Microscopic calculations of the electric Quadrupole transition strengths of Be isotopes (9, 10, 12, 14)

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Abstract

Electric Quadrupole transitions are calculated for beryllium isotopes (9, 10, 12 and 14). Calculations with configuration mixing shell model usually under estimate the measured *E2* transition strength. Although the consideration of a large basis no core shell model with $2\hbar\omega$ truncations for 9,10,12 and14 where all major shells *s*, *p*, *sd* are used, fail to describe the measured reduced transition strength without normalizing the matrix elements with effective charges to compensate for the discarded space. Instead of using constant effective charges, excitations out of major shell space are taken into account through a microscopic theory which allows particle–hole excitations from the core and model space orbits to all higher orbits with $2\hbar\omega$ excitations which are called core-polarization effects. The two body Michigan sum of three ranges Yukawa potential (M3Y) is used for the core-polarization matrix element. The simple harmonic oscillator potential is used to generate the single particle matrix elements of all isotopes considered in this work. The b value of each isotope is adjusted to reproduce the experimental matter radius, These size parameters of the harmonic oscillator almost reproduce all the root mean square (rms) matter radii for ^{9,10,12,14}Be isotopes within the experimental errors. Almost same effective charges are obtained for the neutron- rich *Be* isotopes which are smaller than the standard values. The major contribution to the transition strength comes from the core polarization effects. The present calculations of the neutron-rich ^{12,14}Beisotopes show a deviation from the general trends in accordance with experimental and other theoretical studies. The configurations arises from the shell model calculations with core-polarization effects reproduce the experimental *B*(E2) values.

Key words

electric Quadrupole, transition strength of exoticnuclei, shell model calculations.

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حسابات مجهرية للأنتقالات الرباعية الكهربائية لنظائر البريليوم(9,10,12,14)

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الخلاصـــــــــة

حسبت الانتقالات الكهربائية الرباعية القطب لنظائر البريليوم (9,10,12,14) استنادا على انموذج القشرة مع فضاءات انموذج القشرة *psd, p*. حسابات انموذج القشرة ذو التشكيلات المختلطة مع انموذج الفضاءات المحدودة عادة تقلل من قوة تقدير الانتقال E2 المقاس, استخدم انموذج القشرة بدون قلب الموسع مع قطع $2\hbar w$ لـ 9,10,12,14 حيث اخذنا بنظر الاعتبار كل القشرة الرئيسية s, p ,sd.

حيث فشلت في وصف قوة الانتقال المختزل المقاس بدون معايرة عناصر المصفوفة مع الشحنات الفعاله للتعويض عن الفضاءات المهملة بدلا من استخدام شحنات فعاله ثابتة. فقد اخذت التهيجات خارج فضاءات القشرة

الرئيسية بنظر الاعتبار من خلال النظرية المايكروية التي تسمح بتهيجات جسيمة – فجوة من القلب ومدارات انموذج الفضاء لكل المدارات الاعلى مع تهيجات $2\hbar w$ الذي يسمى تأثير استقطاب القلب كذالك تم حساب عناصر المصفوفة لاستقطاب القلب بواسطة جهد M3Y, استخدم جهد المتذبذب التوافقي البسيط لتوليد عناصر المصفوفة للجسيمة المفردة لكل النظائر المعتبرة في هذا العمل. افترض قيمة الـ b لكل نظير لأعادة نصف القطر الكتلي العملي. هذه المعلمات الحجمية للمتذبذب التوافقي على الأغلب تعيد توليد كل انصاف الاقطارالكتلية لنظائر البريليوم ^{9,10,12,14}Be ضمن الأخطاء العملية. استحصلت نفس الشحنات الفعالة لنظائر البريليوم الغنية بالنيوترونات والتي هي اصغر من القيم القياسية. قيم الـ*B*(E2) المحسوبة لنظائر البريليوم ^{10,12}Be تتفق بصورة جيدة جدا مع القيم العملية, تأتي المساهمة الرئيسية لقوة الانتقال من تأثيرات استقطاب القلب. حساباتنا لنظائر البريليوم ^{12,14}Be تبين انحراف عن الميول الرئيسية للحسابات العملية والاخرى النظرية. تنشأ التشكيلات من حسابات انموذج القشرة من تاثير استقطاب القلب التي تولد قيم الـB(E2) العملية.

