

## Quadrupole moment of $^{14}\text{B}$ exotic nucleus

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### Abstract

The quadrupole moment of  $^{14}\text{B}$  exotic nucleus has been calculated using configuration mixing shell model with limiting number of orbital's in the model space. The core- polarization effects, are included through a microscopic theory which considers a particle-hole excitations from the core and the model space orbits into the higher orbits with  $6\hbar\omega$  excitations using M3Y interaction. The simple harmonic oscillator potential is used to generate the single particle wave functions. Large basis no-core shell model with  $(0+2)\hbar\omega$  truncation is used for  $^{14}\text{B}$  nucleus. The effective charges for the protons and neutrons were calculated successfully and the theoretical quadrupole moment was compared with the experimental data, which was found to be in a good agreement.

### Key words

Quadrupole moment, exotic nucleus, root mean square.

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### عزم رباعي القطب الكهربائي للنواة الغريبة $^{14}\text{B}$

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### الخلاصة

تم حساب عزم رباعي القطب الكهربائي للنواة الغريبة  $^{14}\text{B}$  باستخدام تأثير استقطاب القلب من خلال النظرية المجهرية التي تعتمد استثارة جسيم- فجوة من القلب ومدارات فضاء الأنموذج والمتضمنة المدارات العليا مع طاقة تهيج  $6\hbar\omega$  وباستخدام تفاعل M3Y. أستخدم جهد المتذبذب التوافقي لتوليد دالة الموجة للجسيم المنفرد. استخدم فضاء الأنموذج بدون قلب مع  $(0+2)\hbar\omega$  حسب الشحنة الفعالة للبروتونات والنيوترونات بنجاح وقورنت قيم عزم رباعي القطب الكهربائي أنظريه بالبيانات العملية التي بينت أن هناك اتفاق جيد.

### Introduction

The study of exotic nuclei, i.e. nuclei with extreme properties, such as an extreme ratio of the number of neutrons and protons, excitation energy or total nuclear spin provides an attractive testing of nuclear structure studies. The interest for exotic nuclei has increased enormously since the recent progress in nuclear accelerators. This made it possible to produce exotic nuclei with sufficient rates.

The study of neutron rich nuclei revealed unexpected phenomena such as weakening

of closed shells and the formation of neutron halos [1].

The quadrupole and magnetic moment of exotic nuclei are serious tests for these new developed nuclear patterns. They contain a lot of information about the structure of the nuclear state: the magnetic dipole moment is sensitive to the orbitals of nucleons that are not dual to zero spin [2]. The electric quadrupole moment shows information on the deformation of the charge distribution of the nucleus. The

electromagnetic moments are one of the main probes to gain information about the nuclear structure throughout the entire nuclear chart.

Ozawa et al.[3] reviewed experimental studies on nuclear sizes and related topics. The recent development of radioactive nuclear beams has enabled them to study the nuclear sizes of unstable nuclei. The nuclear sizes for unstable nuclei, which are deduced by the interaction cross sections and reaction cross sections, are mainly reviewed. From a theoretical view point, a Glauber-model analysis is important to deduce nuclear sizes. Also they discussed halo and skin from a nuclear size point of view. Sumikama et al. [4] have detected the  $\beta$ -NQR (nuclear quadrupole resonance) signals of  ${}^8\text{B}$  ( $J^\pi = 2^+, \tau_{1/2} = 770$  ms) implanted in  $\text{TiO}_2$  to determine the electric quadrupole moment of  ${}^8\text{B}$  with high precision. The ratio of the quadrupole moments of  ${}^8\text{B}$  and  ${}^{12}\text{B}$  was determined as  $|Q({}^8\text{B})/Q({}^{12}\text{B})|=4.88\pm 0.04$ . Combined with the known sign, the quadrupole moment of  ${}^8\text{B}$  was obtained as  $Q({}^8\text{B}) = +(64.5\pm 1.4)$  mb, which is consistent with more precise than the previously reported value. The experimental values of the  $Q$  moment, the proton and neutron radii and the density distribution of  ${}^8\text{B}$  were compared with several theoretical predictions and were found to be best reproduced by a microscopic cluster model, which suggests the existence of a proton halo.

