Nucleon momentum distributions and elastic electron scattering form factors for ^{48}Ti and ^{54}Fe nuclei

Ghaith N. Flaiyh, Ahmed A. Hamza

Department of physics, College of Science, University of Baghdad, Iraq

E-mail: Kaithnaima@yahoo.com

Abstract

The nucleon momentum distributions (NMD) for the ground state and elastic electron scattering form factors have been calculated in the framework of the coherent fluctuation model and expressed in terms of the weight function (fluctuation function). The weight function has been related to the nucleon density distributions of nuclei and determined from theory and experiment. The nucleon density distributions(NDD) is derived from a simple method based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states. The feature of long-tail behavior at high momentum region of the NMD has been obtained using both the theoretical and experimental weight functions. The observed electron scattering form factors for ^{48}Ti and ^{54}Fe nuclei are in reasonable agreement with the present calculations throughout all values of momentum transfer q.

Key words

Nucleon density distributions, Nucleon momentum distributions, Elastic electron scattering and fp shell nuclei.

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توزيعات زخم النيكليون وعوامل التشكل للاستطارة الالكترونية المرنه للنوى ^{48}Ti و ^{54}Fe

غيث نعمة فليح, احمد عدنان حمزه

قسم الفيزياء, كلية العلوم, جامعة بغداد, العراق

الخلاصة

تم أستخدام أنموذج التموج المتشاكه في حساب كل من توزيعات زخم النيكليونات النووي للحالة الأرضية وعوامل التشكل للأستطارة الألكترونية المرنة, حيث تم التعبير عنهما بدلالة دالة تسمى دالة التموج. لقد تم التعبير عن دالة التموج بواسطة توزيعات كثافة النيكليونات وتم حسابها من النتائج النظرية والعملية لتوزيعات كثافة النيوكليونات. ان حساب توزيعات كثافة النيكليونات يعتمد بالأساس على كل من اعداد اشغال الحالات النووية وعلى الدوال الموجية للجسيمة المنفردة المتواجدة في الجهد التوافقي.

تميزت نتائج توزيعات زخوم النيكليونات (المستندة على دالة التموج النظرية والعملية) بصفة الذيل الطويل عند منطقة الزخم العالي. أظهرت هذه الدراسة بأن عوامل التشكل النظرية تتفق مع النتائج العملية للنوى (^{54}Fe و ^{48}Ti).

Introduction

The study of momentum wave functions is a powerful tool in studying the ground state properties of nuclei, particularly the momentum distribution of nucleons. The quantitative knowledge of the momentum distribution is very important for revealing the proper mechanism of the nuclear reactions and for their complete description and also it is important in studying the bulk properties of the nucleus such as: total binding and kinetic energies and many other properties of the nucleus [1]. At present, there is no method for directly measuring the NMD in nuclei. The quantities that are measured by particle-nucleus and nucleus-nucleus collisions are the cross sections of different reactions, and these contain information on the *NMD* of target nucleons. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei established the existence of long-tail behavior of the *NMD* at high momentum region ($k \geq 2\,fm^{-1}$) [2-6]. In principle, the mean field theories cannot describe correctly *NMD* and form factors simultaneously [7] and they exhibit a steep-slope behavior of *NMD* at high momentum region. In fact, the *NMD* depends a little on the effective mean field considered [7] due to its sensitivity to short-range and tensor nucleon-nucleon correlations [8] which are not included in the mean field theories.

