Derivative of an image that corrupted by additive noise

Suhad. A. Hamdan

Department of Physics, College of Science, University of Baghdad

E-mail: Suhadalani 2010@yahoo.com

Abstract

In this paper the behavior of the quality of the gradient that implemented on an image as a function of noise error is presented. The cross correlation coefficient (ccc) between the derivative of the original image before and after introducing noise error shows dramatic decline compared with the corresponding images before taking derivatives. Mathematical equations have been constructed to control the relation between (ccc) and the noise parameter.

Key words

Gradient, additive noise, first derivative.

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اشتقاق صورةٍ أفسدتْ بضوضاءٍ مضافةٍ

سهاد عبد الكريم حمدان

قسم الفيزياء، كلية العلوم، جامعة بغداد

الخلاصة

في هذه البحث، سلوك نوعية الميل الذي طبّق على صورة مشوبة بالضوضاء مقدّمة. معامل الإرتباط المتقاطع بين إشتقاق الصورة الأصلية قبل وبعد اضافة الضوضاء لاحظنا هبوط مثير في المشتقة مقارن بمعامل الإرتباط المتقاطع بين الصورة الأصلية والصورة بعد الحصول على خطأ ضوضاء. من هذه النتائج معادلات رياضية بنيت التي ربطت بين معامل الإرتباط المتقاطع ومعاملات الضوضاء.

Introduction

An image may be subject to noise and interference from several sources, including electrical sensor noise, photographic grain noise, and channel errors. These noise effects can be reduced by classical statistical filtering techniques. Image noise arising from a noisy sensor or channel transmission errors usually appears as discrete isolated pixel variations that are not spatially correlated. Pixels that are in error often appear visually to be markedly different from their neighbors [1].

An image gradient is a directional change in the intensity or color in an image. Image gradients may be used to extract information from images [2]. Mathematically, the gradient of a two-variable function (here the image intensity function) at each image point is a 2D vector with the components given by the derivatives in the horizontal and vertical directions. At each image point, the gradient vector points in the direction of largest possible intensity increase [3].

In 2012 various linear, non-linear and fuzzy techniques for impulse noise detection and reduction are discussed and compared them by Anuj Goel and Vikas Mittal [4]. Fuzzy techniques present an efficient approach to deal with uncertain data in order to remove the impulse noise to a great extent. NAFSM (Noise Adaptive Fuzzy Switching Median) filter is able to outperform other fuzzy techniques and works efficiently up to the noise levels of about 50-60%. The processing time taken by NAFSM algorithm is also less compared to other techniques.

Nafis uddin Khan, K. V. Arya, Manisha Pattanaik [5] proposed a method of image enhancement and de-noising by applying traditional singular value decomposition on partial differential equation based diffused images. Linear non-homogeneous isotropic diffusion and inverse heat diffusion method has been applied in the first stage of the proposed process to generate the diffused versions of the original noisy image in the form of smoothed image and edge enhanced image respectively. Singular value decomposition is then applied to each of the two diffused versions of the image with a fixed threshold individually which leads to the enhancement of sharp features of the image as well as noise reduction.

Celia A. Zorzo Barcelos a, Marcos Aure´lio Batista b[6] presented a method for image restoration. This method simultaneously fills in missing, corrupted, or undesirable information while it removes noise. The denoising is performed by the smoothing equation working inside and outside of the inpainting domain but in completely different ways.

In [2004] a new method for filtering MR images with spatially varying noise levels is presented by Alexei A. Samsonov and Chris R. Johnson[7]. In this method, a priori information regarding the image noise level spatial distribution is utilized for the local adjustment of the anisotropic diffusion filter. The noise-adaptive method was demonstrated to outperform the standard anisotropic diffusion filter in both image error reduction and image signal-to-noise ratio (SNR) improvement.

Jiang Tao，Zhao Xin [8] puts forward an improved method based on curvelet transform because certain regions of the image have the ringing and radial stripe after curvelet transform. The experimental results indicate that the improved curvelet transform has a abroad future for eliminating the noise of images. It suits not only the ordinary visual image, but also remote sensing image.

Aliaa A.A.Youssif A.A.Darwish A.M.M.Madbouly [9] proposed an adaptive threshold method for image denoising based on curvelet transform to estimate noise and remove it from digital images in order to achieve a good performance

Mathematical formulation

The image corrupted by noise could be written as follows [10]:

$$g(x,y) = O(x,y) + Q * r(x,y) \qquad (1)$$

where:

$\boldsymbol{O(x,y)}$: is the original image.

