Nuclear level density with proton resonance using Gaussian orthogonal ensemble theory

M.H.Jasim, Z. A.Dakhil, R.J.Kadhim

Department of physics, college of science, University of Baghdad

E-mail: rasha_alwan@yahoo.com

Abstract

The Gaussian orthogonal ensemble (GOE) version of the random matrix theory (RMT) has been used to study the level density following up the proton interaction with ^{44}Ca,^{48}Ti and ^{56}Fe.

A promising analysis method has been implemented based on the available data of the resonance spacing, where widths are associated with Porter Thomas distribution. The calculated level density for the compound nuclei ^{45}Sc,^{49}Vand ^{57}Co shows a parity and spin dependence, where for Sc a discrepancy in level density distinguished from this analysis probably due to the spin misassignment .The present results show an acceptable agreement with the combinatorial method of level density.

Key words

angular momentum, level density, RMT, GOE, Spin misassignment.

Article info

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كثافة المستويات النووية مع حسابات رنين البروتون بأستخدام نظرية مجموعة جاوس المتعددة

مهدي هادي جاسم، زاهدة احمد دخيل، رشا جواد كاظم

قسم الفيزياء، كلية العلوم، جامعة بغداد

الخلاصة

استخدمت مجموعة جاوس المتعددة التي هي احد صيغ نظرية المصفوفة العشوائية لدراسة كثافة المستويات في تفاعل البروتون مع انوية الكالسيوم 44، التيتانيوم 48 والحديد 56، طبقت طريقة تحليل واعدة مستندة على البيانات المتوفرة للمسافات(المباعدة) الرنينية والتي تكون عروضها الطاقية متوافقة مع توزيعة بورتر- ثوماس.

بينت كثافة المستويات المحسوبة للانوية المركبة co, v, sc اعتماد الزخم والبارتي و اوضحت النتائج تناقضاً في كثافة المستويات لنواة sc وهو ناتج عن اختفاء اللف المغزلي. والنتائج اظهرت توافق بارز مع نتائج نظرية التجميع لكثافة المستويات.

Introduction

The modern statistical approach to nuclear physics utilizes RMT. A number of seminal papers are included in the compilation by Porter [1]. This new kind of statistical mechanics views the nucleus as a system in which a large number of particles are interacting according to unknown laws. The problem is how to describe an ensemble of systems in which all possible laws of interaction are equally probable. [2].

Although the initial theory was established in 1962, the high-quality data required to test the predictions of random matrix theory firstly became available in the 1970s. Haq et al. [3] analyzed a collection of neutron and proton resonance data (the Nuclear Data

Ensemble) and found an excellent agreement between theory and experiment. The data agree very well with predictions based on an ensemble of real symmetric matrices whose elements are independent random variables. This GOE makes striking, parameter-free predictions for the eigen value distributions (including short- and long-range correlations), which are confirmed by the data.

The GOE theory makes striking predictions for the fluctuation properties and has no adjustable parameters. The key predictions of the GOE are level repulsions. Eigenvalues are usually examined with the nearest-neighbor spacing (NNS) distribution and the Dyson-Mehta Δ3 statistic [4]. The crucial practical issues in the analysis of experimental data are sample size, purity and completeness. The most striking difference between the two extremes, GOE and Poisson, is the presence or absence of level repulsion .Anther differences that GOE is characterized by much smaller fluctuations than Poisson and more sensitivity to missing levels. The effects of small sample sizes and of missing and spurious levels on the standard tests are described by Shriner and Mitchell [5].

Theory

1- Width Analysis of Imperfect Sequences

The Gaussian assumption for the distribution of reduced width amplitudes leads to the Porter-Thomas distribution [4]:

$$p(\gamma^2) = \frac{1}{\sqrt{2\pi\gamma^2/\langle\gamma^2\rangle}} \exp(-\frac{\gamma^2}{2\langle\gamma^2\rangle}) \qquad (1)$$

In terms of a dimensionless variable, where γ is the reduced width and $\langle\gamma\rangle$ is the average reduced width:

$$y = \frac{\gamma^2}{\langle\gamma^2\rangle} \qquad (2)$$

The Porter-Thomas distribution becomes:

$$p(y) = \frac{1}{\sqrt{2\pi y}} e^{\frac{-y}{2}} \qquad (3)$$

According to this distribution, the smallest widths are most frequent. Since these weak levels may not be observed experimentally when an experimental level sequence is usually incompleted, and therefore the width distribution is distorted. While the absence of weak levels causes the sequence to be incomplete, [6, 7].There are various other effects that can also distort the observed width distribution. Non-statistical phenomena such as doorway states can affect the sequence. Another cause is spin misassignments. This leads to an impure sequence, which will have a different distribution. One must consider these effects when analyzing the observed resonance widths.

