# Quadrupole moments of exotic carbon isotopes ($^{9}C$, $^{11}C$, $^{17}C$ and $^{19}C$) including core polarization effect

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**Abstract**

Quadrupole *Q* moments and effective charges are calculated for $^{9}C$, $^{11}C$, $^{17}C$ and $^{19}C$ exotic nuclei using shell model calculations. Excitations out of major shell space are taken into account through a microscopic theory which are called core-polarization effects. The simple harmonic oscillator potential is used to generate the single particle matrix elements of $^{9,11,17,19}C$. The present calculations with core-polarization effects reproduced the experimental and theoretical data very well.

**Key words**

*Quadrupole moments, effective charges, exotic nuclei.*

**Article info.**

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## عزوم رباعية القطب والشحنة الفعالة للـ ($^{9}C$ , $^{11}C$, $^{17}C$ و $^{19}C$)

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**الخلاصة**

تم حساب عزوم رباعية القطب والشحنات الفعالة للنوى الغريبة $^{9}C$, $^{11}C$ , $^{17}C$ و$^{19}C$ باستخدم أنموذج القشرة مع الأخذ بنظر الاعتبار التهيجات خارج فضاء القشرة الرئيسية من خلال النظرية المايكروية التي تسمى بتأثير استقطاب القلب. أستخدم جهد المتذبذب التوافقي البسيط لتوليد عناصر المصفوفة للجسيمة المفردة لنظائر الكاربون. اتفقت هذه الحسابات مع القيم العملية والنظرية بصورة جيدة عند ادخال تأثير استقطاب القلب.

## Introduction

Nuclei far from the stability lines open a new test ground for nuclear models. Recently, many experimental and theoretical efforts have been paid to study structure and reaction mechanism in nuclei near drip lines. Modern radioactive nuclear beams and experimental detectors reveal several unexpected structure of light nuclei with the mass number A, 10–24 such as the existence of halo and skins [1], modifications of shell closures [2], and Pigmy resonances in electric dipole transitions [3]. The physical mechanism of these phenomena might originate from a large asymmetry of mean fields between protons and neutrons as well as the large extension of the wave functions.

Electromagnetic observables will provide useful information to study the structure of nuclei, not only ground states but also excited states. Namely, these observables are expected to pin down precise information of deformations and unknown spin parities of both stable and unstable nuclei since the deformation is intimately related to observables such as *Q* moments and *E*2 transitions[4]. We will study deformations of C isotopes as typical light open shell nuclei. Calculated *Q* moments are compared with experimental data to establish the isotope dependence of deformations in these nuclei.

In light and medium mass nuclei, the shell model has been known as one of the most successful models in describing the nuclear structure in both the ground state and the excited states [5]. The effective charges have been used commonly in the shell model calculations to study *Q* moments. In this paper, shell model calculations are performed with two effective interactions, Millener-Kurath (MK) in (1975) [6], and Cohen-Kurath (CK) interactions in (1965) [7].

Halo nuclei are weakly bound and spatially extended systems; they are threshold phenomenon, as the binding energy of the last nucleon(s) becomes small, the nucleon(s) becomes in the proximity of the particle continuum, the tail of the wave function extends more and more outward the central nuclear confining potential well which leads to the formation of a diffuse nuclear cloud due to quantum-mechanical penetration (the so-called nuclear halo); in turn such large diffusivity causes unusual spatial properties of the nucleon density distribution, leading to nuclear sizes deviating substantially from the $R \approx rA^{1/3}$ rule. Halo nuclei are fragile and oversized, they are expected to appear along the driplines, their structure are imagined to be composed of a tightly bound core surrounded by one or few loosely nucleons (two-nucleon halo is called Borromean [8]; where none of the binary subsystems of the core plus two-nucleons are found in bound structure). Halo nucleon(s) prefers to occupy orbits with low orbital quantum numbers, in s- or p-orbital; to lower the confining effect coming from Coulomb and centrifugal barrier which push or suppress the tail of the radial wave function toward core; leading to non-halo behavior. The half-life time for halo nuclei are in general less than one second. Because of the rapid decay of these nuclei, it is rather difficult to make targets with them, therefore, experiments have been done in inverse kinematics (i.e., the role of target and projectile are exchanged) with a beam of exotic nuclei incident on a stable target at radioactive ion beam (RIB) facilities.

The nuclear deformation can be investigated through the measurements of electromagnetic transitions and moments. The quadrupole (*Q*) moment data of exotic nuclei provide guides to the structure of associated nuclear state which give serious tests of nuclear structure studies [1].