Introduction

The study of the properties of extremely neutron or proton rich nuclei of light elements is considered as an important and exciting research topic in modern nuclear physics. The term " halo" refers to the weakly bound nucleon or nucleons forming a cloud of low density around a core of normal density. It appeared first in a paper by Hansen and Jonson in 1987 [1]. Since then it has become the label for a few light exotic nuclei with weakly bound nucleons in spatially extended states where the radius of the halo system is significantly larger than the normal nuclear radius. A nuclear halo is a structure with a dilute matter distribution which extends far beyond the core of the nucleus. All halo nuclei have the same features as having an extended low-density distribution (or a large nuclear matter radius) and low binding energies of valence nucleons surrounding the core. The study of structure of halo nuclei which are near or at the drip lines (including neutron-drip line and proton- drip line on the Z-N plane) has attracted the interests of many scientists and researchers all over the world. The electric quadrupole moment *Q*, representing a deviation from a spherical distribution of the electric charges in a nucleus, is sensitive to the admixture of collective components. In particular, if the valence nucleons are of neutron type, the observation of *Q* gives a useful measure of how the core is polarized by the presence of the added particles, since in this case the valence particles themselves are neutral and should not directly contribute to the electric quadrupole moment. The ground state quadrupole moment of exotic nuclei can be obtained using several different techniques. As an example, the quadrupole moment can be deduced from the reduced transition probabilities, *B*(E2). For even-even nuclei, rotational energy levels are much simpler than in odd-even or odd-odd nuclei. Therefore, even-even nuclei are preferentially used to extract *B*(E2) values and to deduce the intrinsic quadrupole moment. However, the *B*(E2) connects two states and extraction of quadrupole moment of a respective state is theory dependent.Effective charges were introduced for evaluating E2 transitions in shell-model studies to take into account effects of model-space truncation. A systematic analysis had been made for observed B(E2) values with shell-model wave functions using a least-squares fit with two free parameters gave proton and neutron effective charges, $e_p^{eff.} = 1.3\ e$ and $e_n^{eff.} = 0.5\ e$[2] in sd-shell nuclei.

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which allows one particle–one hole (1*p*–1*h*) excitations of the core and also of the model space to describe these E2 excitations. These effects provide a more practical alternative for calculating nuclear collectivity. These effects are essential in describing transitions involving collective modes such as E2 transition between states in the ground-state rotational band, such as in ^{18}O [3].

The structure of the beryllium (Z = 4) isotopic chain is dominated by the presence of clusterization and neutron halos. The noteworthy features of each isotope on the neutron-rich side of this chain will be described here.

Beryllium-9 is the only stable isotope, being bound by the presence of an additional neutron. It retains the highly deformed double-alpha shape; due to the proximity of the low-lying $2\alpha - 1n$ threshold at 1.67 MeV (α is a ^4_2He nucleus,^{9}Be has a good structureα+α+$1n$in a cluster model). Two rotational bands have been observed in its spectrum, built on the ground and first-excited states [4]. The most deeply bound isotope is Beryllium-10, which has a neutron separation energy of6.8 MeV and a β $-$decay half life of 1.5×10^{-6}years. The $\alpha - \alpha$ clusterization continues in this nucleus near the corresponding threshold at 8.48 MeV [5]. The ground state is also strongly deformed, with a quadrupole deformation parameter $\beta_2$of 1.14 [6]. Beryllium-12 provides an excellent benchmark for the structure of neutron-rich nuclei because the neutron number forms a closed shell in stable nuclei. Shell closures define the framework of the nuclear chart, occurring at the magic numbers 2, 8, 20, 28, 50, 82, and 126. Magic nuclei have several characteristic qualities, including spherical shape, strong binding energy, and elevated excitation energies for the first 2^+ states. The first excited state of ^{12}Be lies at 2.10 MeV. Less is known about the heavier isotopes of beryllium. Barker predicted that low lying 0^+ states in ^{12}Be should be formed with the same ^{10}Be core and a pair of neutrons in the $2s_{1/2}$, $1p_{1/2}$, or $1d_{1/2}$ orbit [7]. Theoretical calculations which study the mixing with intruder orbitals from the sd shell also exist.

Clustering structure [8, 9, 10] or few body structures with a ^{12}Be composed from a^{10}Be core plus two valence neutrons [11,12] have been investigated as well.