There are several theoretical methods used to study elastic electron-nucleus scattering, such as the plan-wave Born approximation (PWBA), the Eikonal approximation and the phase-shift analysis method [9-15]. The PWBA method can give qualitative results and has been used widely for its simplicity. To include the Coulomb distortion effect, which is neglected in PWBA, the other two methods may be used. In the past few years, some theoretical studies of elastic electron scattering off exotic nuclei have been performed. Wang et al. [11, 12] studied such scattering along some isotopic and isotonic chains by combining the Eikonal approximation with the relativistic mean field theory. And, very recently, Roca-Maza et al. [13] systematically investigated elastic electron scattering off both stable and exotic nuclei with the phase-shift analysis method. Karataglidis and Amos [14] have studied the elastic electron scattering form factors, longitudinal and transverse, from exotic (*He* and *Li*) isotopes and from ^{8}B nucleus using large space shell model functions. Very recently, Chu et al. [15] have studied the elastic electron scattering along *O* and *S* isotopic chains and shown that the phase-shift analysis method can reproduce the experimental data very well for both light and heavy nuclei.

In the coherent fluctuation model (CFM), which is exemplified by the work of Antonov et. al. [4, 16], the local *NDD* and the *NMD* are simply related and expressed in terms of an experimentally obtainable fluctuation function (weight function) $|f(x)|^2$. They [4, 16] investigated the *NMD* of (^{4}He and ^{16}O), ^{12}C and (^{39}K, ^{40}Ca and ^{48}Ca) nuclei using weight functions $|f(x)|^2$ expressed in terms of, respectively, the experimental two parameter Fermi ($2PF$) *NDD* [17], the experimental data of Ref. [18] and the experimental model-independent *NDD* [17]. It is important to point out that all above calculations obtained in the framework of the *CFM* proved a high momentum tail in the *NMD*. Elastic electron scattering from ^{40}Ca nucleus was also investigated in Ref. [16], where the calculated elastic differential cross sections ($d\sigma/d\Omega$) were found to be in good agreement with those of experimental data.

The aim of the present work is to derive an analytical form for the *NDD* applicable throughout all $1f-2p$ shell nuclei based on the use of the single particle harmonic oscillator wave functions and the occupation numbers of the states. The derived *NDD* is employed in determining the theoretical weight function $|f(x)|^2$ which is used in the *CFM* to study the *NMD* and elastic form factors for some $1f-2p$ shell nuclei, such as ^{48}Ti and ^{54}Fe nuclei. We shall see later that the theoretical $|f(x)|^2$, based on the derived *NDD*, is capable to give information about the *NMD* and elastic electron scattering form factors.

Theory

In the *CFM* [4, 16], the mixed density is given by

$$\rho(r,r') = \int_0^\infty |f(x)|^2 \rho_x(r,r')dx \qquad (1)$$

where

$$\rho_x(r,r') = 3\rho_0(x)\frac{j_1(k_F(x)|\vec{r}-\vec{r}\,'|)}{k_F(x)|\vec{r}-\vec{r}\,'|} \times \theta\left(x - \frac{|\vec{r}+\vec{r}\,'|}{2}\right) \qquad (2)$$

is the density matrix for A nucleons uniformly distributed in the sphere with radius x and density $\rho_0(x) = 3A/4\pi x^3$. The Fermi momentum is defined as[4,16]

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} = \left(\frac{9\pi A}{8}\right)^{1/3}\frac{1}{x} = \frac{\alpha}{x}\ ; \qquad \alpha = \left(\frac{9\pi A}{8}\right)^{1/3} \qquad (3)$$

and the step function θ is defined by

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \qquad (4)$$

Equation (1) corresponds to the general statement of the *CFM* in which the *NDD* of the nuclear matter fluctuates around the average distribution, keeping spherical symmetry and uniformity. The diagonal element of Eq.(1) gives the one-particle density as

$$\rho(r) = \rho(r, r' = r) = \int_0^\infty |f(x)|^2 \rho_x(r)dx \qquad (5)$$

In Eq.(5), $\rho_x(r)$ and $|f(x)|^2$ have the following forms [16]

$$\rho_x(r) = \rho_0(x)\theta(x - |\vec{r}|) \qquad (6)$$

$$|f(x)|^2 = -\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}\Big|_{r=x} \qquad (7)$$

The weight function $|f(x)|^2$ of (7), determined in terms of the *NDD* $\rho(r)$, satisfies the normalization condition

$$\int_0^\infty |f(x)|^2 dx = 1 \qquad (8)$$

and holds only for monotonically decreasing *NDD*, i.e. $\frac{d\rho(r)}{dr} < 0$.