The structure of light neutron-rich nuclei can be understood within the shell model. Shell model within a restricted 1*p* model space is not appropriate to describe *Q* moments and transitions for light *p*-shell nuclei. Expanding the model space to include $2\hbar w$ configurations in describing the form factors, Cichoki et al. [9] found that only 10% improvement was realized. The electromagnetic properties can be supplemented to the usual shell model treatment by allowing excitations from the core and model space orbits into higher orbits. A perturbative theory treatment was made of core-polarization effects in electromagnetic and inelastic scattering transitions due to high-lying collective excitations, which showed that there was a natural disparity in neutron and proton polarizations[10].

The conventional approach to supply this added ingredient to shell model wave functions is to redefine the properties of valence nucleons from those exhibited by actual nucleons in free space to model-effective values [11]. Effective charges are introduced for evaluating *E*2 transitions in shell-model studies to take into account effects of model-space truncation. A systematic analysis has been made for observed B(*E*2) values with shell-model wave functions using a least-squares fit with two free parameters gave standard proton and neutron effective charges, $e_p^{eff.}=1.3$, $e_n^{eff.}=0.5$ (e) [12], in *sd*-shell nuclei.

An interpretation of effective charge in valence nuclear models was proposed in which they are seen as proportional to derivatives of the "collectivity" with charges in proton or neutron number [13].

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which allows one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these *Q* properties. These effects provide a more practical alternative for calculating nuclear collectivity. These effects are essential in describing transitions involving collective modes such as $E2$ transition between states in the ground-state rotational band, such as in $^{18}$O[14].

**The aim of the present work**

In the present work, we will adopt a harmonic oscillator (HO) model space to calculate the quadrupole ($Q$) moment of some light exotic nuclei ($^{9,11,17,19}$C isotopes ). The nuclear shell model calculations are performed using the OXBASH shell model code [15], where the one body density matrix (*OBDM* ) elements of the core and halo parts in spin-isospin formalism are obtained. One particle-one hole (1p-1h) excitations from the core and model space will be taken into consideration through first-order perturbation theory. These 1p-1h excitations from the core and model space orbits are considered into all higher allowed orbits with 6 $\hbar w$ excitation. Excitations up to 6 $\hbar w$ seem to be large enough for sufficient convergence [16]. These excitations are essential in obtaining a reasonable description of the data.

Effective charges are calculated in this work for different model spaces used in this work and compared with those of stable nuclei. The standard constant effective charges, are $e_p^{eff.}$ =1.3 e and $e_n^{eff.} = 0.5$ e [12], with the harmonic oscillator wave functions which explain successfully $Q$ moments and $B(E2)$ transitions in many stable *p*- and *sd*-shell nuclei [17]. Our calculations depend on the first order core-polarization.

**Theory**

The one-body density matrix elements *(OBDM)* contains all the information about transitions of given multiplicities, which is imbedded in the model space wave function and is given in second quantization as:

$$OBDM(J_f, J_i, \alpha, \beta, J, t_z) = \frac{\langle J_f \| [a^+_{\alpha,t_z} \otimes \tilde{a}_{\beta,t_z}]^J \| J_i \rangle}{\sqrt{2J+1}} \quad (1)$$

where $J_i$ , $J_f$ initial and final total angular momentum (spin) respectively and $t_z$ = 1/2 for a proton and -1/2 for a neutron. The initial and final single particle states ($n\ l\ j\ t_z$) are denoted by $\alpha$ and $\beta$, respectively.

As the nuclear shell wave functions have good isospin, it is appropriate to evaluate the *OBDM* elements by means of isospin-reduced matrix elements. The relation between tripled-reduced *OBDM* and the proton or neutron *OBDM* is given by [18]:

$$OBDM(J_f, J_i, \alpha, \beta, J, t_Z) = (-1)^{T_f - T_Z} \left[ \begin{pmatrix} T_f & 0 & T_i \\ -T_Z & 0 & T_Z \end{pmatrix} \sqrt{2} \times \frac{OBDM(J_f, T_f, J_i, T_i, \alpha, \beta, J, T=0)}{2} + (2t_Z)\sqrt{6} \begin{pmatrix} T_f & 1 & T_i \\ -T_Z & 0 & T_Z \end{pmatrix} \frac{OBDM(J_f T_f, J_i T_i, \alpha, \beta, J, T=1)}{2} \right] \quad (2)$$

where $T_i$ , $T_f$ are initial and final isospin respectively and $T_z = \frac{Z-N}{2}$ , where Z, N are protons and neutrons numbers, respectively.