Neugart et al. [5] measured the electric quadrupole moment and the magnetic moment of the  ${}^{11}\text{Li}$  halo nucleus with more than an order of magnitude higher precision than before,  $Q = 33.3(5)$  mb and  $\mu = 3.6712 \mu_N$ , revealing a 8.8 (1.5)% increase of the quadrupole moment relative to that of  ${}^9\text{Li}$ . This result is compared to various models that aim at describing the halo properties. In the shell model an increased quadrupole moment points to a

significant occupation of the  $1d$  orbits, whereas in a simple halo picture this can be explained by relating the quadrupole moments of the proton distribution to the charge radii. Advanced models so far fail to reproduce simultaneously the trends observed in the radii and quadrupole moments of the lithium isotopes.

Douici et al. [6] studied the effect of the particle-number projection on the electric quadrupole moment ( $Q_2$ ) of even-even proton-rich nuclei in the isovector neutron-proton pairing case. As a first step, an expression of the electric quadrupole moment, which takes into account the isovector neutron-proton pairing effect and which conserves the particle number, is established within the Sharp-Bardeen-cooper-Schrietter (SBCS) method. This expression does generalize the one used in the pairing between like-particles case. As a second step,  $Q_2$  is calculated for even-even proton-rich nuclei using the single-particle energies of a Woods-Saxon mean-field. The obtained results are compared with experimental data when available as well as with the results obtained in the pairing between like-particles case. It is shown that the neutron-proton pairing effect, as well as the projection one, is maximal when  $N = Z$ .

### Theory

The reduced one-body matrix element for shell-model wave functions of initial spin  $J_i$  and final spin  $J_f$  for a given multipolarity can be expressed as a linear combination of the single-particle matrix elements

$$\langle J_f \| \hat{L}_{J_z}(qr) \| J_i \rangle = \sum_{\alpha\beta} \langle \alpha \| \hat{L}_{J_z}(qr) \| \beta \rangle OBDM(J_f, J_i, \alpha, \beta, J, t_z) \quad (1)$$

The reduced single-particle matrix element of the Coulomb (Longitudinal) operator is given by [7]:

where as:

$$\langle n_\alpha l_\alpha \| j_J(qr) \| n_\beta l_\beta \rangle = \int_0^\infty dr r^2 j_J(qr) R_{n_\alpha l_\alpha}(r) R_{n_\beta l_\beta}(r) \quad (2)$$

where  $j_J(qr)$  is the spherical Bessel function and  $R_{nl}(r)$  is the single-particle radial wave function. Electron scattering Coulomb form factor involving angular momentum  $l$  and momentum transfer  $q$ , between initial and final nuclear shell model states of spin  $J_i$  and  $J_f$ , are [7]:

$$|F_J(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \langle J_f \| \hat{L}_{J_z}(q) \| J_i \rangle \right|^2 |F_{fs}(q)|^2 |F_{cm}(q)|^2 \quad (3)$$

where the finite size of the nucleon  $F_{fs}(q)$  is given by [8]:

$$F_{fs}(q) = [1 + (q/4.33 \text{ fm}^{-1})^2]^{-2}$$

$$\text{and } F_{cm}(q) = e^{q^2 b^2 / 4A}$$

The reduced transition probability is related to the form factor at the photon point, which is given by [9]:

$$B(CJ) = \frac{[(2J+1)!!]^2 Z^2 e^2}{4\pi k^{2J}} |F_J(q=k)|^2 \quad (4)$$

where  $q=k = \frac{E_x}{\hbar c}$  is the momentum transfer at the photon point.

The quadrupole moment is related to the reduced transition probability  $B(J)$ , as [9]:

$$Q = J \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{(2J+1)}} \sqrt{(2J_i+1)B(CJ)} \quad (5)$$

Using wigner-Eckart theorem, The Quadrupole moment can be written in terms of that in spin-isospin space [9,10]:

$$Q = 2 \begin{pmatrix} J_i & 2J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T_i, T_z} (-1)^{T_i-T_z} \begin{pmatrix} T_i & T & T_i \\ -T_i & 0 & T_i \end{pmatrix} \langle J_f T_f \| \hat{O}_{2T} \| J_i T_i \rangle \quad (6)$$

