
| $_{60}$Nd | 144 | 15.31 | 15.2680614 | 9.34067368 | 15.15257359 | 16.2113957 |
| | 146 | 16.47 | 15.3901835 | 9.47040526 | 15.363026 | 16.2228746 |
| $_{62}$Sm | 148 | 15.63 | 15.5042121 | 9.60013684 | 15.57347842 | 17.11393 |
| | 150 | 17.72 | 15.6100793 | 9.72986842 | 15.78393083 | 17.8295009 |
| | 152 | 17.04 | 15.7077585 | 9.8596 | 15.99438324 | 17.9051928 |
| $_{64}$Gd | 156 | 16.79 | 15.8786673 | 10.1190632 | 16.41528806 | 17.6566381 |
| | 158 | 16.42 | 15.9520718 | 10.2487947 | 16.62574047 | 17.5527322 |
| $_{66}$Dy | 162 | 16.44 | 16.0755968 | 10.5082579 | 17.04664529 | 17.8704107 |
| | 164 | 15.23 | 16.1262057 | 10.6379895 | 17.2570977 | 17.9941847 |
| $_{68}$Er | 168 | 16.58 | 16.2067705 | 10.8974526 | 17.67800253 | 18.0873278 |
| $_{70}$Yb | 172 | 17.3 | 16.2627243 | 11.1569158 | 18.09890735 | 19.2029519 |
| | 174 | 16.9 | 16.2828158 | 11.2866474 | 18.30935976 | 18.9371323 |
| $_{72}$Hf | 178 | 18.06 | 16.3105234 | 11.5461105 | 18.73026458 | 19.4614311 |
| | 180 | 17.76 | 16.3196784 | 11.6758421 | 18.94071699 | 18.4981708 |
| $_{74}$W | 184 | 18.08 | 16.3330176 | 11.9353053 | 19.36162181 | 19.5080585 |
| $_{76}$Os | 188 | 18.07 | 16.3465804 | 12.1947684 | 19.78252664 | 20.9440512 |
| | 190 | 18.37 | 16.3563343 | 12.3245 | 19.99297905 | 20.9111294 |

| Even-even nuclei | A | $a_{\exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}$ (present work) |
|---|---|---|---|---|---|---|
| $_{78}$Pt | 196 | 17.95 | 16.4135013 | 12.7136947 | 20.62433628 | 21.5174676 |
| $_{80}$Hg | 200 | 14.86 | 16.4901024 | 12.9731579 | 21.0452411 | 18.4552726 |
| | 202 | 14.68 | 16.5450396 | 13.1028895 | 21.25569351 | 19.2160412 |
| $_{82}$Pb | 208 | 8.53 | 16.7995572 | 13.4920842 | 21.88705075 | 16.3075695 |
| $_{90}$Th | 230 | 23.62 | 19.8127868 | 14.9191316 | 24.20202727 | 23.3804865 |
| $_{92}$U | 234 | 23 | 20.9152557 | 15.1785947 | 24.62293209 | 24.6700114 |
| | 236 | 24.58 | 21.5549377 | 15.3083263 | 24.8333845 | 25.2157611 |
| $_{94}$Pu | 240 | 24.72 | 23.0314807 | 15.5677895 | 25.25428932 | 25.0089065 |
| $_{96}$Cm | 246 | 22.61 | 25.8046002 | 15.9569842 | 25.88564656 | 27.6892429 |

| Odd-odd Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{21}$Sc | 46 | 5.72 | 4.9412812 | 2.98382632 | 4.840405454 | 4.59567797 |
| $_{23}$V | 52 | 5.24 | 5.6400768 | 3.37302105 | 5.471762687 | 6.35421687 |
| $_{25}$Mn | 56 | 5.83 | 6.0875981 | 3.63248421 | 5.892667509 | 6.33120716 |
| $_{27}$Co | 60 | 6.41 | 6.5272652 | 3.89194737 | 6.313572331 | 7.36658187 |