Beryllium-14 is bound with respect to two neutron emissions by 1.26 MeV. Total interaction cross-section measurements for this nucleus are consistent with the presence of a two-neutron halo or skin [13]. Since the first observation of ^{14}Be and ^{17}Bin 1973 [14] and ^{12}Be in 1965 [15], interest in these nuclei has greatly increased. Some properties, such as the mass and matter radius have been studied and reported by Liatard et al. [16], Tanihata et al. [17] and Ozawa et al. [18]. The halo structure of ^{14}Be has been confirmed by Zahar et al. [19] in fragmentation experiments of ^{14}Be on a ^{12}C target. They suggest a strong correlation between the two external neutrons. The ^{14}Be nucleus has been investigated in the three cluster generated coordinate method, involving several^{12}Be+n+n configurations by Descouvemout [20]. The ^{12}Be core nucleus was described in the harmonic oscillator model with all possible configurations in the p-shell. A strong enhancement of the root mean square (rms) radius with respect to the ^{12}Be core was obtained, in agreement with experiment. The microscopic wave functions were used to investigate several aspects of the ^{14}Be spectroscopy.

Aim of this work

In this present work, the fundamental relations are used to get the reduced transition strength *B*(E2) for some Beryllium isotopes in the range A=9,10,12,14.A good imagination for the nuclear structure of these isotopes has adopted using different model spaces and interactions. These *B*(E2) values represent basic nuclear information complementary to the knowledge of the energies of low-lying levels in these nuclides. As well, the calculations are depending on basic equations which explained the relation between different parameters to reproduce the root mean square radius (rms) to fix the size parameter b of the single-particle (HO) wave function. In this study, the size parameter plays the role of a characteristic length of the harmonic-oscillator potential. Also, the rms matter radius for these isotopes is reproduced. For the quadrupole transition strength (J=2), excitation from the core and model space will be taken into consideration through first-order perturbation theory, where (1*p*-1*h)* excitations are taken into considerations. These (1*p*-1*h)* excitations from the core and model space orbital are considered into all higher allowed orbits with excitations up to $6\hbar\omega$. Effective charges are calculated in this work through the microscopic theory discussed above for the different model spaces used, and compared with each of the stable nuclei and the standard effective charges $e_p^{eff.} = 1.3\ e, e_n^{eff.} = 0.5$. The calculated *B* (E2) values will be compared with the most recent experimental data. The nuclear shell model calculations were performed using the OXBASH shell model code [21], where the one body density matrix (*OBDM*) elements of the core and halo parts in spin-isospin formalism are obtained. For halo nuclei, the two frequency harmonic oscillator single-particle wave functions are used for the core and the halo nucleon, to reproduce the rms matter radii. Calculations are presented with model space (MS) only, and with core-polarization (CP) effects. Instead of using effective charges for the model space nucleons to account for the core-polarization effects, a microscopic calculation are adopted to include these effects, and to calculate the photon point effective charges.

Theory

The many-particles reduced matrix elements $\hat{O}_{J,t_z}$ operator can be expressed as the sum of the product of the elements of the one-body density matrix (OBDM) times the single-particle matrix elements:

$$\langle J_f|||\hat{O}_{JT}|||J_i\rangle = \sum_{\alpha,\beta} OBDM^{J,T}(i,f,j_\alpha,j_\beta)\langle\alpha|||\hat{O}_{JT}|||\beta\rangle \qquad 1$$

The single particle states are designated by α and β for initial and final states, respectively.

The reduced single-particle matrix element in spin-isospin space is given as:

$$\langle\alpha|||\hat{O}_{JT}|||\beta\rangle = \sqrt{\frac{2T+1}{2}}\sum_{t_z} I_T(t_z)\langle\alpha||\hat{O}_{JT}||\beta\rangle \qquad 2$$

where,

$$I_T(t_z) = \begin{cases} 1 & for\ T=0 \\ (-1)^{\frac{1}{2}-t_z} & for\ T=1 \end{cases} \quad 3$$

The single-particle matrix element $\langle\alpha|||\hat{O}_{Jt_z}|||\beta\rangle$ for the electric transition operator can be calculated as [22].

$$\left\langle \alpha \left\| \hat{O}_{J t_z} \right\| \beta \right\rangle = e_{t_z} \left\langle l_\alpha \tfrac{1}{2} j_\alpha \left\| Y_J(\Omega_r) \right\| l_\beta \tfrac{1}{2} j_\beta \right\rangle \times \left\langle n_\alpha l_\alpha | r^J | n_\beta l_\beta \right\rangle \quad 4$$

where

$t_z = \frac{1}{2}$ *for proton* & $t_z = -\frac{1}{2}$ *for neutron*

l_α, l_β denote quantum numbers (orbital angular quantum number) in coordinate space and isospace.