$\boldsymbol{Q}$: is a noise parameter $(0 < Q \leq 1)$.

$\boldsymbol{r(x,y)}$: is a random error that to be added to image.

The first derivative of an image $f(x,y)$ is defined by:

$$\nabla f(x,y) = \begin{cases} f_x(x,y) = \dfrac{\partial f(x,y)}{\partial x} \dots\dots\dots\dots\dots (2) \\ f_y(x,y) = \dfrac{\partial f(x,y)}{\partial y} \dots\dots\dots\dots\dots (3) \end{cases}$$

The gradient magnitude is defined as:

$$|\nabla f(x,y)| = \sqrt{\left(f_x(x,y)\right)^2 + \left(f_y(x,y)\right)^2} \dots\dots\dots\dots (4)$$

The Cross Correlation Coefficient (CCC) which is used to measure the quality of an image. If we have two images, the original image f(x,y) and the processed image g(x,y), the (ccc) is given by :

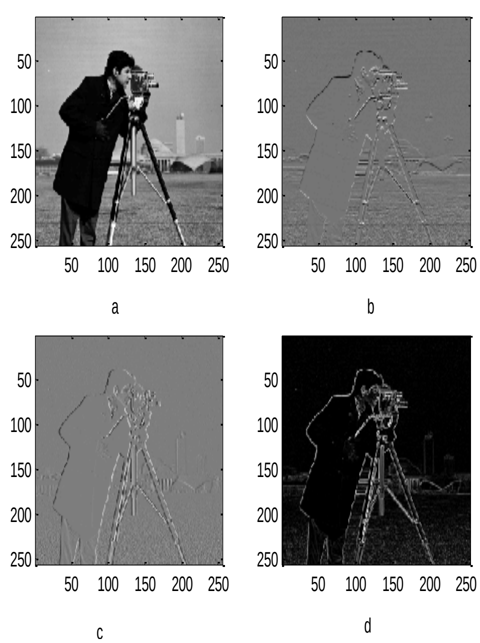
$$CCC = \frac{<\{f(x,y)g(x,y) - <f(x,y)><g(x,y)>\}>}{[<\{f(x,y) - <f(x,y)>\}^2><\{g(x,y - <g(x,y)>\}>]^{1/2}} \ldots\ldots(5)$$

where $0 < ccc \le 1$, if ccc=1 means a perfect correlation, but when $ccc \approx 0$ means totally uncorrelated image**.**

Experimental Work and Results

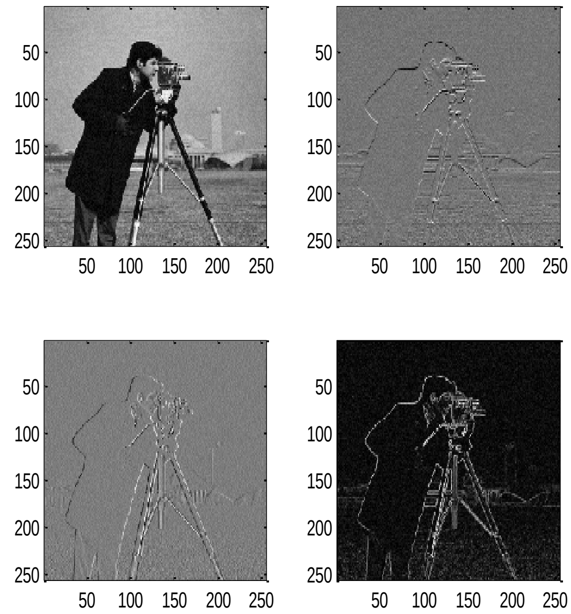
To implement the above equations using a MATLAB, the original image is taken to be a "cameraman image" of size 254 by254 pixels $f(x,y)$ is used as shown in Fig. 1a.

The first derivative of the image before and after noise addition using Q=0.1 & Q=0.5 are shown in Figs. 1, 2, and 3 respectively.