2- The standard iterative method

Most of the levels that are missed are below the threshold of experimental observability in a particular experiment. Therefore, the simplest assumption is normally adopted.

One assumes that all of the levels with γ^2 smaller than the minimum are observed and reduced width, γ^2, which are not detected and that all resonances with widths larger than the minimum value are observed. Usually the cutoff parameter y_0 is taken to be the smallest of all the observed widths divided by the average reduced width.

The observed average reduced width $\langle\gamma^2\rangle_{obs}$ of a sequence of given total angular momentum – parity, J^π, is [4]:

$$\langle\gamma^2\rangle_{obs} = \sum_{i=1}^{N_{obs}} \frac{\gamma_i^2}{N_{obs}} \qquad (4)$$

where N_{obs} is the number of observed levels and the cutoff for that sequence is:

$$y_0 = \frac{\gamma^2_{\min}}{\langle \gamma^2 \rangle_{obs}} \quad (5)$$

The observed fraction, f, of the sequence is obtained by:

$$1 - f = \int_0^{y0} p(y)\, dy \quad (6)$$

where f is the fraction of levels missed.
The number of observed levels N_{obs} must be corrected by this missing fraction. The corrected number N_{new} is closer to the true number of levels. Because of missing levels, the observed strength is smaller than the actual strength.
The observed strength f_s can be found from:

$$1 - f_s = \int_0^{y0} y p(y)\, dy \quad (7)$$

A new corrected number of levels in the sequence is determined from:

$$N_{new} = \frac{N_{obs}}{f} \quad (8)$$

and the total strength is corrected as:

$$\sum \gamma_i^2 = \frac{\sum_{obs} \gamma_i^2}{f_s} \quad (9)$$

Using this new average reduced width, a new value of y_0 is defined, and the above steps are repeated and after a few iterations a constant value of (1 – f) is obtained.
One can then determine the average level spacing D or level density ρ from

$$D = \frac{1}{\rho} = \frac{E_{\max} - E_{\min}}{N - 1} \quad (10)$$

where E_{max},E_{min} maximum and minimum value of incident particle energy .

3-Angular momentum dependence

The observed angular momentum dependence of the proton resonance level densities in the three nuclei under consideration is compared with the conventional spin dependence of the level density formula [4]

$$f(J) \cong \frac{(2J+1)}{2\sigma^2} \exp[-J(J+1)/2\sigma^2] \quad (11)$$

where σ is the spin cut-off parameter.

Results and Discussion

1. Total level density

By reference to the experimental results for p+^{44}Ca [^{45}Sc], p+^{48}Ti [^{49}V] and p+^{56}Fe [^{57}Co] reactions from [4], the observed resonance energies have been used to calculate the total level density for the given nuclei and certain spins.
Comparison indicated close results with ref.[4] within an error mentioned in Table1.

2. Angular momentum dependence f (J)

The angular momentum dependence of the level densities in p +^{44}Ca, p +^{48}Ti, and p +^{56}Fe are shown in Figs.1, 2, and 3, respectively. The solid lines represents f(J), and the level densities for a positive (negative) parity are indicated by colored circles. The parameter σ is taken to be 2.9, where the value is taken from [8] for all three nuclei. The agreement was good for p +^{48}Ti and fair for p +^{56}Fe. There is a large deviation for J = 1/2 and 3/2 in p +^{44}Ca. We believe that this discrepancy is due to a large number of spin misassignments. A large number of misassigned levels will also affect the observed parity dependence in this nucleus.

Table 1: Level densities for p +^{44}Ca, p +^{48}Ti, and p +^{56}Fe via width analysis.

j^{π}[4]	N_{obs}[4]	$N_{missing}$	E_P range(MeV)	f(pw)*	ρ MeV^{-1} (pw)*	ρ MeV^{-1} [4]
p+^{44}Ca						
$1/2^+$	116	18	2.9561-3.7095	0.844	181±15	177±15
$1/2^-$	132	24	2.9584-3.7099	0.797	236±20	242±26
$3/2^-$	79	26	2.9565-3.6913	0.675	158±24	165±17
$3/2^+$	125	40	2.9545-3.7109	0.679	242±24	247±24
$5/2^+$	129	31	2.9557-3.7125	0.759	225±19	227±20
p+^{48}Ti						
$1/2^+$	103	12	3.0850-3.8574	0.879	151±10	152±13
$1/2^-$	105	20	3.0802-3.8568	0.812	166±16	159±17
$3/2^-$	175	47	3.0873-3.8584	0.734	308±21	295±22
$3/2^+$	139	38	3.0913-3.8395	0.727	254±21	251±22
$5/2^+$	180	49	3.0816-3.8595	0.726	318±21	313±24
p+^{56}Fe						
$1/2^+$	56	8	3.1212-4.0005	0.859	73±12	67±6
$1/2^-$	35	7	3.1084-3.9965	0.813	47±15	52±7
$3/2^-$	66	16	3.1055-3.9993	0.759	96±19	105±13
$3/2^+$	35	10	3.1221-3.9614	0.713	57±22	56±8
$5/2^+$	91	17	3.1117-4.0016	0.816	124±15	128±13