| $_{29}$Cu | 64 | 7.55 | 6.9636352 | 4.15141053 | 6.734477153 | 7.32868565 |
| | 66 | 7.94 | 7.1817162 | 4.28114211 | 6.944929564 | 7.76507802 |
| $_{31}$Ga | 70 | 8.53 | 7.6193901 | 4.54060526 | 7.365834386 | 9.41714445 |
| | 72 | 8.68 | 7.8394958 | 4.67033684 | 7.576286797 | 9.02410688 |
| $_{33}$As | 76 | 9.78 | 8.2831181 | 4.9298 | 7.997191619 | 9.48550276 |
| $_{35}$Br | 80 | 9.94 | 8.7318633 | 5.18926316 | 8.418096441 | 9.86274546 |
| | 82 | 10.72 | 8.9581975 | 5.31899474 | 8.628548852 | 10.2368361 |
| $_{37}$Rb | 86 | 7.84 | 9.4144962 | 5.57845789 | 9.049453674 | 9.77488798 |
| $_{39}$Y | 90 | 7.93 | 9.8747974 | 5.83792105 | 9.470358496 | 9.63092983 |
| $_{41}$Nb | 94 | 11.42 | 10.33762 | 6.09738421 | 9.891263318 | 11.0268624 |
| $_{45}$Rh | 104 | 13.35 | 11.491930 | 6.74604211 | 10.94352537 | 12.6915937 |
| $_{47}$Ag | 108 | 13.49 | 11.945789 | 7.00550526 | 11.3644302 | 13.814527 |
| | 110 | 14.39 | 12.169586 | 7.13523684 | 11.57488261 | 14.0946985 |
| $_{49}$In | 114 | 14.46 | 12.608976 | 7.3947 | 11.99578743 | 13.5640243 |
| | 116 | 14.53 | 12.823783 | 7.52443158 | 12.20623984 | 14.3401311 |

| Odd-odd Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{51}$Sb | 122 | 13.72 | 13.443913 | 7.91362632 | 12.83759707 | 14.8739111 |
| | 124 | 12.99 | 13.641232 | 8.04335789 | 13.04804948 | 14.7635785 |
| $_{53}$I | 128 | 13.73 | 14.019541 | 8.30282105 | 13.46895431 | 14.626706 |
| $_{55}$Cs | 134 | 12.95 | 14.541004 | 8.69201579 | 14.10031154 | 14.8392116 |
| $_{57}$La | 140 | 12.68 | 15 | 9.08121053 | 14.73166877 | 15.6008468 |
| $_{59}$Pr | 142 | 14.19 | 15.137952 | 9.21094211 | 14.94212118 | 15.3496082 |
| $_{63}$Eu | 152 | 19.66 | 15.707758 | 9.8596 | 15.99438324 | 18.4297763 |
| | 154 | 19.55 | 15.797266 | 9.98933158 | 16.20483565 | 18.1883277 |
| $_{65}$Tb | 160 | 17.46 | 16.017642 | 10.3785263 | 16.83619288 | 17.9033118 |
| $_{67}$Ho | 166 | 16.74 | 16.169799 | 10.7677211 | 17.46755011 | 18.5551092 |
| $_{69}$Tm | 170 | 17.68 | 16.237571 | 11.0271842 | 17.88845494 | 18.6996719 |
| $_{71}$Lu | 176 | 18.27 | 16.298504 | 11.4163789 | 18.51981217 | 20.0597577 |
| $_{73}$Ta | 182 | 18 | 16.326854 | 11.8055737 | 19.1511694 | 19.7648887 |
| $_{75}$Re | 186 | 18.64 | 16.339215 | 12.0650368 | 19.57207423 | 20.7978174 |
| | 188 | 18.92 | 16.346580 | 12.1947684 | 19.78252664 | 21.1246786 |
| $_{77}$Ir | 192 | 19.56 | 16.369787 | 12.4542316 | 20.20343146 | 21.2498897 |