The *OBDM* elements for ($J$=0) is given by:

$$OBDM(T=0) = n_j(p+n)\sqrt{\frac{2T_i+1}{2}}\sqrt{\frac{2J_i+1}{2j+1}}$$

$$OBDM(T=1) = n_j(p-n) \times \frac{\sqrt{T_i(2T_i+1)(T_i+1)}}{\sqrt{6}T_z}\sqrt{\frac{2J_i+1}{2j+1}} \quad (3)$$

where $n_j(p+n)$ and $n_j(p-n)$ are the occupation numbers of the single particle state $j$.

The average occupation numbers in each $j$ is given by**:**

$$n(j_a, t_z) = OBDM(a, a, t_z, J=0) \times \sqrt{(2j_a+1)/(2J_i+1)} \quad (4)$$

The root mean square radius in terms of occupation numbers for the harmonic oscillator potential with size parameter is given by [19]:

$< r^2 > = 1/A \sum_{tz} \sum_a n\,(j_a\,,\,t_z)\,[(2n+l-1/2)\,b^2]$ (5)

where *n* is occupation number and A is mass number A=Z+N.

The electric quadrupole moment *Q*, representing a deviation from a spherical distribution of the electric charges in a nucleus, is sensitive to the admixture of collective components. In particular, if the valence nucleons are of neutron type, the observation of *Q* gives a useful measure of how the core is polarized by the presence of added particles, since in this case the valence particles themselves are neutral and should not directly contribute to the electric quadrupole moment. The quadrupole moment is an excellent tool to study the deformation of nuclei.

The electric quadrupole operator is defined by[20].

$$\hat{Q} = e_{tz}\,(3z^2 - r^2) \quad (6)$$

where $e_{tz}$ are the charges for the proton and neutron in units of e. For the fee-nucleon charge we would take $e_p$ =1 and $e_n$ =0, for the proton and neutron, respectively. Quadrupole moments are usually quoted in units of e $fm^2$ or e barn where the "barn" =100 $fm^2$ [21].

The one-body electric multipole transition operator with multipolarity *J* for a nucleon is give by [19]:

$$\hat{O}_{JM}(\vec{r})_k = e(k)\, r_k^J\, Y_{JM}(\Omega_k) \quad (7)$$

and quadrupole moment operator for a nucleon *k* is

$$Q(\vec{r})_k = \sqrt{\frac{16\pi}{5}}\, e(k)\, r_k^2\, Y_{20}(\Omega_k) \quad (8)$$

where *e*(*k*) is the electric charge for the *k*-th nucleon**.** Since *e*(*k*) = 0 for neutron, there should appear no direct contribution from neutrons; however, this point requires further attention: The addition of a valence neutron will induce polarization of the core into configurations outside the adopted model space. Such core polarization effect is included through perturbation theory which gives effective charges for the proton and neutron.

For *n* nucleons, the electric mutipole transition operator is [19]

$$\hat{O}_{JM}(\vec{r}) = \sum_{k=1}^{n} e(k)\, r_k^J\, Y_{JM}(\Omega_k) \quad (9)$$

$$\hat{O}_{JM}(\vec{r}) = \sum_{k=1}^{n} \left( e_{\mathrm{p}} \frac{1+\tau_z(k)}{2} + e_{\mathrm{n}} \frac{1-\tau_z(k)}{2} \right) r_k^J\, Y_{JM}(\Omega_k) \quad (10)$$

where $\tau_z|\mathrm{p}\rangle = |\mathrm{p}\rangle$, $\tau_z|\mathrm{n}\rangle = -|\mathrm{n}\rangle$**.** Equation (10) can be rearranged to

$$\hat{O}_{JM}(\vec{r}) = \sum_{k=1}^{n} \left( \frac{e_{\mathrm{p}} + e_{\mathrm{n}}}{2} + \frac{e_{\mathrm{p}} - e_{\mathrm{n}}}{2}\, \tau_z(k) \right) r_k^J\, Y_{JM}(\Omega_k) \quad (11)$$

which can be written as

$$\hat{O}_{JM}(\vec{r}) = e_{\mathrm{IS}} \sum_{k=1}^{n} r_k^J\, Y_{JM}(\Omega_k) + e_{\mathrm{IV}} \sum_{k=1}^{n} r_k^J\, Y_{JM}(\Omega_k)\, \tau_z(k) \quad (12)$$

where $e_{\mathrm{IS}} = \frac{e_{\mathrm{p}} + e_{\mathrm{n}}}{2}$ and $e_{\mathrm{IV}} = \frac{e_{\mathrm{p}} - e_{\mathrm{n}}}{2}$, are the isoscalar and isovector charges, respectively**.** The bare proton and neutron charges are denoted by $e_{\mathrm{p}}$ and $e_{\mathrm{n}}$, respectively**.**