**pw: present work*

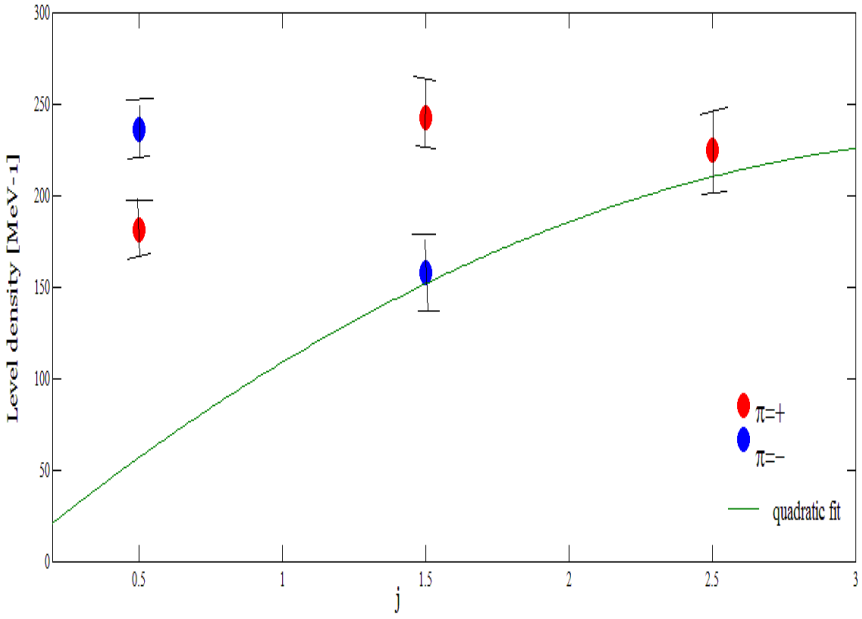

Fig.1: J dependence of level densities in p+^{44}Ca, the line represents the quadratic fit for Eq. (11).

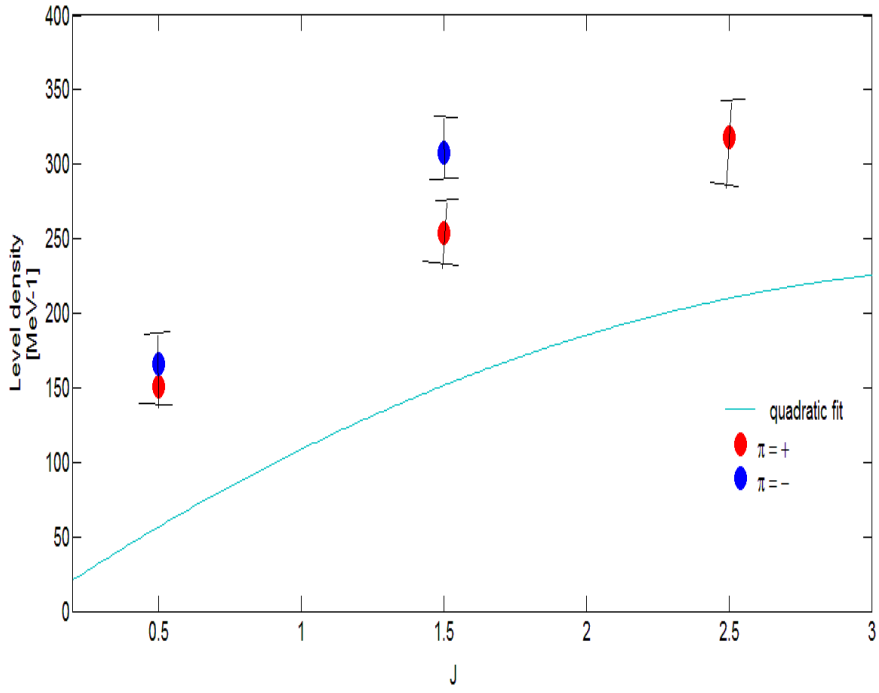

Fig.2: J dependence of level densities in p+^{48}Ti, the line represents the quadratic fit for Eq. (11).