| | 194 | 17.99 | 16.388343 | 12.5839632 | 20.41388387 | 20.4985626 |
| $_{79}$Au | 198 | 16.26 | 16.446855 | 12.8434263 | 20.83478869 | 19.8182868 |
| $_{81}$Tl | 204 | 11.26 | 16.613569 | 13.2326211 | 21.46614592 | 18.5632163 |
| | 206 | 10.21 | 16.697702 | 13.3623526 | 21.67659834 | 16.3255224 |
| $_{83}$Bi | 210 | 10.12 | 16.921365 | 13.6218158 | 22.09750316 | 17.8411221 |
| $_{93}$Np | 238 | 24.71 | 22.258944 | 15.4380579 | 25.04383691 | 25.6354378 |

| Odd-mass Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{18}$Ar | 41 | 4.52 | 4.3180224 | 2.6594973 | 4.314274426 | 5.142583 |
| $_{19}$K | 41 | 3.9 | 4.3180224 | 2.6594973 | 4.314274426 | 5.232031 |
| $_{20}$Ca | 41 | 4.12 | 4.3180224 | 2.65949737 | 4.314274426 | 5.680418 |
| | 43 | 4.63 | 4.57296784 | 2.78922895 | 4.524726837 | 5.357583 |
| | 45 | 4.92 | 4.82020416 | 2.91896053 | 4.735179248 | 5.849749 |
| $_{22}$Ti | 47 | 4.68 | 5.06084032 | 3.04869211 | 4.945631659 | 5.006182 |
| | 49 | 5.37 | 5.2959008 | 3.17842368 | 5.15608407 | 6.221961 |
| $_{23}$V | 49 | 4.85 | 5.2959008 | 3.17842368 | 5.15608407 | …… |
| | 51 | 7.23 | 5.52632801 | 3.30815526 | 5.366536481 | 7.12776 |
| $_{24}$Cr | 51 | 4.76 | 5.52632801 | 3.30815526 | 5.366536481 | 6.130819 |
| | 53 | 5.09 | 5.75298473 | 3.43788684 | 5.576988892 | 5.827369 |
| | 55 | 5.22 | 5.97665659 | 3.56761842 | 5.787441303 | 6.152112 |

| Odd-mass Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{25}$Mn | 51 | 4.23 | 5.52632801 | 3.30815526 | 5.366536481 | …… |
| | 53 | 4.64 | 5.75298473 | 3.43788684 | 5.576988892 | …… |
| | 55 | 4.74 | 5.97665659 | 3.56761842 | 5.787441303 | …… |
| $_{26}$Fe | 55 | 4.87 | 5.97665659 | 3.56761842 | 5.787441303 | 6.347724 |
| | 57 | 5.22 | 6.19805446 | 3.69735 | 5.997893714 | 6.170524 |
| $_{27}$Co | 55 | 4.9 | 5.97665659 | 3.56761842 | 5.787441303 | …… |
| | 57 | 5.21 | 6.19805446 | 3.69735 | 5.997893714 | …… |
| | 59 | 5.5 | 6.41781692 | 3.82708158 | 6.208346125 | …… |
| $_{28}$Ni | 59 | 4.91 | 6.41781692 | 3.82708158 | 6.208346125 | 6.85504 |
| | 61 | 5.69 | 6.63651273 | 3.95681316 | 6.418798536 | 6.948118 |
| | 63 | 6.67 | 6.8546432 | 4.08654474 | 6.629250947 | 7.857594 |
| | 65 | 6.9 | 7.07264469 | 4.21627632 | 6.839703358 | 7.91924 |
| $_{29}$Cu | 61 | 5.03 | 6.63651273 | 3.95681316 | 6.418798536 | …… |
| | 63 | 5.74 | 6.8546432 | 4.08654474 | 6.629250947 | …… |
| | 65 | 5.47 | 7.07264469 | 4.21627632 | 6.839703358 | …… |
| $_{30}$Zn | 65 | 7.07 | 7.07264469 | 4.21627632 | 6.839703358 | 7.837973 |