The reduced matrix element in both spin-isospin spaces of the electric transition operator $\hat{O}_\Lambda$ is expressed as the sum of the product of the elements of the one-body density matrix (OBDM) $X^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha,\beta)$ times the single-particle matrix elements, and is given by:

$$\left\langle \Gamma_f \,|||\, \hat{O}_\Lambda \,|||\, \Gamma_i \right\rangle = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha,\beta) \left\langle \alpha \,|||\, \hat{O}_\Lambda \,|||\, \beta \right\rangle \quad (13)$$

where $\alpha$ and $\beta$ label single-particle states (isospin is included) for the shell model space**.** The states $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\Lambda \equiv JT$ .

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation to describe *EJ* excitations: these are called core polarization effects. These effects can be included directly by including effective chares for the protons and neutrons rather than the bare charges [11].

Using Wickner-Eckart theorem, the single particle matrix elements reduced in both spin and isospin, are written in terms of the single-particle matrix elements reduced in spin only

$$\left\langle \alpha_2 \, ||| \, \hat{O}_\Lambda \, ||| \, \alpha_1 \right\rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \left\langle j_2 \, || \, \hat{O}_{Jt_z} \, || \, j_1 \right\rangle \quad (14)$$

with $I_T(t_z) = \begin{cases} 1 & \text{for} \quad T=0 \\ (-1)^{1/2-t_z} & \text{for} \quad T=1 \end{cases}$ (15)

The single particle matrix element of the electric transition operator reduced in spin is

$$\left\langle j_2 \, || \, \hat{O}_{Jt_z} \, || \, j_1 \right\rangle = e_{t_z} \left\langle j_2 \| Y_J \| j_1 \right\rangle \left\langle n_2 \ell_2 \left| r^J \right| n_1 \ell_1 \right\rangle \quad (16)$$

where $\left| n\ell \right\rangle$ is the single-particle radial wave function.

The reduced single-particle matrix element of the Electric transition operator in isospin formalism becomes

$$\left\langle \alpha_2 \, ||| \, \hat{O}_\Lambda \, ||| \, \alpha_1 \right\rangle = e_T \sqrt{2(2T+1)} \left\langle j_2 \| Y_J \| j_1 \right\rangle \left\langle n_2 \ell_2 \left| r^J \right| n_1 \ell_1 \right\rangle \quad (17)$$

where $e_T$ is the isoscalar ($T$=0) and isovector ($T$=1) charges.

The Quadrupole moment is given by [19],

$$Q(J=2) = J \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \left\langle J_i T_i \, ||| \, \hat{O}_{JT} \, ||| \, J_i T_i \right\rangle \quad (18)$$

which can be written as

$$Q(J=2) = J \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{JT} \quad (19)$$

where $M_{JT} = \left\langle J_f T_f \, ||| \, \hat{M}_{JT} \, ||| \, J_i T_i \right\rangle$. The isoscalar ($T$=0) and isovector ($T$=1) charges are given by:

$$e_0 = e_{\mathrm{IS}} = \frac{1}{2} e, \quad e_1 = e_{\mathrm{IV}} = \frac{1}{2} e. \quad (20)$$

To include core polarization effects, the Quadrupole moment can be represented in terms of only the model space matrix elements using effective charges as.

$$Q = 2 \begin{pmatrix} J_i & 2 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{5}} \sum_{T=0,1} e_T^{\mathrm{eff}} (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{JT} \quad (21)$$

The proton and neutron effective charges are.

$$e_{\mathrm{p}}^{\mathrm{eff}} = e + \delta e_{\mathrm{p}}, \quad e_{\mathrm{n}}^{eff} = \delta e_{\mathrm{n}} \quad (22)$$

## Results and discussion

New exotic feature in nuclear physics appeared, when some neutron-rich isotopes of light elements, such as $^{19}C$, were found to have exceptionally large radii. The new exotic feature in nuclear physics is of great interest not only because it constitutes a stringent test for the available nuclear models but also because it opens up new research fields in nuclear science. Upon discovery in 1985, this phenomenon was attributed to either large deformation on to a long tail in the matter distribution [1].