The reduced matrix element of the spherical harmonic is given by [22]:

$$\left\langle l_\alpha \tfrac{1}{2} j_\alpha \| Y_J(\Omega_r) \| l_\beta \tfrac{1}{2} j_\beta \right\rangle = \tfrac{1}{2}(-1)^{j_\alpha + \frac{1}{2}} \left[1 + (-1)^{l_\alpha + l_\beta + J}\right] \left[\frac{(2j_\alpha+1)(2j_\beta+1)(2J+1)}{4\pi}\right]^{\frac{1}{2}} \times \begin{pmatrix} j_\alpha & J & j_\beta \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \quad 5$$

$$\langle n_\alpha l_\alpha | r^J | n_\beta l_\beta \rangle = \int_0^\infty dr\, r^J r^2 R_{n_\alpha l_\alpha}(r) R_{n_\beta l_\beta}(r) \quad 6$$

$$\int_0^\infty dr\ r^4 R_{n_\alpha l_\alpha}(r) R_{n_\beta l_\beta}(r) = b^2 \left(N + \frac{3}{2}\right) \quad 7$$

where N=2(n-1)+ ℓ, with n is the principalquantum number.
The One-Body Density Matrix Elements (OBDM) in isospin representation is obtained from the value of $OBDM^{J t_z}$ as[23]:

$$OBDM^{J,t_z} = (-1)^{T_f - T_z} \sqrt{2} \begin{pmatrix} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \frac{OBDM\,(T=0)}{2} + (2t_z)\sqrt{6} \begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \frac{OBDM\,(T=1)}{2} \quad 8$$

where $T_z = \frac{Z-N}{2}$

The OBDM contains all the information about transitions of given multiplicities, which is imbedded in the model wave functions.

Root mean square radius in terms of occupation number

The average occupations number in each subshell j is given by:

$$occ\#(j, t_z) = OBDM\,(a, b, t_z, J=0) \sqrt{\frac{2j+1}{2J_i+1}} \quad 9$$

The root mean square radius for (p/n)is:

$$\langle r^2 \rangle_{t_z} = \frac{1}{N_{t_z}} \sum_a occ\#(t_z)_a \left(b^2\left(N_a + \frac{3}{2}\right)\right) \quad 10$$

and the matter rms radius is:

$$\langle r^2 \rangle_m = \frac{1}{A} \sum_a occ\#(a) \left(b^2\left(N_a + \frac{3}{2}\right)\right) \quad 11$$

where A is matter number A=Z+ N and b is the size parameter.

The reduced electric transition strength

The reduced electric transition strength is given by [22]:

$$B(EJ)=\frac{1}{(2J_i+1)}\left|\sum_{T=0,1}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix}\left\langle J_f T_f \,|||\tilde{\tilde{O}}_{JT}|||\, J_i T_i\right\rangle\right|^2 \quad 12$$

where the reduced matrix element of $\tilde{\tilde{O}}$ operator contain model space (MS). and the core polarization (CP) reduced matrix element, as given in Ref. [24]. Equation (12) can be written as:

$$B(EJ)=\frac{1}{(2J_i+1)}\left|\sum_{T=0,1}e_T\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix}\tilde{M}_{JT}\right|^2 \quad 13$$

where $\tilde{M}_{JT}=\left\langle J_f T_f \,|||\tilde{\tilde{M}}_{JT}|||\, J_i T_i\right\rangle$

The isoscalar (T=0) and sovector(T=1) charges are given by:

$$e_0=e_{IS}=\frac{1}{2}e \quad e_1=e_{IV}=\frac{1}{2}e$$

The B (E2) value can be represented in terms of only the model space matrix elements as:

$$B(EJ)=\frac{1}{(2J_i+1)}\left|\sum_{T=0,1}e_T^{\mathrm{eff}}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix}M_{JT}\right|^2 \quad 14$$

Then the isoscalar and isovector effective charges are given by**:**

$$e_T^{\mathrm{eff}}=\frac{M_{JT}+\Delta M_{JT}}{2M_{JT}}e=\frac{e_p+(-1)^T e_n}{2} \quad 15$$

The proton and neutron effective charges can be obtained as follows:

$$e_p=e_0^{eff}+e_1^{eff} \quad e_n=e_0^{eff}-e_1^{eff}$$

The above effective charges work for mixed isoscalar and isovector transitions.