The reduced matrix elements of the electron scattering operator  $\hat{O}_A$  is expressed as a sum of the model space (MS) contribution and the core polarization (CP) contribution, as follows:

$$\langle \Gamma_f \| \hat{O}_A \| \Gamma_i \rangle = \langle \Gamma_f \| \hat{O}_A \| \Gamma_i \rangle_{\text{MS}} + \langle \Gamma_f \| \Delta \hat{O}_A \| \Gamma_i \rangle_{\text{CP}} \quad (7)$$

which can be written as

$$\langle \Gamma_f \| \hat{O}_A \| \Gamma_i \rangle = \sum_{\alpha\beta} \text{OBDM}(\Gamma_f, \Gamma_i, \alpha, \beta, \Lambda) [\langle \alpha \| \hat{O}_A \| \beta \rangle + \langle \alpha \| \Delta \hat{O}_A \| \beta \rangle] \quad (8)$$

The states  $|\Gamma_i\rangle$  and  $|\Gamma_f\rangle$  are described by the model space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e  $\Gamma_i \equiv J_i T_i$ ,  $\Gamma_f \equiv J_f T_f$  and  $\Lambda \equiv J T$ .

According to the first-order perturbation theory, the single particle core polarization term is given by [10]:

$$\begin{aligned} \langle \alpha \| \Delta \hat{O}_A \| \beta \rangle = & \left\langle \alpha \| \hat{O}_A \frac{Q}{E_i - H_0} V_{res} \| \beta \right\rangle \\ & + \left\langle \alpha \| V_{res} \frac{Q}{E_f - H_0} \hat{O}_A \| \beta \right\rangle \end{aligned} \quad (9)$$

where the operator  $Q$  is the projection operator onto the space outside the model space. The single particle core-polarization terms given in equation (8) are written as [10]:

$$\begin{aligned} \langle \alpha \| \Delta \hat{O}_A \| \beta \rangle = & \sum_{\alpha_1 \alpha_2 \Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{\varepsilon_\beta - \varepsilon_\alpha - \varepsilon_{\alpha_1} + \varepsilon_{\alpha_2}} (2\Gamma+1) \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha,\alpha_1})(1+\delta_{\alpha,\beta})} \\ & \times \langle \alpha \alpha_1 | V_{res} | \beta \alpha_2 \rangle_{\Gamma} \langle \alpha_2 \| \hat{O}_A \| \alpha_1 \rangle \end{aligned}$$

+terms with  $\alpha_1$  and  $\alpha_2$  exchanged with an overall minus sign, (10)

where the index  $\alpha_1$  runs over particle states and  $\alpha_2$  over hole states and  $\varepsilon$  is the single-particle energy, and is calculated according to [10]:

$$\varepsilon_{nlj} = (2n+l-1/2)\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r) \rangle_{nl} & \text{for } j=l-\frac{1}{2} \\ \frac{1}{2}l\langle f(r) \rangle_{nl} & \text{for } j=l+\frac{1}{2} \end{cases} \quad (11)$$

with  $\langle f(r) \rangle_{nl} \approx -20A^{-2/3}$  and  $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$

Higher energy configurations are taken into consideration through  $1p-1h$   $2\hbar\omega$  excitations. For the residual two-body interaction  $V_{res}$ , the M3Y interaction of Bertsch *et al.* [11] is adopted. The form of the potential is defined in equations (1-3) in Ref. [11]. The parameters of 'Elliot' are used which are given in Table1 of the mentioned reference. A transformation between  $LS$  and  $jj$  is used to get the relation between the two-body shell model matrix elements and the relative and center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation. The Quadrupole moment is given by [10]:

$$Q = 2 \begin{pmatrix} J_i & 2J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} (-1)^{T-T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \langle J_i T_i \parallel \tilde{O}_{2T} \parallel J_i T_i \rangle \quad (12)$$

which can be written as:

$$Q = 2 \begin{pmatrix} J_i & 2J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \tilde{M}_{2T} \quad (13)$$

where  $\tilde{M}_{JT} = \langle J_f T_f \parallel \tilde{M}_{JT} \parallel J_i T_i \rangle$ . The isoscalar ( $T = 0$ ) and isovector ( $T = 1$ ) charges are given by  $e_0 = e_{is} = \frac{1}{2}e$ ,

$$e_1 = e_{iv} = \frac{1}{2}e.$$

The quadrupole moment can be represented in terms of only the model space matrix elements as:

$$Q = 2 \begin{pmatrix} J_i & 2J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T^{\text{eff}} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{2T} \quad (14)$$