In the description of the halo nuclei it is important to take into account a model space for the core different from that of the halo neutrons (protons) which have to be treated separately in order to explain their properties. This assumption is supported by the fact that the valence neutrons (protons) are distributed in a spatial region which is much larger than the core.

In this paper, we study the properties of the ground states of C isotopes by performing shell model calculations. Special emphasis will be put on the quadrupole (*Q)* moments for odd C isotopes, which will manifest their new shell structure. The core-polarization (CP) contributions are calculated with microscopic theory with one particle-one hole excitations up to all higher allowed orbits with 6 $\hbar w$ excitations, as described in Ref. [16].

**$^9$C nucleus ($J^\pi T = \frac{3}{2}^- \frac{3}{2}$), Z=6 , N=3**

Recently, Matsuta et al. [22], studied several properties regarding this nucleus, such as interaction cross sections, magnetic moment and effective root mean square matter radius (effective charges radii). It is believed that $^9$C has a proton skin and a large matter radius due to the large difference between the proton and neutron numbers. In fact, these quantities may relate to the puzzle of the large isoscalar spin expectation value deduced from the magnetic moment [23]. In (2004), Utsuno suggested that the shell quenching in $^9$C accounts for the anomalous magnetic moment and breaks the mirror symmetry in the ground-state wave functions [24]. Therefore, $^9$C is an interesting nucleus among carbon isotopes. In addition, this isotope located at the proton drip line has no bound excited states below the one- and two-proton separation energies. They have developed a recoil proton spectrometer to measure the elastic scattering of protons with radioactive ion beams. Using the spectrometer, cross sections for proton elastic scattering from $^9$C at 277–300 MeV/nucleon were measured. The main purposes of their work are to measure the unstable $^9$C nucleus and deducing the matter radius. The matter radius was deduced with the relativistic folding model formulated by Murdock and Horowitz [25]. The root-mean-square matter radius was deduced to be 2.43 fm. This value is consistent with the radii deduced from the measurements of the interaction cross sections and the reaction cross sections. It also agrees with the experimental radii of the mirror nucleus $^9$Li, which is equal to 2.44 fm [22].

The ground state is $3/2^-$ [26]; it is described in terms of five nucleons distributed in p-shell (model-space) outside a closed 1s-shell. Our calculations are performed with Cohen-Kurath (CK) interaction[7]. The experimental matter rms radius of $^9$C, which is used in the calculations is 2.42 fm [27], which gives b= 1.695 fm for the size parameter of the HO potential. The calculated quadrupole moments for ground state are -2.059 e fm$^2$ with no core polarization effects, -3.319 e fm$^2$ with CP effect with effective charges $e_p^{eff.} = 0.978$, $e_n^{eff.} = 0.560$(e) and -3.842 e fm$^2$ with standard effective charges $e_p^{eff.}$ =1.3(e), $e_n^{eff.} = 0.5$(e)[12]. No experimental data are available till now. So, we will resort to compare the results with other theoretical results.

Other theoretical calculations performed by Sagawa at al. [26], for the same purpose. The shell model calculations are performed with two effective interactions: Millener-Kurath (MK)[6] and Warburton-Brown (WBP) [28] interactions. They applied the isospin-depended core polarization charges with Hartree-Fock (HF) wave functions to study *Q* moments of odd C isotopes. Both interactions are commonly used in mean field calculations and also random phase approximations for the excited states. Essential differences of the two interactions are nuclear incompressibility and spin properties. The WBP Hamiltonian is designed to reproduce systematically the energies of the ground and excited states of stable *p*- and *sd*-shell nuclei. The MK Hamiltonian is also constructed for stable *p*- and *sd*-shell nuclei. They expected some differences in the results of the two interactions in the magnitude and sign of deformations especially, in odd nuclei. They found that the two interactions give essentially the same results as far as the deformations are concerned [26,29].

The values of *Q* moments are -4.63 e fm$^2$ for MK interaction and -3.88 e fm$^2$ for WBP interaction; these values are taken from Ref.[26], and -3.61 e fm$^2$ for WBP interaction from [29]. These results are compared with those obtained by a standard shell model calculation with harmonic oscillator (HO) wave functions and constant effective charges. We see that the three values of *Q* moments of $^9$C have same signs as our calculated value. Also, we see that, our value of *Q* moment with CK interaction is very close to the calculated value of *Q* moment of Ref. [26,29]. The results are listed in the Table I.