On the basis of Eq.(5), the *NMD*, $n(k)$, is expressed as [16]

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k)dx \qquad (9)$$

where

$$n_x(k) = \frac{4}{3}\pi x^3 \theta(k_F(x) - |\vec{k}|) \qquad (10)$$

is the Fermi-momentum distribution of the system with density $\rho_0(x)$. By means of Eqs.(7), (9) and (10), an explicit form for $n(k)$ is expressed in terms of $\rho(r)$ as

$$n(k) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{A} \times \left[6\int_0^{\alpha/k} \rho(x)x^5 dx - \left(\frac{\alpha}{k}\right)^6 \rho\left(\frac{\alpha}{k}\right)\right] \qquad (11)$$

with normalization condition

$$\int n(k)\frac{d^3k}{(2\pi)^3} = A$$

The *NMD* of $1f-2p$ shell nuclei is also determined by the shell model using the single particle harmonic oscillator wave function in momentum representation and is given by

$$n(k) = \frac{b^3}{\pi^{3/2}} e^{-b^2k^2}\left\{10 + 8(bk)^4 + (A-40)\frac{8}{105}(bk)^6\right\} \qquad (12)$$

where b is the harmonic oscillator size parameter.

The form factor $F(q)$ of the nucleus is also expressed in the CFM and is given by [16]

$$F(q) = \frac{1}{A}\int |f(x)|^2 F(x,q)dx \qquad (13)$$

where $F(x,q)$ is the form factor of uniform charge density distribution given by

$$F(x,q) = \frac{3A}{(qx)^2}\left[\frac{\sin(qx)}{qx} - \cos(qx)\right] \qquad (14)$$

Equation (14) reflects the physical scattering picture, inherent by the CFM, in which the scattering amplitude is a superposition of different uniform charge distributions. The nucleon finite size (fs) form factor is defined by $F_{fs}(q) = Exp(-0.43q^2/A)$ [10] and $F_{cm}(q) = Exp(q^2b^2/4A)$ is the correction for the lack of translational invariance in the shell model (center of mass correction) [10]. Inclusion of $F_{fs}(q)$ and $F_{cm}(q)$ in the calculations requires multiplying the form factor of Eq.(13) by these corrections.

It is important to point out that all physical quantities studied above in the framework of the CFM such as $n(k)$ and $F(q)$ are expressed in terms of the weight function $|f(x)|^2$. Therefore, it is worthwhile trying to obtain the weight function firstly from the NDD of $2PF$ model extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The NDD of $2PF$ model is given by [17]

$$\rho_{2PF}(r) = \rho_0 \Big/ \left(1 + e^{\frac{r-c}{z}}\right);$$

$$\rho_0 = \frac{A}{4\pi}\frac{1}{\int_0^\infty \left(1+e^{\frac{r-c}{z}}\right)^{-1} r^2 dr} \qquad (15)$$

The experimental weight functions $|f(x)|^2_{2PF}$ is obtained by introducing eq. (15) into eq.(7)

Theoretically, the NDD of one body operator can be written as [10]

$$\rho(r) = \frac{1}{4\pi}\sum_{nl}\zeta_{nl}\,4(2l+1)\phi^*_{nl}(r)\phi_{nl}(r) \qquad (16)$$

Here ζ_{nl} is the nucleon occupation probability of the state nl ($\zeta_{nl} = 0$ or 1 for closed shell nuclei and 0< ζ_{nl} <1 for open shell nuclei) and $\phi_{nl}(r)$ is the radial part of the single particle harmonic oscillator wave function.