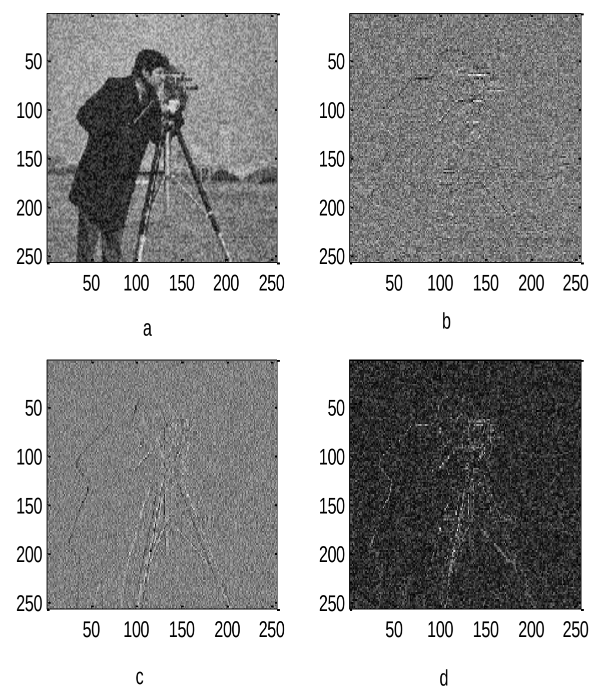
(a) $f(x,y)$ (b) $f_x(x,y)$ (c) $f_y(x,y)$ (d) $|\nabla f(x,y)|$

Fig. 1: Derivatives of the image(free noise images).



(a) $f_{noise}(x,y)$ (b) $f_{noise\,(x)}(x,y)$ (c) $f_{noise\,(y)}(x,y)$ (d) $|\nabla f_{noise}(x,y)|$

Fig. 2: Derivatives of the noisy images (Q=10%).



(a) $f_{noise}(x,y)$ (b) $f_{noise\,(x)}(x,y)$ (c) $f_{noise\,(y)}(x,y)$ (d) $|\nabla f_{noise}(x,y)|$

Fig. 3: Derivatives of the noisy (Q=50%).

It should be pointed out here that r(x,y) is used to be a uniform random distribution whose histogram obey uniform distribution as shown in Fig.4.
This figure demonstrated clearly that as the length of the vector rand (1, n) increases the histogram become smoother and when the length approaches infinity, the distribution becomes perfectly uniform.