| | 67 | 7.74 | 7.29089103 | 4.34600789 | 7.050155769 | 7.882864 |
| | 69 | 7.32 | 7.50969597 | 4.47573947 | 7.26060818 | 8.346354 |
| $_{32}$Ge | 71 | 8.33 | 7.72931561 | 4.60547105 | 7.471060591 | 9.000851 |
| | 73 | 8.74 | 7.94995084 | 4.73520263 | 7.681513002 | 8.681587 |
| | 75 | 8.03 | 8.17174982 | 4.86493421 | 7.891965413 | 8.296081 |
| | 77 | 9.26 | 8.39481035 | 4.99466579 | 8.102417824 | 8.688087 |
| $_{34}$Se | 75 | 9.29 | 8.17174982 | 4.86493421 | 7.891965413 | 9.221092 |
| | 77 | 9.25 | 8.39481035 | 4.99466579 | 8.102417824 | 9.143392 |
| | 79 | 9.78 | 8.61918238 | 5.12439737 | 8.312870235 | 8.634957 |
| | 81 | 10.2 | 8.84487042 | 5.25412895 | 8.523322646 | 10.35598 |
| | 83 | 9.35 | 9.07183597 | 5.38386053 | 8.733775057 | 10.11008 |
| $_{38}$Sr | 85 | 9.95 | 9.3 | 5.51359211 | 8.944227469 | 10.58398 |
| | 87 | 9.97 | 9.52924536 | 5.64332368 | 9.15467988 | 9.352705 |
| | 89 | 7.63 | 9.75941922 | 5.77305526 | 9.365132291 | 10.32116 |
| $_{40}$Zr | 91 | 9 | 9.99033553 | 5.90278684 | 9.575584702 | 10.81066 |
| | 93 | 10.89 | 10.2217775 | 6.03251842 | 9.786037113 | 10.90037 |
| | 95 | 10.99 | 10.4534998 | 6.16225 | 9.996489524 | 11.68625 |
| $_{42}$Mo | 93 | 9.19 | 10.2217775 | 6.03251842 | 9.786037113 | 10.96484 |
| | 95 | 10.02 | 10.4534998 | 6.16225 | 9.996489524 | 10.93223 |
| | 97 | 10.76 | 10.6852315 | 6.29198158 | 10.20694193 | 10.90001 |
| | 99 | 12.05 | 10.9166781 | 6.42171316 | 10.41739435 | 11.62667 |
| | 101 | 13.48 | 11.1475238 | 6.55144474 | 10.62784676 | 11.59007 |
| $_{44}$Ru | 103 | 11.37 | 11.3774347 | 6.68117632 | 10.83829917 | 11.36869 |
| | 105 | 13.63 | 11.6060603 | 6.81090789 | 11.04875158 | 12.15483 |

| Odd-mass Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{48}$Cd | 113 | 13.36 | 12.5002773 | 7.32983421 | 11.89056122 | 13.45191 |
| | 115 | 14.74 | 12.7168288 | 7.45956579 | 12.10101363 | 14.2347 |
| $_{50}$Sn | 113 | 14.58 | 12.5002773 | 7.32983421 | 11.89056122 | 14.0506 |
| | 115 | 12.85 | 12.7168288 | 7.45956579 | 12.10101363 | 14.30474 |
| | 117 | 13.84 | 12.9297913 | 7.58929737 | 12.31146604 | 14.15689 |
| | 119 | 13.92 | 13.1387699 | 7.71902895 | 12.52191846 | 14.41021 |
| | 121 | 14.77 | 13.3433731 | 7.84876053 | 12.73237087 | 12.54215 |
| | 123 | 13.95 | 13.5432152 | 7.97849211 | 12.94282328 | 13.80684 |
| | 125 | 13.09 | 13.7379189 | 8.10822368 | 13.15327569 | 13.93031 |
| $_{52}$Te | 123 | 14.33 | 13.5432152 | 7.97849211 | 12.94282328 | 14.90146 |