**$^{11}$C nucleus ($J^{\pi}T = \frac{3-}{2}\ \frac{1}{2}$)**, Z=6 , N=5

General information about this nucleus $^{11}$C, like all the other carbon isotopes described in Antisymmetrized Molecular Dynamics (AMD) theory are expected to have a proton density for the ground state with an oblate deformation, and a different shape for the neutron density with an triaxial deformation [30]. $^{11}$C is short-lived nucleus, with half-life of 20 min. It's low-lying spectroscopy is relatively well-known , where the excited state below the proton separation threshold is $S_p$= 8.69 MeV [31].

The ground state is 3/2$^-$. The nucleus $^{11}$C is considered as a $1s_{1/2}$ core with four nucleons, plus seven active nucleons outside the core distributed over the p-shell orbits. Our calculations are performed with Cohen-Kurath (CK) interaction. The experimental matter rms radius of $^{11}$C, which is used in the calculations is 2.12 fm [27]. The calculated quadrupole moment for ground state is 1.162 e fm$^2$ with bare charges for proton and neutron (no core polarization effects). This value underestimates the experimental value 3.426 e fm$^2$ [26]. Core-polarization calculation with effective charges $e_p^{eff.} = 1.143,\ e_n^{eff.} = 0.489\ (e)$ for the proton and neutron, respectively, the value of quadrupole moment becomes 2.426 e fm$^2$ and 2.633 e fm$^2$ with standard effective charges, which is close to the experimental value by about a factor of 0.8, as shown in the Table I.

If we compare this result with that of its mirror nucleus $^{11}$B, we found that the two results for two quadrupole moments are very close to each other considering only the model space contributions. The experimental value of the *Q* moment of $^{11}$C is 3.426 e fm$^2$, which is about a factor of 0.6 of the experimental value of $^{11}$B, which is 4.070 e fm$^2$ [32]. Also the interaction cross sections of $^{11}$B is consistent with that of $^{11}$C [33], and root mean square deduced by a Glauber-model analysis with an optical-limit (OL) of $^{11}$B is 2.09 fm which also consistent with that of $^{11}$C [27].

Also we compared the results with those obtained by Sagawa at al. [26] and Suzuki et al [29]; Our *Q* value is very close to that of Suzuki et al. [29], as shown in the Table I.

**$^{17}$C nucleus ($J^{\pi}T = \frac{3+}{2}\ \frac{5}{2}$)**, Z=6, N=11

Early experimental studies suggested not a possible halo structure for $^{17}$C. The momentum distribution of the fragment $^{16}$C from $^{17}$C was found to be relatively broad[34-36]. The interaction cross section ($\sigma_I$) at 965 MeV/A did not show a significant enhancement to its neighbors [1]. These indicated that there was no halo-structure for $^{17}$C. However, subsequent experimental studies gave a conflicting result. The measurement of the reaction cross section ($\sigma_R$) by Wu et al. [37] for $^{17}$C on $^{12}$C at 79 MeV/A suggested that $^{17}$C was a one-neutron halo nucleus. Finally, they showed us the necessity of a long tail structure for $^{17}$C by use of the Glauber-type analysis.

$^{17}$C, with small one-neutron separation energy $S_n$ = 0.729 ±0.018 MeV and large two-neutron separation energy $S_{2n}$ = 4.979 ±0.018 MeV [38], is an interesting candidate for a one-neutron halo nucleus. Since without the Coulomb barrier, the valence neutron separation energy could mostly confirm neutron-halo structure. $^{17}$C is a typical psd-shell nucleus, the valence neutron radial wave function exhibits configuration mixing of the s and d-wave. If the valence neutron has a d-dominant configuration, the radial extension of the wave function will not be significant[39]. Density distribution of $^{17}$C deduced by modified Glauber with deformed Woods-Saxon (WS) core plus single particle model (SPM) type functional shape. The center of mass effect was taken into account [37]. The results show that $^{17}$C has a tail structure, though a d-wave dominant configuration hinders the radial extension of the wave function. Although, the definition of halo structure is still ambiguity, we can conclude that $^{17}$C is a mostly halo-like nucleus. The deformation may explain the broad momentum distribution of the fragment $^{16}$C from $^{17}$C.