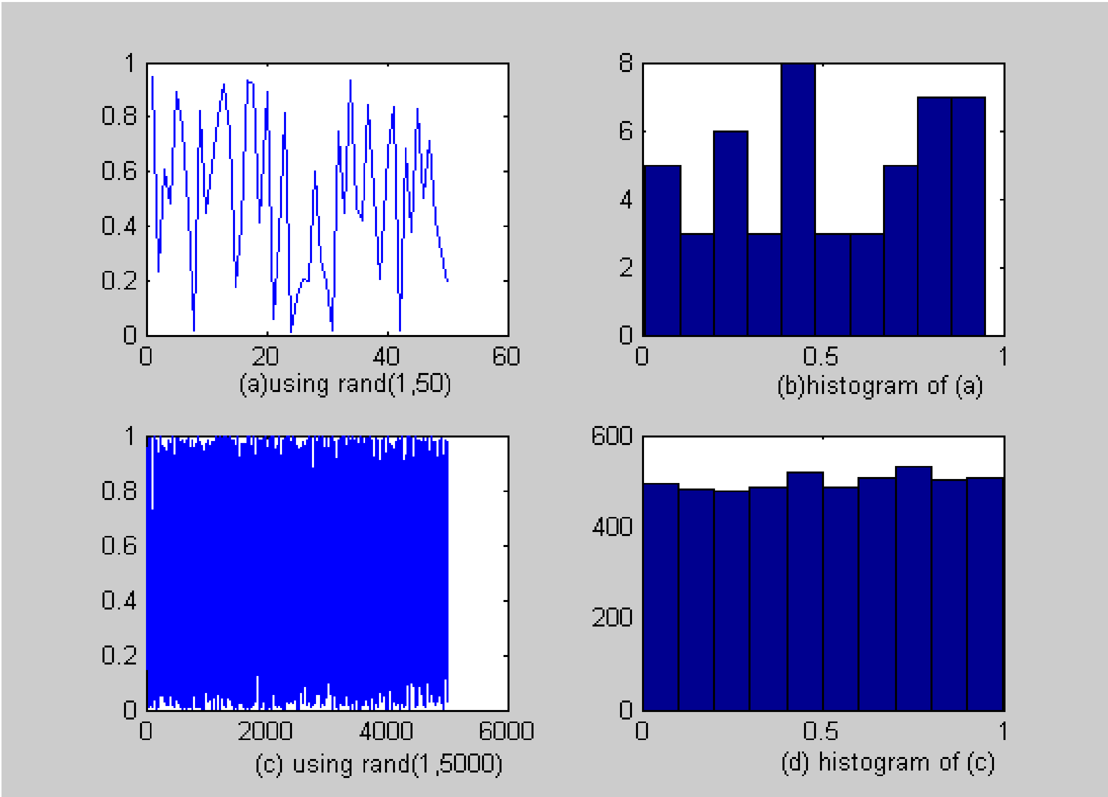


Fig. 4: Uniform random distribution.

Table 1: Demonstrated the results of computing ccc for all cases presented in Figures (1) to (3).

Percentage Noise Addition (Q)	**CCC Between original image and noisy image**	**CCC Between gradient of original image and gradient of noisy image**
0	1	1
.1	.9936	.9414
.2	.9751	.7985
.3	.9460	.6272
.4	.9097	.4929
.5	.8697	.3694
.6	.8252	.2911
.7	.7824	.2258
.8	.7383	.1828
.9	.6981	.1639
1	.6621	.1331

The block diagram of the whole processing that taking place is shown in Fig.5.

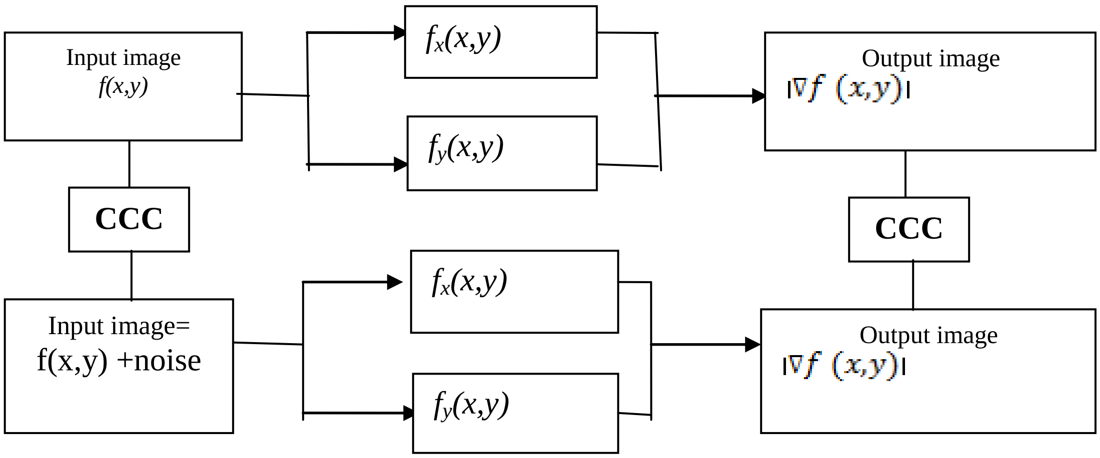


Fig.5: The block diagram of the basic image processing algorithm.

The data of the Table 1 are plotted in Fig. 6. It shows the behavior of derivatives before and after adding noise error. Mathematical equations are constructed via polynomial fitting using MATLAB package. The equations that shown on each sub figures are describe the relationship between ccc & Q.