| | 125 | 13.9 | 13.7379189 | 8.10822368 | 13.15327569 | 15.73198 |
| | 127 | 14.61 | 13.9271175 | 8.23795526 | 13.3637281 | 14.81972 |
| | 129 | 14.26 | 14.1104576 | 8.36768684 | 13.57418051 | 16.36709 |
| | 131 | 14.16 | 14.2876012 | 8.49741842 | 13.78463292 | 18.25256 |
| $_{56}$Ba | 135 | 14.1 | 14.6220402 | 8.75688158 | 14.20553774 | 14.24978 |
| | 137 | 13.86 | 14.7787598 | 8.88661316 | 14.41599016 | 16.13866 |
| | 139 | 12.76 | 14.9281363 | 9.01634474 | 14.62644257 | 15.34556 |
| $_{57}$La | 139 | 12.34 | 14.9281363 | 9.01634474 | 14.62644257 | 14.25224 |
| $_{58}$Ce | 137 | 14.65 | 14.7787598 | 8.88661316 | 14.41599016 | 16.96715 |
| | 141 | 15.43 | 15.0699465 | 9.14607632 | 14.83689498 | 17.53128 |
| | 143 | 16.51 | 15.2039972 | 9.27580789 | 15.04734739 | 16.91696 |
| $_{60}$Nd | 143 | 17.67 | 15.2039972 | 9.27580789 | 15.04734739 | 16.98009 |
| | 145 | 15.47 | 15.3301283 | 9.40553947 | 15.2577998 | 17.63495 |
| | 147 | 16.4 | 15.4482146 | 9.53527105 | 15.46825221 | 16.6986 |
| | 151 | 17.59 | 15.6599424 | 9.79473421 | 15.88915703 | 16.69925 |
| $_{62}$Sm | 151 | 16.56 | 15.6599424 | 9.79473421 | 15.88915703 | 17.34688 |
| | 153 | 16.69 | 15.7535312 | 9.92446579 | 16.09960944 | 16.86351 |
| | 155 | 15.49 | 15.8389747 | 10.0541974 | 16.31006185 | 16.86013 |
| $_{64}$Gd | 153 | 18.01 | 15.7535312 | 9.92446579 | 16.09960944 | 17.95326 |
| | 155 | 18.2 | 15.8389747 | 10.0541974 | 16.31006185 | 18.24469 |
| | 157 | 16.35 | 15.9163601 | 10.1839289 | 16.52051427 | 17.77053 |
| | 159 | 16.38 | 15.9858242 | 10.3136605 | 16.73096668 | 17.50515 |
| | 161 | 16.3 | 16.0475563 | 10.4433921 | 16.94141909 | 17.48096 |
| $_{66}$Dy | 157 | 19.08 | 15.9163601 | 10.1839289 | 16.52051427 | 18.36803 |
| | 159 | 15.84 | 15.9858242 | 10.3136605 | 16.73096668 | 16.90873 |
| | 161 | 18.41 | 16.0475563 | 10.4433921 | 16.94141909 | 17.63037 |
| | 163 | 15.68 | 16.1018001 | 10.5731237 | 17.1518715 | 17.34935 |
| | 165 | 15.42 | 16.1488566 | 10.7028553 | 17.36232391 | 17.23237 |

| Odd-mass Nuclei | A | $a_{exp.}$ $I_{eff}=0.5I_{rigid}$ [10] | $a_{polynomial}$ (present work) | $a=\left(\frac{\pi^2}{6}\right)g_0$ $g_0=3A/2E_F$ | $a=\left(\frac{\pi^2}{6}\right)g_0\sqrt{\varepsilon/E_F}$ $\varepsilon=100MeV$ $E_F=38MeV$ | $a_{BSFGED}(MeV^{-1})$ (present work) |
|---|---|---|---|---|---|---|
| $_{86}$Er | 163 | 18.21 | 16.1018001 | 10.5731237 | 17.1518715 | 18.47276 |
| | 165 | 17 | 16.1488566 | 10.7028553 | 17.36232391 | 17.98798 |