The NDD of $1f-2p$ shell nuclei is derived on the assumption that there is a core of filled $1s$, $1p$ and $1d$ shells and the occupation numbers of nucleons in $2s$, $1f$ and $2p$ shells are equal to, respectively, $4-\delta$, $A-40-\delta$ and δ. Using this assumption with the help of Eq. (16), an analytical form for $\rho(r)$ is obtained as

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{10 - \frac{3\delta}{2} + \frac{11}{3}\delta\left(\frac{r}{b}\right)^2 + (8-2\delta)\left(\frac{r}{b}\right)^4 + \left(\frac{8}{105}(A-40) + \frac{4}{15}\delta\right)\left(\frac{r}{b}\right)^6\right\} \qquad (17)$$

Here, the parameter δ characterizes the deviation of the nucleon occupation numbers from the prediction of the simple shell model (δ=0). The central NDD, $\rho(0)$, is obtained from Eq.(18) as

$$\rho(0)=\frac{1}{\pi^{3/2}b^{3}}\left\{10-\frac{3\delta}{2}\right\} \qquad (18)$$

Therefore, δ can be obtained from (18) as

$$\delta=\frac{2}{3}\left\{10-\pi^{3/2}b^{3}\rho(0)\right\} \qquad (19)$$

Using Eq.(17) in Eq.(7), an analytical expression for theoretical weight function $|f(x)|^2$ is obtained as

$$|f(x)|^2=\frac{8\pi}{3Ab^2}x^4\rho(x)-\frac{16}{3A\pi^{1/2}b^5}x^4 \times e^{-x^2/b^2}\left\{\frac{11}{6}\delta+(8-2\delta)\frac{x^2}{b^2}+\left(\frac{4}{35}(A-40)+\frac{2}{5}\delta\right)\frac{x^4}{b^4}\right\} \qquad (20)$$

Results and discussion

The nucleon momentum distributions $n(k)$ and elastic form factors for $1f-2p$ shell nuclei are studied by means of the CFM. The distribution $n(k)$ of Eq.(9) is calculated in terms of the weight function obtained firstly from the fit to the electron-nuclei scattering experiments, $|f(x)|^2{}_{2PF}$ and secondly from theory, as in Eq.20.

The harmonic oscillator size parameters b are chosen in such a way as to reproduce the measured root mean square radii (rms) of nuclei. The parameters δ are determined by introducing the chosen values of b and the experimental densities $\rho_{\exp}(0)$ into Eq.(20). The values of b and δ together with the other parameters employed in the present calculations for ^{48}Ti and ^{54}Fe nuclei are listed in Table 1. The calculated rms $\langle r^2\rangle_{cal}^{1/2}$ and those of experimental data $\langle r^2\rangle_{\exp}^{1/2}$ [17] are displayed in this table as well for comparison. The comparison shows a remarkable agreement between $\langle r^2\rangle_{cal}^{1/2}$ and $\langle r^2\rangle_{\exp}^{1/2}$ for all considered nuclei.

The dependence of the NDD on $r(fm)$ for ^{48}Ti and ^{54}Fe nuclei is shown in Fig.1. The solid circles are experimental nucleon density distributions of two parameter Fermi [17] whereas the solid and dotted curves are the calculated NDD of considered nuclei when $\delta\neq0$ and $\delta=0$, respectively. These figures show a very clear discrepancy between the dotted curve and the experimental data(solid circles curve)especially at the central region. Taking the effect of the occupation number of higher orbits δ into consideration leads to reduce this discrepancy significantly at the central region and subsequently improves the result by brining the distribution of solid curves closer to the experimental data.