Results and discussion

^{9}Be nucleus $(J^{\pi}_{g.s}T=3/2^-,1/2)$

The ground state of ^{9}Be is specified by $J^{\pi}T=(3/2^-,1/2)$, this is the only stable isotope. This nucleus considered as an inert ^{4}He core plus five nucleons are distributed over $1p_{3/2}-1p_{1/2}$ shell.

In this work we study the transition $J^{\pi}=3/2^-, T=1/2$, $J^{\pi}=5/2^-, T=1/2$ and $J^{\pi}=7/2^-, T=1/2$ states with *b*=1.66 fm.

The calculations are performed with *p*-shell model space ($0\hbar\omega$) with Cohen-Kurath interaction (CKI) [25].

The calculated *B*(E2) for $3/2^-1/2\rightarrow 3/2^-1/2$ state is 3.78 e^2fm^4 with no core polarization effects, and with CP effect 7.45 e^2fm^4This result compared with experimental value 17.1±0.03 [26] .The calculated effective charges are 1.178e, 0.408e for the proton and neutron, respectively. The calculated value underestimates the measured values even with core-polarization effects. Glickman *et al*. [26] predicted the value 9.72 e^2fm^4 using effective charges 1.15 and 0.45e for the proton and neutron respectively with full p-shell wave function of Cohen Kurath interaction. The calculated *B*(E2) depends strongly on the *b* value of the HO potential. The present choice for *b*=1.66 fm is chosen to reproduce the value of R_m=2.38±0.01fm[27]. Including CP effects gives the value of *B*(E2)9.4 $e^2fm^4$with the standard effective charge 1.3e, 0.5e for the proton and neutron, respectively, which still underestimates the measured value. The calculated *B*(E2) for the first excited state $3/2^-1/2\rightarrow 5/2^-\,1/2$ is10.43 e^2fm^4

with no core polarization effects and including CP effects gives 25.87$e^2fm^4$for $e_p^{eff.}$ =1.121e,$e_n^{eff.}$ =0.504e and 31.95 $e^2fm^4$for $e_p^{eff.}$ =1.3, $e_n^{eff.} = 0.5$ and this result is compared with the experimental value46.0$\pm$0.05 [26] and 40.65$\pm$0.03[28].

For the second excited state $3/2^- 1/2 \rightarrow 7/2^- 1/2$ the calculated *B*(E2) is 5.25 $e^2fm^4$with no core polarization effects. Including CP effects gives the value9.75 $e^2fm^4$for $e_p^{eff.}$ =1.157e, $e_n^{eff.} = 0.408e$ and 12.64$e^2fm^4$for $e_p^{eff.}$=1.3e,$e_n^{eff.} = 0.5e$.these results are compared with the experimental values 33$\pm$0.01[26] and 10.5$\pm$0.045[29]. Thus the present data for the ground state and {first, second} excited states appear to be in a little agreement with experimental data.

Large –basis model space with partially inert core was adopted by Radhi et al. [24] to study elastic and inelastic electron scattering from ^{9}Be. All major shells *s*, *p*,*sd* and *pf*were considered in their study, using core polarization effect with one particle – one hole excitation from all major shell orbits into all higher allowed orbits with excitation up to 10 $\hbar\omega$. Their result for the *B*(E2) values are 8.74, 37.11 and 13.37 e^2fm^4 for the $3/2^-$, $5/2^-$ *and* $7/2^-$ states, respectively. So, even with enlarging the model space, no major differences, are noticed in comparison with the result of *p-shell* model, especially with effective charges $e_p^{eff.} = 1.3e$ and $e_n^{eff.} = 0.5e$. The results of *B*(E2) and effective charges are tabulated in Table1.

Table 1:The calculated effective charges and B(E2) values of ^{9}Be compared with the experimental data.