The 1/2$^+$ state as a candidate for the ground state, while the 3/2$^+$ and 5/2$^+$ states show good agreement with the experimental value within a few percent. Complementary experimental data are available for the selection rule on $\beta$ decay from $^{17}$C to $^{17}$N, which favor the spin $3/2^+$ assignment for the ground state of $^{17}$C [29],[40]. The measurement of *Q* moment will be the most decisive experiment to assign the spin and the parity of the ground state of $^{17}$C , and will provide experimental support of the shell model predictions.

$^{17}$C is described in terms of 13 nucleons distributed in psd-shell model-space outside a closed 1s-shell. The calculations are performed with MK interaction. This information was confirmed by Elekes et al. [41] measurement. The experimental matter rms radius of $^{17}$C is 2.72 fm [27]. Using the configuration discussed above, we calculated quadrupole *Q* moment, which is found to be 0.5084 e fm$^2$, for state $3/2^+$, with bare charges for proton and neutron (no core polarization effects) and 2.554 e fm$^2$, with standard effective charges for proton and neutron.

In comparison to Sagawa et al. calculations [26], the *Q* moment value of $^{17}$C was 2.89 e fm$^2$. Their calculations are performed with MK interaction. We see that the value of *Q* moment of $^{17}$C close to that we calculated by about a factor of 0.3 and have same signs. Also Suzuki et al. [29] calculated the quadrupole *Q* moments for carbon isotopes, but their calculations are performed with WBP interaction. For $^{17}$C the *Q* value is 2.63 e fm$^2$, for state $3/2^+$ too. It is close and has same sign. The result are listed in the Table I together with available *Q* moments.

**$^{19}$C nucleus ($J^{\pi}T = \frac{3+}{2}\ \frac{7}{2}$), Z=6, N=13:**

$^{19}$C is a one-neutron halo nucleus composed of the core $^{18}$C nucleus plus one loosely bound neutron surrounding the core; the one-neutron halo is considered to be in $2s_{1/2}$ orbit. The separation energy of the outer (halo) neutron is $S_{1n}$=0.16(11) MeV. Many theoretical and experimental studies discussed and confirmed the halo structure in $^{19}$C [26,27,42-45].

The effects of halo [1] are studied. The single-particle energy of the neutron $2s_{1/2}$ orbit obtained in Woods-Saxon potentials is adjusted to reproduce the experimental separation energy of 0.16 MeV [46,47], for $^{19}$C. The one neutron separation energy of $^{19}$C is still under dispute experimentally, and is between 0.16 MeV and 0.5 MeV [36]. In C isotopes, the halo consists mainly of the 2*s* orbit and there is no way to get coherence in the dipole transitions. The effects of the halo or skin are, thus, found to be rather small in the Suzuki et al. calculations [29], for $^{17}$C, and $^{19}$C.

The spin and the parity of the ground state is $1/2^+$ or $5/2^+$ [36]. According to the shell model and the deformed HF calculations, the lowest $3/2^+$ state is also close to the lowest $1/2^+$ and $5/2^+$ states in energy. The neutron and the proton contributions to the *Q* moments are −15.9 mb and −14.0 mb in the $3/2^+$ state, while they are 16.3 mb for neutrons and 14.9 mb for protons in the $5/2^+$ state of $^{19}$C. Notice that the proton and neutron contributions are very different from the single-particle value of the pure $1d_{5/2}$ state. These results suggest the large configuration mixing in the lowest $5/2^+$ state of $^{19}$C. Since the magnetic moment and *Q* moment are very different for the three configurations in $^{19}$C, measurements of these moments will give decisive information on the spin assignment of the ground state of $^{19}$C. The calculations are performed with the model space $P_{1/2}$-sd, outside the $^{12}$C closed core. The valence 5 neutron are distributed with four neutrons over the d orbit, and one over the $2s_{1/2}$ orbits. Reehal-Wildenthal interaction is used [48]. The experimental matter rms radius of $^{19}$C, which is used in the calculations is 3.13 fm [27], which gives oscillator size parameter $b_{halo}$ equals to 1.944 fm. The matter rms radius for $^{18}$C core nucleus is 2.82 fm [27].

Since no valence proton are present in the model space, the *Q* moment is zero for $1/2^+$ state. Using the standard effective charge, which give the contribution of the core proton and neutron. The *Q* moments are -2.645 e fm$^2$ and 3.21 e fm$^2$ for the $3/2^+$ and $5/2^+$ state, respectively.