The dependence of the $n(k)$ (in fm^3) on k (in fm^{-1}) for ^{48}Ti and ^{54}Fe nuclei is shown in Fig.2. The dash-dotted distributions are the NMD's of (12) obtained by the shell model calculation using the single particle harmonic oscillator wave functions in the momentum representation. The solid circles and solid curves distributions are the NMD's obtained by the CFM and expressed in terms of the experimental and theoretical weight functions, respectively. It is clear that the behavior of the dash-dotted distributions reproduced by the shell model calculations is in contrast with those of the dashed and solid distributions reproduced by the CFM. The important feature of the dash-dotted distributions is the steep slope behavior when k increases. This behavior is in disagreement with other studies [4, 5, 7, 8] and it is attributed to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into account the important effects of the short range dynamical correlation functions. Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the *NMD* [5, 7]. It is seen that the dashed and solid distributions deviate slightly from each other around the region of momentum $k > 3\,fm^{-1}$. It is also noted that the general structure of the dotted and solid distributions at the region of high momentum components is almost the same for ^{48}Ti and ^{54}Fe nuclei, where these distributions have the feature of long-tail behavior at momentum region $k \geq 2\,fm^{-1}$. In fact, the feature of long-tail behavior obtained by the *CFM*, which is in agreement with other studies [4, 5, 7, 8], is related to the existence of high densities $\rho_x(r)$ in the decomposition (5), though their weight functions $|f(x)|^2$ are small.

The elastic electron scattering form factors from the considered spin zero nuclei are calculated in the framework of the *CFM* as given by Eq.13.

The present results for elastic form factors are plotted versus the momentum transfer q for ^{48}Ti and ^{54}Fe nuclei as shown in Fig.3. The dotted and solid curves are the calculated results obtained, respectively, without and with including the corrections of $F_{fs}(q)$ and $F_{cm}(q)$. The solid circles are the experimental data of elastic form factors for considered nuclei. It is clear that the experimental data [17] are in reasonable agreement with both calculations of the solid and dotted curves throughout all values of q. Including $F_{fs}(q)$ and $F_{cm}(q)$ corrections in the calculations of the ^{48}Ti and ^{54}Fe nuclei leads to slight reduction in the calculated elastic form factors throughout all values of q. All the first and second diffraction minima are reproduced in the correct places.

Summary and Conclusions

The *NMD* and elastic electron scattering form factors, calculated in the framework of the *CFM*, are expressed by means of the weight function $|f(x)|^2$. The weight function, which is connected with the local density $\rho(r)$, was determined from experiment and from theory. The feature of the long-tail behavior of the *NMD*, which is in accordance with the other studies [4, 5, 7, 8], is obtained by both theoretical and experimental weight functions and is related to the existence of high densities $\rho_x(r)$ in the decomposition Eq.(5), though their weight functions are small. The observed elastic electron scattering form factors from ^{48}Ti and ^{54}Fe nuclei are in reasonable agreement with the present calculations of the *CFM* throughout all values of q. It is noted that the theoretical *NDD* Eq.(18) employed in the determination of the theoretical weight function Eq.(21) is capable of reproducing information about the *NMD* and elastic form factors.

Table 1: The values of various parameters employed in the present calculations together with $\langle r^2 \rangle_{cal}^{1/2}$ and $\langle r^2 \rangle_{exp}^{1/2}$.

Nuclei	$2PF$ [17]		Experimental central NDD [17]	Calculated parameters and rms of the present work			Experimental rms [17]
	c (fm)	z (fm)	$\rho_{exp}(0)$ (fm^{-3})	b (fm)	δ	$\langle r^2 \rangle_{cal}^{1/2}$ (fm)	$\langle r^2 \rangle_{exp}^{1/2}$ (fm)
^{48}Ti	3.843	0.588	0.1636795	1.990	1.875	3.622	3.691
^{54}Fe	4.075	0.506	0.1652131	2.000	1.757	3.699	3.710