Then the isoscalar and isovector effective charges are given by:

$$e_T^{\text{eff}} = \frac{M_{JT} + \Delta M_{JT}}{2M_{JT}} e = \frac{e_p^{\text{eff}} + (-1)^T e_n^{\text{eff}}}{2} \quad (15)$$

The proton and neutron effective charges can be obtained as follows:

$$e_p^{\text{eff}} = e_0^{\text{eff}} + e_1^{\text{eff}} \quad \text{and} \quad e_n^{\text{eff}} = e_0^{\text{eff}} - e_1^{\text{eff}}.$$

## Results and Discussion

$^{14}\text{B}$  nucleus is an exotic system, Poskanzer *et al.* [12] discovered  $^{14}\text{B}$  in 1966.  $^{14}\text{B}$  nucleus ( with valence neutron separation energy of 0.97 MeV ) is of particular interest as it is the lowest mass bound system among the  $N = 9$  isotones. The nucleus  $^{14}\text{B}$  has been controversially predicted to be particle unstable by Ball *et al.* [13].

In the simplest model, the  $J^\pi = 2^-$  ground state of  $^{14}\text{B}$  results from the coupling of a  $2s1d$ -shell neutron to the  $^{13}\text{B}$  ground state. The nucleus  $^{14}\text{B}$  has been controversially predicted to be particle unstable by Ball *et al.* [13], whereas Garvey and Kelson [14] predicted it to be particle stable by about 400 keV. The mass excess of  $^{14}\text{B}$  has been determined for the first time by Ball *et al.*  $^{14}\text{B}$  is stable with respect to the decay into  $^{13}\text{B} + n$  by about 0.97 MeV. Shell-model calculations [15] give 0.986 MeV in good agreement with the experimental value for the stability of  $^{14}\text{B}$ .

The  $1p$ -shell has been a testing ground for nuclear models [2] but in spite of its success in describing the static properties of nuclei it failed to describe electron scattering data without including the core - particle effects. These effects are included by giving the model space nucleons effective charges, different from their bare values to account for the discarded space. These effects are called core-polarization effects. Large-basis no core model calculation have been performed [16] for  $p$ -shell nuclei using six major shells (from  $1s$

to  $3p-2f-1h$ ). In these calculations all nucleons are active. However, constrained by computer capabilities, one can use a truncated no-core calculations, where only those configurations are retained from the full no-core case in which there are up and including few  $\hbar\omega$  levels excitations of the lowest unperturbed. As the number of  $\hbar\omega$  levels increases, the result will converge and approach those of the full no-core calculations. It was observed that the E2 transition rates obtained in the 4  $\hbar\omega$  calculations for  ${}^6\text{Li}$  are weaker than those calculated in the 6  $\hbar\omega$  space. Shell model structure of low-lying excited states in  ${}^{6,7}\text{Li}$  have been studied [17] using multi-  $\hbar\omega$  excitations. However, it was found that the result of the quadrupole moments were far from the experimental values, even with  $(0+2+4+6)$   $\hbar\omega$  wave functions. A clear improvement in most observables was evident for the calculation of the  ${}^{10}\text{C} \rightarrow {}^{10}\text{B}$  Fermi matrix elements [18], where the size of the large model space was increased from 2  $\hbar\omega$  to 4  $\hbar\omega$ . Calculations of E2 transitions and quadrupole moments for  $A=7-11$  underestimated the data [19] and there were still need for effective charges despite the large model space, 6  $\hbar\omega$  for  $A=7$  nuclei, and 4  $\hbar\omega$  for  $A=8-10$  nuclei. Convergent results were obtained for  $A=3$  and  $A=4$  with 5 $\hbar\omega$  and 16  $\hbar\omega$ , respectively. Radhi et al. [20].

For this nucleus, we use a large HO model space considering the major shell,  $1s$ ,  $1p$ ,  $2s-1d$  and  $2p-1f$ , with partially inert core, considering  $(0+2)$   $\hbar\omega$  truncated  $spsdpf$  model space used by Radhi et al. for  ${}^9\text{Be}$  [20].