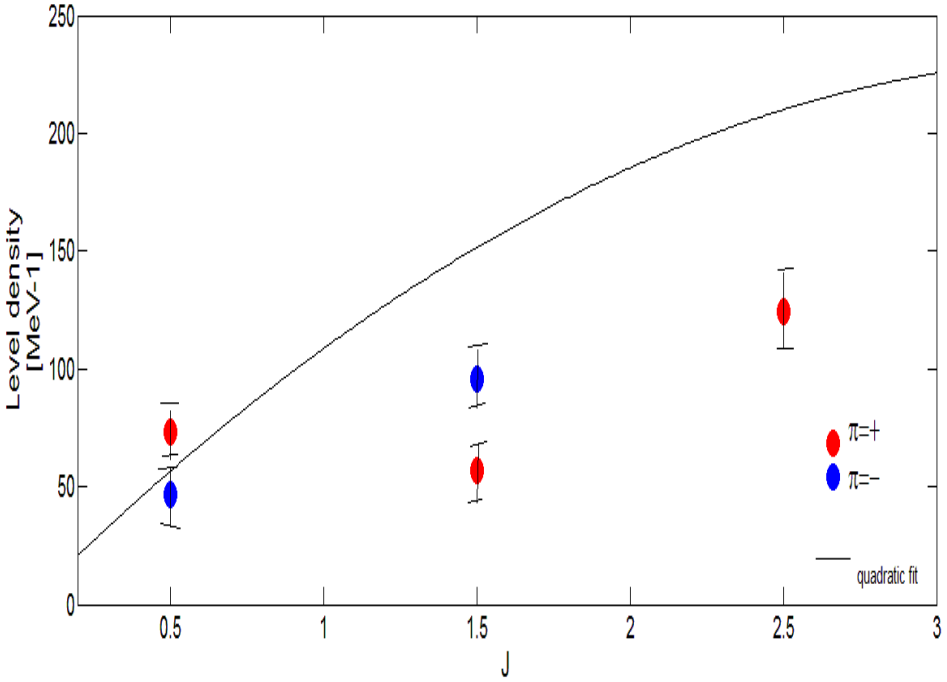

Fig.3: J dependence of level densities in p+^{56}Fe, the line represents the quadratic fit for Eq. (11).

The results of the present work are compared with the results of the combinatorial method .The differences between results come from constancy of parameter σ in present work but such feature is not found in ref. [9]. This can be shown in Fig.4.

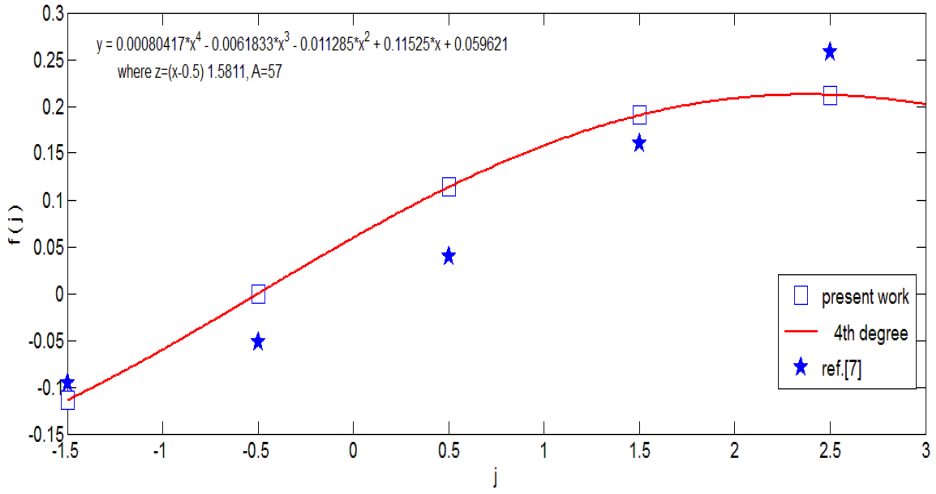

Fig.4: J dependence in two methods, squares represent our work, star points are computational method and red line is 4-polynomial fitting for Eq. (11).

3-Parity dependence

It is convenient to introduce a parameter α for the parity asymmetry of the level density[8]:

$$\alpha = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-} 100\% \qquad (12)$$

where ρ_+ and ρ_- are the positive and negative parity level densities for a given value of J. The values for α are also shown in Fig.5, where the upper graph is for J = 1/2, and the lower graph is for J = 3/2. Since α is proportional to ρ_+, ρ_-, positive values for α means that there are more positive parity states than negative for a given J value.