In comparison to Sagawa et al. calculations [26], the *Q* moment value of $^{19}$C was -3.21 e fm$^2$. For state $3/2^+$ their calculations are performed with MK interaction. Also Suzuki et al. [29] calculated the quadrupole *Q* moments for $^{19}$C, but their calculations are performed with WBP interaction. The *Q* value is -3.31 e fm$^2$, for state 3/2$^+$ too. It is close and has same sign too. They also calculated the *Q* moment value of $^{19}$C for state 5/2$^+$. In Sagawa et al. calculations, the *Q* moment value of $^{19}$C was 3.13 e fm$^2$, with WBP interaction. This value is close to that we calculated and has same sign. Also they calculated *Q* moment of $^{19}$C but this time their calculations are performed with MK interaction and the value of *Q* moment was -0.051 e fm$^2$, for state 5/2$^+$. It underestimates other calculations. The result are listed in the Table I together with available *Q* moments.

***Table I: Matter rms radii , oscillator parameter b and the quadrupole Q moment calculated and experimental values (with and without effective charges).***

| A $J^{\pi}$ T | $J_f^{\pi}$ | rms (fm) Exp. | b (fm) Calc. | $e_p^{eff.}$ | $e_n^{eff.}$ $(e)$ | $Q\,(efm^2)$ Our Cal. | Exp. | $Q\,(efm^2)$ with (CP) MK. [26] | WBP. [26,29] |
|---|---|---|---|---|---|---|---|---|---|
| $^9$C (3/2$^-$ 3/2) | 3/2$^-$ | 2.42 | 1.695 | 1.0<br>.978<br>1.3 | 0 .0<br>0.560<br>0 .5 | -2.05904<br>-3.31975<br>-3.8425 | | -4.63 | -3.61<br>-3.88 |
| $^{11}$C (3/2$^-$ 1/2) | 3/2$^-$ | 2.12 | 1.450 | 1.0<br>1.143<br>1.3 | 0.0<br>0.489<br>0.5 | 1.162286<br>2.425859<br>2.633024 | \|3.426\| | 4.76 | 2.89<br>4.27 |
| $^{17}$C (3/2$^+$ 5/2) | 3/2$^+$ | 2.72 | 1.741 | 1.0<br>1.3 | 0.0<br>0.5 | 0.50847<br>2.5545 | | 2.89 | 2.63<br>2.39 |
| $^{19}$C (3/2$^+$ 7/2) HALO | 3/2$^+$ | $^{18}$C = 2.82<br>$^{19}$C = 3.13 | 1.784 | 1.0<br>1.3 | 0.0<br>0.5 | 5.578 E-06<br>-2.645 | | -3.21 | -3.31<br>-3.00 |
| $^{19}$C (5/2$^+$ 7/2) HALO | 5/2$^+$ | $^{18}$C = 2.82<br>$^{19}$C = 3.13 | 1.784 | 1.0<br>1.3 | 0.0<br>0.5 | 1.565 E-06<br>3.210 | | -0.051 | 0.11<br>3.13 |

## Conclusions

Shell model calculations are performed for ($^9$C, $^{11}$C, $^{17}$C and $^{19}$C) including core-polarization effects through first-order perturbation theory. In general, there are some notes have been indicated from the present work which can be explained as:

The measurement of *Q* moment will be the most decisive experiment to assign the spin and the parity of the ground state of carbon isotopes and other exotic nuclei, and will provide experimental support of the shell model predictions. The effect in light nuclei is unique compared with that in rare-earth nuclei

The long tail behavior, considered as a distinctive feature of halo nuclei, is evidently in the sense that the prolate and oblate deformations appear clearly in the ground states of the isotopes.

Our calculations refer to all odd carbon isotopes that we studied have prolate deformations except $^9$C have oblate deformations. Also we compared the results of $^9$C and $^{11}$C with their mirror nuclei $^9$Li and $^{11}$B, where a reasonable agreement are obtained. The results of *Q* moments for ($^9$C, $^{11}$C, $^{17}$C and $^{19}$C) compared with that of other studies which also agrees with their calculations.

revealed in the calculated neutron and matter density distributions. Besides, the noticeable difference that is found between the calculated overall proton and neutron rms radii of $^{19}C$ also indicates a halo structure. Small effective charges are obtained for neutron rich nuclei, and their values are close to each other. Average effective charges can be deduced from this study, to be used for future work for other neutron rich nuclei, and also for studying the truncation rates and electron scattering form factors.

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