The  $0\hbar\omega$  configuration is  $[(1s)^4(1p)^{10}]$ , while the  $2\hbar\omega$  configurations are  $[(1s)^3(1p)^{10}(2s1d)^1]$  and  $[(1s)^4(1p)^9(2s1d)^0(2p1f)^1]$  for one particle-one hole ( $1p-1h$ ) excitations. Also the configuration  $[(1s)^2(1p)^{12}]$  and  $[(1s)^4(1p)^8(2s1d)^2]$  are allowed

for two particle-two hole  $2p-2h$  excitation, these excitations form the model space. Shell model interaction encompassing the four oscillator shells have been constructed by Warburton and Brown [21].

These interactions are based on interactions for the  $1p-2s1d$  shell determined by a least-square fit to 216 energy levels in the  $A=10-22$  region assuming no mixing of  $n\hbar\omega$  and  $(0+2)$   $\hbar\omega$  configurations. The  $1p2s1d$  part of the interaction (cited in Ref. [21] as WBP) model space was expanded to include  $1s$  and  $2p1f$  major shells by adding the appropriate  $2p1f$  and cross-shell  $2s1d-2p1f$  two-body matrix element of the Warburton-Becker-MillinerBrown (WBMB) interaction [22] and all the necessary matrix elements from the bare G-matrix potential are of Hosaka, Kubo and Toki [23]. The  $2s1d$  shell interaction is the USD Hamiltonian [24].

The calculated rms matter radius which is 2.48 fm, using the single-particle wave functions of a Harmonic oscillator potential with size parameter  $b=1.64$  fm is chosen to reproduce the root mean square charge radius  $2.44 \pm 0.06$  fm [25]. Shell model calculation was performed with the shell model code OXBASH [26] when the *OBDM* elements were obtained.

The occupation numbers for the ground state is calculate to be equal to  $(1s_{1/2})^4 (1p_{3/2})^{6.972} (1p_{1/2})^{2.0} (1d_{5/2})^{1.02}$ . Using the *OBDM* elements of the model space discussed above, with the single-particle wave functions of a Harmonic oscillator potential with size parameter  $b=1.64$ fm, the calculated quadrupole moment value is  $1.927\text{efm}^2$ . This value is calculated without core polarization effect, i.e. with bare charges, and it is underestimates the measured value  $2.98 \pm 0.075\text{efm}^2$  [27] by around a factor of 1.5, when core polarization effects are taken into consideration, the quadrupole moment becomes  $2.22\text{efm}^2$ , which still

underestimates the experimental value by around factor 1.3. The effective charges are 1.1e and 0.06e, for the proton and neutron, respectively.

Using the effective charges calculated for the stable  $^{10}\text{B}$  nucleus, which is equal to 1.284e and 0.284e for the proton and

neutron, respectively. The quadrupole moment of  $^{14}\text{B}$  becomes  $2.91\text{efm}^2$ , which is very close to the measured value. The results of effective charges and quadrupole moments are tabulated in Table1.

**Table 1: The calculated and experimental quadrupole moment with effective charges for  $^{14}\text{B}$  nucleus.**

NUCLEUS ( $J^\pi T$ )	$b(\text{fm})$	Core + Halo (ms)Interaction	Effective charge (e) and $Q(\text{efm}^2)$		
			$e_p^{\text{eff}}$ and $e_n^{\text{eff}}$	$Q_{\text{Cal.}}$	$Q_{\text{Exp. (Ref.)}}$
$^{14}\text{B} (2^- 2)$	$b=1.64$	$Spsdpf$ (0+2) $\hbar\omega$	1.0, 0.0 1.1, 0.06	1.927 2.22	$2.98 \pm 0.057$ [27]

## Conclusions

In this work, we have given an overview on a specific topic which attracts much attention in contemporary nuclear structure research, namely the quadrupole moments of  $^{14}\text{B}$  exotic nucleus. We list some of these below.

1. The quadrupole moment increases only with the root mean square matter radii.
2. Excitation from all major shell orbits into all higher allowed orbits are essential in obtaining a reasonable description of the experimented moments.
3. Empirical effective charges used for stable nuclei are not applicable for exotic, neutron rich nuclei.
4. Small effective charges are obtained for neutron rich nuclei, and their values are close to each other.
5. Average effective charges can be deduced from this study, to be used for future work for other neutron rich nuclei, and also for studying the transition rates and electron scattering form factors.

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