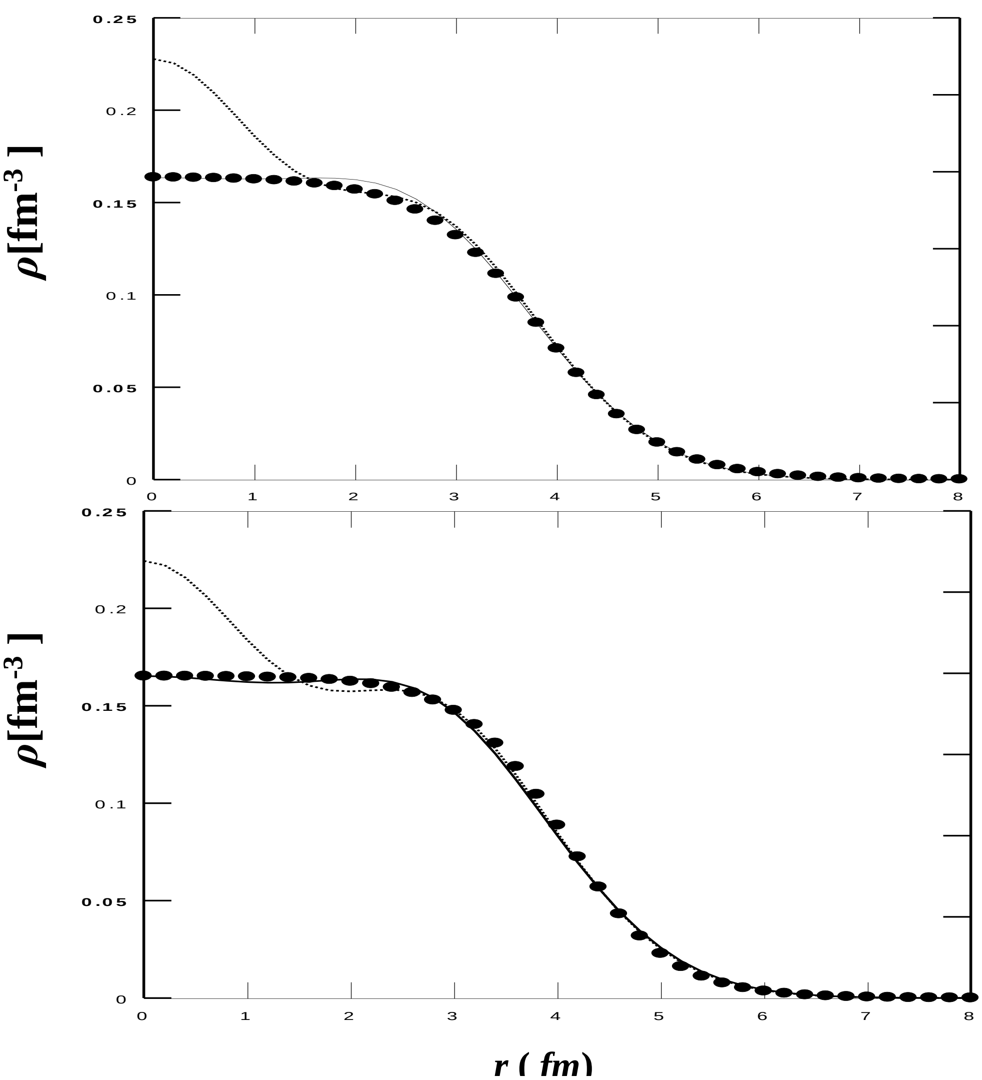

Fig.1: The dependence of the NDD on $r(fm)$ for ^{48}Ti and ^{54}Fe nuclei. The dotted and solid curves are the calculated NDD using eq. (17) when δ =0 and $\delta \neq 0$, respectively. The symbols of solid circles are the experimental data [17].