| | 167 | 17.07 | 16.1890862 | 10.8325868 | 17.57277632 | 18.33755 |
| | 169 | 16.73 | 16.2229114 | 10.9623184 | 17.78322873 | 18.01162 |
| | 171 | 16.59 | 16.2508192 | 11.09205 | 17.99368114 | 18.36604 |
| $_{70}$Yb | 171 | 16.24 | 16.2508192 | 11.09205 | 17.99368114 | 18.59466 |
| | 173 | 16.59 | 16.2733632 | 11.2217816 | 18.20413355 | 18.91443 |
| | 175 | 16.51 | 16.2911666 | 11.3515132 | 18.41458596 | 18.67521 |
| | 177 | 17.16 | 16.3049244 | 11.4812447 | 18.62503838 | 18.94611 |
| $_{71}$Lu | 177 | 18.04 | 16.3049244 | 11.4812447 | 18.62503838 | 18.62813 |
| $_{72}$Hf | 175 | 17.33 | 16.2911666 | 11.3515132 | 18.41458596 | 19.06473 |
| | 177 | 17.77 | 16.3049244 | 11.4812447 | 18.62503838 | 19.47821 |
| | 179 | 17.44 | 16.3154054 | 11.6109763 | 18.83549079 | 19.6769 |
| | 181 | 17.61 | 16.3234554 | 11.7407079 | 19.0459432 | 20.50287 |
| $_{74}$W | 181 | 18.35 | 16.3234554 | 11.7407079 | 19.0459432 | 19.51901 |
| | 183 | 16.74 | 16.3299993 | 11.8704395 | 19.25639561 | 20.02515 |
| | 185 | 16.8 | 16.3360432 | 12.0001711 | 19.46684802 | 20.30986 |
| | 187 | 18.57 | 16.3426776 | 12.1299026 | 19.67730043 | 20.16339 |
| $_{76}$Os | 187 | 17.71 | 16.3426776 | 12.1299026 | 19.67730043 | 19.86439 |
| $_{80}$Hg | 199 | 15.92 | 16.4671325 | 12.9082921 | 20.9400149 | 20.24573 |
| | 201 | 13 | 16.5159933 | 13.0380237 | 21.15046731 | 18.01871 |
| $_{82}$Pb | 205 | 12.09 | 16.6535558 | 13.2974868 | 21.57137213 | 19.42503 |
| | 207 | 8.94 | 16.7462777 | 13.4272184 | 21.78182454 | 17.53924 |
| | 209 | 8.48 | 16.8578231 | 13.55695 | 21.99227695 | 24.10089 |
| $_{90}$Th | 231 | 24.33 | 20.0675084 | 14.9839974 | 24.30725347 | 25.42159 |
| | 233 | 24.07 | 20.6182632 | 15.1137289 | 24.51770588 | 25.00017 |
| $_{92}$U | 233 | 24.2 | 20.6182632 | 15.1137289 | 24.51770588 | 24.83092 |
| | 235 | 23.69 | 21.2273131 | 15.2434605 | 24.7281583 | 24.57222 |
| | 237 | 24.37 | 21.8986411 | 15.3731921 | 24.93861071 | 25.15343 |
| | 239 | 25.11 | 22.6363776 | 15.5029237 | 25.14906312 | 26.09505 |
| $_{94}$Pu | 239 | 22.28 | 22.6363776 | 15.5029237 | 25.14906312 | 25.21975 |
| | 241 | 23.01 | 23.4448029 | 15.6326553 | 25.35951553 | 25.89626 |
| $_{95}$Am | 243 | 22.05 | 24.3283496 | 15.7623868 | 25.56996794 | 26.2458 |
| $_{96}$Cm | 245 | 23 | 25.2916051 | 15.8921184 | 25.78042035 | 25.29426 |
| | 247 | 21.65 | 26.3393141 | 16.02185 | 25.99087276 | 23.8386 |
| | 249 | 23.14 | 27.4763807 | 16.1515816 | 26.20132517 | 25.4437 |

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