Table2: Parity asymmetry in the pro tonresonance level densities.

J^π	*Reaction*	*α%*
1/2	p+^{44}Ca	-13±15 -18±7*
1/2	p+^{48}Ti	-5±12
1/2	p+^{56}Fe	22±14
3/2	p+^{44}Ca	21±11 23±7*
3/2	p+^{48}Ti	-10±7
3/2	p+^{56}Fe	-25±21

Ref [10].

Conclusions

Nuclear resonances following the predictions of the Gaussian Orthogonal Ensemble (GOE) version of the Random Matrix Theory (RMT) are considered in the present work. RMT predictions have been investigated to calculate the corrections of the proton resonance data. Since no measurements are perfect, then data are corrected for the imperfection of these measurements, where the improvement of the conventional analysis method based on the resonance widths.

Another test has been introduced and showed a promising analysis method based on the resonance spacing, where widths follow the Porter-Thomas distribution. Since the weakest widths are not observed in experiments, this distribution is modified to account for this systematic flaw in data. The level density was calculated with greater confidence and improved precision. However, the disagreement in many cases will probably be due to non-statistical effects in the width distribution.

With the newly obtained level density results, the parity and spin dependences of the level density were addressed. More precise data is needed and/or a careful analysis covering a wide range of data is required, before one can draw a definitive conclusion regarding the parity dependence of the level density. A conventional spin dependence of the level density was confirmed for p +^{48}Ti and p +^{56}Fe. For p +^{44}Ca .There is discrepancy from the analytical expression and one probable cause is spin misassignment.

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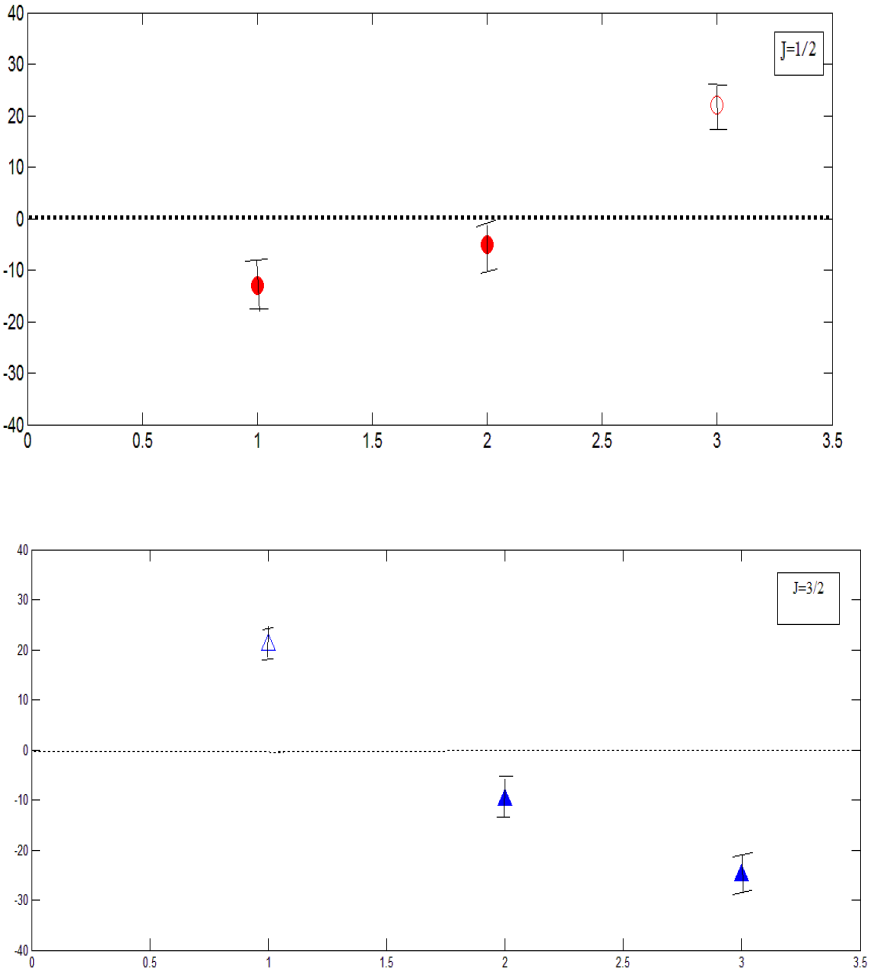

Fig.5: Parity asymmetry in the proton resonance level densities. The upper graph is for the J = 1/2 sequences and the lower graph is for the J=3/2 sequences.