A	$J^\pi\ T(g.s)$ $\tau_{1/2}$	J_f^π	**b** (fm)	**R**$_m$ (fm) [27]	$e_p^{eff.}$, $e_n^{eff.}$ (e)	**B(E2)** (e^2fm^4) Calc.	Exp.
9	$3/2^-\ 1/2$ **stable**	$3/2^-$	1.66	2.38$\pm$0.01	1.0, 0.0 1.178,0.408 1.3,0.5	3.78 7.45 9.40	17.1$\pm$0.03 [26]
		$5/2^-$			1.0,0.0 1.121,0.504 1.3,0.5	10.434 25.868 31.95	46.0$\pm$ 0.05 [26] 40.65$\pm$0.03 [28]
		$7/2^-$			1.0,0.0 1.157,0.408 1.3,0.5	5.249 9.75 12.64	33$\pm$0.01 [26] 10.5$\pm$ 0.045 [29]

^{10}Be **nucleus**$(J^\pi_{g.s}T = 0^+, 1)$

The ground state of ^{10}Be is specified by $J^\pi T = (0^+1)$with half-life=1.5×10^{-6} *years*, is a true bound state. According to the conventional 1*p*-shell model,^{10}Be is considered as a ^{4}He core in the 1*s*-shell and six nucleons outside the core distributed over the 1*p*-shell space.The configuration $(1p)^6$is used for the model space, outside the $(1s_{1/2})^4$inert core, using the Cohen-Kurath interaction (CK) [25]. It is well-known that ^{9}Be has a good $\alpha + \alpha + n$ structure with a ground state three-body separation energy of just 1.6 MeV. However, by adding one extra neutron to make ^{10}Be,the binding is increased to a^{9}Be+ n separation energy of 6.8MeV. This gives the ground and first excited states ($0_1^+, 2_1^+$) reasonably good shell-model-like structure.

We now turn our attention to the transitions in ^{10}Be and consider first the dominant ^{9}Be+ n configurations in the various states of interest as predicted by the microscopic cluster model (MCM) calculations of [30]. The 0^+ ground state of ^{10}Be looks almost entirely shell model like and, in a simple cluster model picture, can be thought of as a $p_{3/2}$ neutron coupled to the $3/2^-$ ground state of ^{9}Be. The first excited state,2_1^+, is also relatively simple to picture; in this case with the $p_{3/2}$neutron mainly coupling to the$5/2^-$ first excited state of ^{9}Be. This explains the very strong collective $B(E2)$ transition between these states, and the MCM and no core shell model (NCSM) predictions are in agreement[31].The calculations are performed with p-shell model space ($0\hbar\omega$) with (CKI) interaction. The size parameter b=1.587fmis chosen to reproduce the rms matter value R_m=2.30$\pm$0.02fm [27]. The calculated B(E2)for the state $0^+1 \rightarrow 2^+1$is 22.72 $e^2fm^4$with no core polarization effects. Including core-polarization effects gives values B(E2)=38.24 e^2fm^4 for $e_p^{eff.}$ =1.16e, $e_n^{eff.} = 0.468e$.(The effective charges in the proton–neutron representation)and B(E2)= 47.16 e^2fm^4 with the standard effective charges $e_p^{eff.}$ =1.3e, $e_n^{eff.} = 0.5e$.

These values are compared with the experimental values 51±0.05[29] and 53±0.06[28]. The most recent experimental value [32] is 46.0±0.15 e^2fm^4, which is very close to the theoretical value with the standard effective charges. The core-polarization effects give a strong modification to the quadrupole transition strength (J=2) where the core polarization effects enhance the quadrupole transition strength and bring the calculated values very close to the experimental data. The quadrupole transition calculated with the single particle harmonic oscillator is very sensitive to the single parameter value b. Exact measurement of the matter radius, will give the reasonable b value, which reflects on the calculated quadrupoletransition. The results of$B$(E2) and effective charges are tabulated in Table 2.

Table 2: The calculated effective charges and B(E2) values of ^{10}Be compared with the experimental data.

A $J^\pi\ T(g.s)$ $\tau_{1/2}$	J_f^π	b (fm)	R_m (fm) [27]	$e_p^{eff.}$, $e_n^{eff.}$ (e)	B(E2)(e^2fm^4) Calc.	Exp.
10 0^+1 **1.51e+6y**	2^+	1.587	2.30 $\pm$0.02	1.0,0.0	22.719	51$\pm$0.05[29]
				1.3,0.5	47.16	53$\pm$0.06[28]
				1.16,0.468	38.24	46.0$\pm$1.16[32]