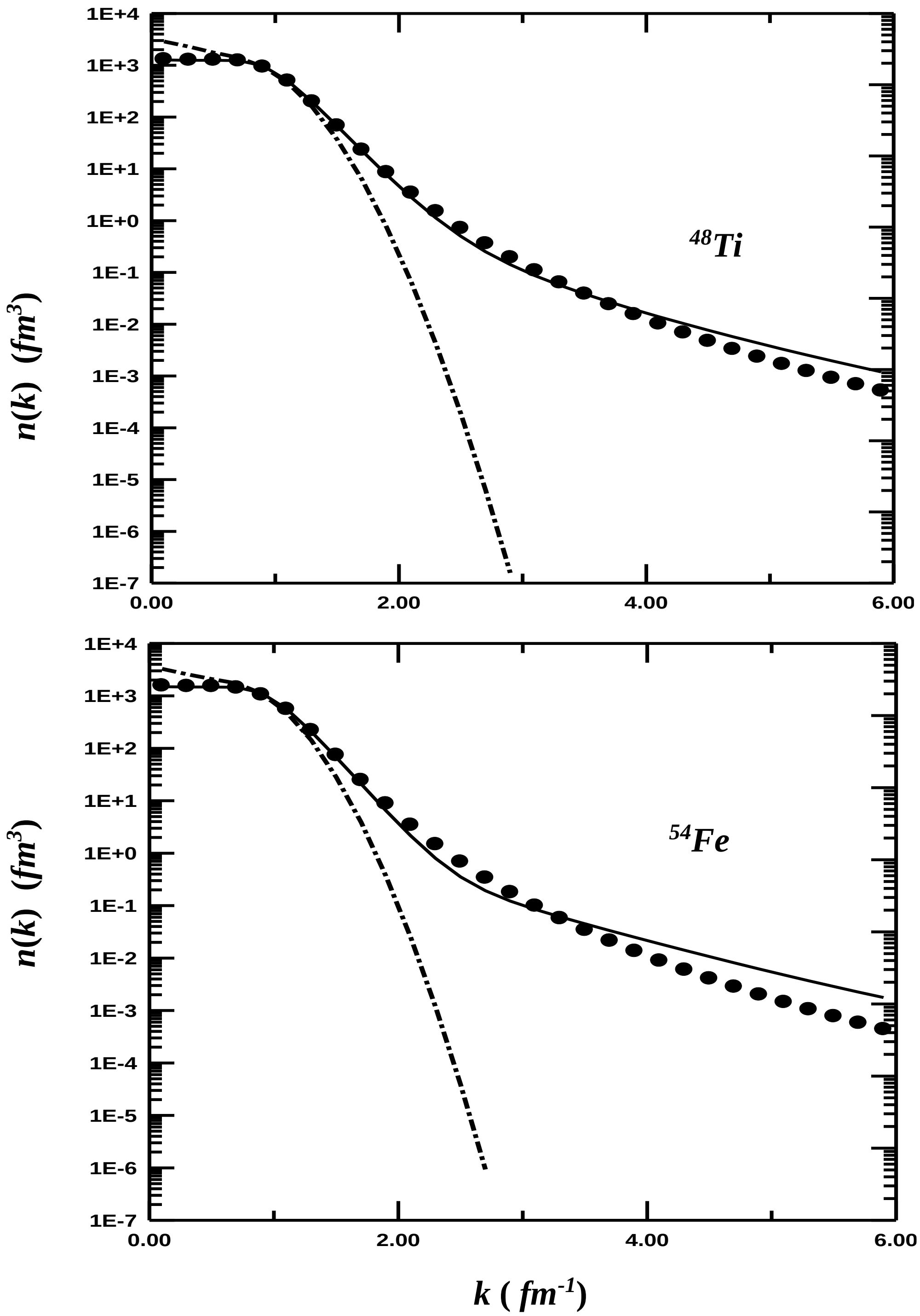

Fig.2. The nucleon momentum distributions (NMD's) for ^{48}Ti and ^{54}Fe nuclei. The dashed-dotted distributions are the results obtained by the shell model calculation using the single particle harmonic oscillator wave functions in the momentum representation. The solid circles and solid distributions are the calculated results expressed by the CFM using the experimental and theoretical weight functions respectively.