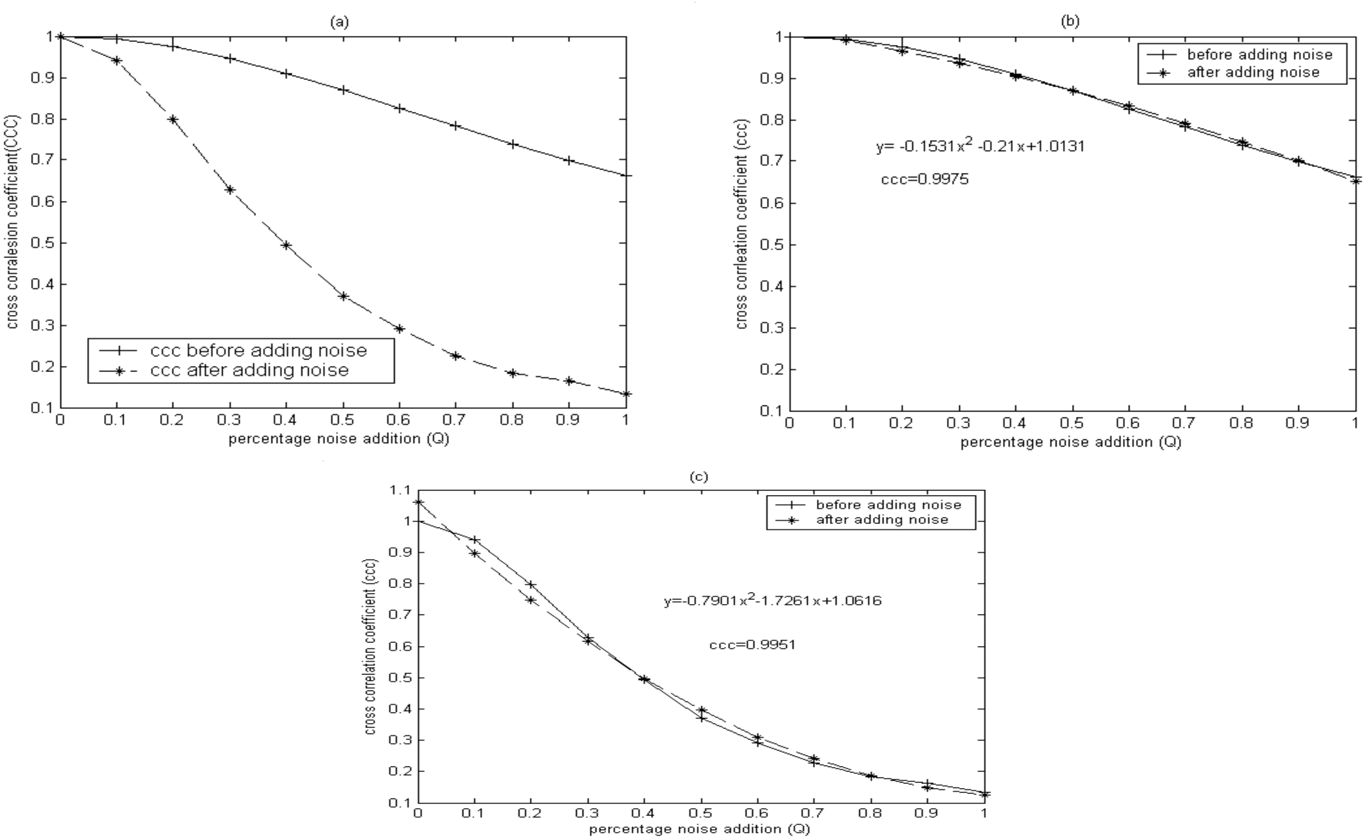


Fig.6: The relation between cross correlation coefficient (ccc) and percentage noise addition (Q.)

Conclusions

The following conclusions could be drawn as follows:

1- We have given mathematical equations represented the behavior of adding percentage error on to quality of the first derivative of an image.

2- The quality of the derivative of the images after adding noise decline vary rapidly compared with the corresponding image that subjected to the same amount of noise before taking derivative.

References

[1] Digital Image Processing: PIKS Inside, Third Edition. William K. Pratt copyright © John Wiley & Sons, Inc., 2001

[2] R C. Gonzalez, Richard E. Woods. Digital image processing, 3rd edition, Pearson education, 2008

[3] N. Puri, A.P. Manjeet Singh, A.P. Sumit Kaushik, International Journal of Advanced Research in Computer Science and Software Engineering, 2, 10 (October 2012) 43-52.

[4]A. Goel and Vikas Mittal, International Journal of Applied Engineering Research, 7, 11 (2012) 1-6.

[5] N. Uddin Khan, K. V. Arya, Manisha Pattanaik, International Journal of Computer Applications, 32, 8, October (2011) 14-22

[6] C. A. Zorzo Barcelos, Marcos Aure´lio Batista, Image and Vision Computing, 25 (2007) 61–69,

[7] A. A. Samsonov and C. R. Johnson, Magnetic Resonance in Medicine, 52 (2004) 798–806

[8] J. Tao，Zhao Xin, "The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences". Vol. XXXVII. Part B2. Beijing, (2008) 363-368.

[9] A.A.A.Youssif A.A.Darwish A.M.M.Madbouly, IJCSNS International Journal of Computer Science and Network Security, 10, 1, January (2010), 322-328

[10] J.Frank, L. Al-Ali, Nature, 256, Juley (1975) 376-379.