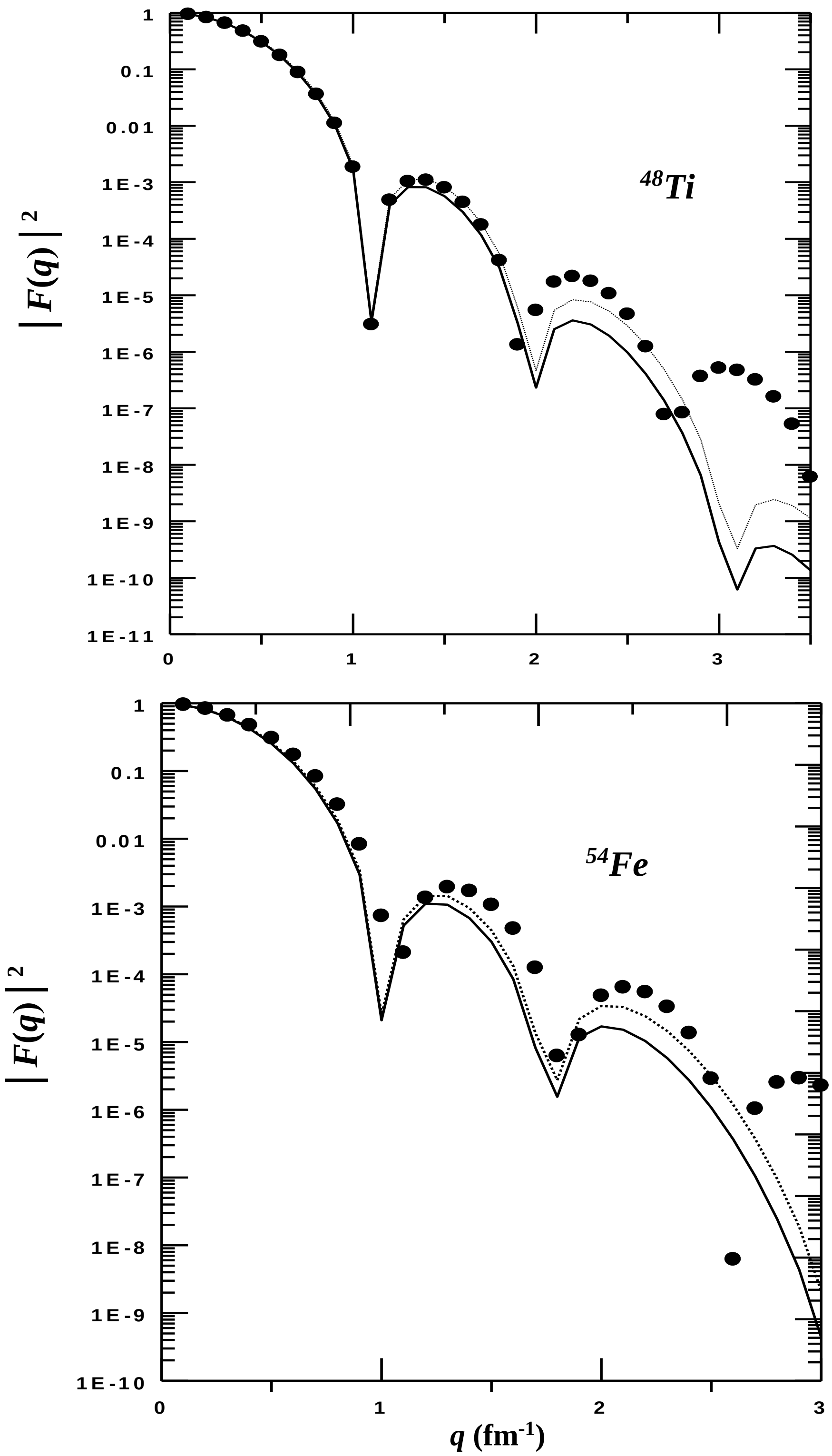

***Fig.3:** Elastic electron scattering is drawn as a function of momentum transfer q for ^{48}Ti and ^{54}Fe nuclei. The dotted and solid curves are the calculated results without and with including the corrections (nucleon finite size and center of mass corrections), respectively. The solid circles are the experimental data, taken from Ref. [17].*

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