^{12}Be nucleus$(J_{g.s}^\pi T = 0^+2)$

The ground state of ^{12}Be is with $J^\pi T = (0^+2)$ with half-life=23.6ms, ^{12}Be is also particle stable, but unstable with respect to$\beta -$decay. The number of neutrons in ^{12}Be is the “magic number” N = 8 but unlike heavier isotones,^{12}Be does not seem to have a closed neutron shell. Assuming a prolate deformation with an axis ratio of roughly 1: 2 shows that in the Nilsson model [33, 34] $1d_{5/2}$ level is coming down very strongly. From a Nilsson model point of view N = 8 cannot be expected to show any shell closure at large prolate deformation. The deformation is caused by the strong α clustering [35]. The last neutron pair is stated to be dominantly in the $(1s^2 + 0d^2)$ intruder configuration [36, 37]. A second 0^+state at 2.24 ± 0.02 MeV was found recently and the first excited state of ^{12}Be lies at 2.10 MeV. Its existence underlines the missing of the "magic" neutron number N = 8 [38]. In highly excited states ^{6}He+^{6}He, ^{5}He+^{7}He or α+^{8}He structures are expected [39].The neutron drip line ^{12}Be$(J^\pi T = 0^+2)$ is considered as a^{10}Be core $(J^\pi T = 0^+1)$ coupled to outer two neutrons $(J^\pi T = 0^+1)$. The calculations are performed with ***PSD***- model space with ***PSDMK***-interaction [40]. The configuration $(1s)^4(1p)^6$ is considered for^{10}Be,and $(sd)^{2\nu}$for the two halo neutrons. The size parameter b=1.587 fm is chosen to reproduce the rms matter radius 2.30±0.02 [27] for ^{10}Be and b=1.99 fm to reproduce the rms matter radius of ^{12}Be[27]. The calculated B(E2) value with bare charges (no CP effect) is 3.75 e^2fm^4 which underestimates the measured value 40±11±4[41].

Using the effective charges deduced from CP effects for^{10}Bethe B(E2) value become 69.81 e^2fm^4. The standard effective charges gives the value 81.74 e^2fm^4 for B(E2).

No core shell model calculations of [42], gives the value of B(E2)=23.0 e^2fm^4using CD-Bonn interaction. This value was obtained without effective charges. Dufour et al. [43] have used microscopic cluster calculations and deduced the value 63.0 e^2fm^4 for the $B(E2, 0^+ \rightarrow 2^+)$.

For non-halo ^{12}Be, the calculation are performed with P-shell model space with CKI-interaction, the size parameter of the HO potential is b=1.76 fm to reproduce the rms matter radius R_m =2.59$\pm$0.06fm[27].The calculated B(E2) in this case is 30.6 $e^2fm^4$with no core polarization effect, The core polarization calculation give the value 39.69 $e^2fm^4$for $e_p^{eff.}$ =1.139e, $e_n^{eff.} = 0.374e$ and also gives B(E2)=51.7 e^2fm^4 with the standard effective charges $e_p^{eff.}$=1.3e, $e_n^{eff.} = 0.5e$.These values of B(E2) are very close to the experimental value $40 \pm 11 \pm 4$ [41], the results of B(E2) and effective charges are tabulated in Table 3.

Table 3: The calculated effective charges and B(E2) values of ^{12}Be compared with the experimental data.

$J^\pi\ T(g.s)$ A $\tau_{1/2}$	J_f^π	**b** (fm)	**R**$_m$ (fm) [27]	$e_p^{eff.}$, $e_n^{eff.}$ (e)	**B(E2)**(e^2fm^4) Calc.	Exp. [41]
12 $0^+\ 2$ **23.6 ms** **Halo nucleus** $(^{10}_4Be(core) + 2n^h)$	2^+	$b_c = 1.587$ $b_h = 1.99$	2.30 ± 0.02 2.59 ± 0.06	1.0,0.0 1.178,0.408 1.3,0.5	3.57 69.81 81.74	$40 \pm 11 \pm 4$
12 $0^+\ 2$ **23.6 ms** **Normal nucleus**	2^+	1.76	2.59 ± 0.06	1.0.0.0 1.3,0.5 1.139,0.374	30.6 51.7 39.69	$40 \pm 11 \pm 4$

^{14}Be **nucleus**$(J^{\pi}_{g.s}T = 0^{+}3)$

The ground state of ^{14}Be is with $J^{\pi}T = (0^{+}3)$ with half-life= 4.35 ms. ^{14}Be also is a Borromean nucleus [44]. That is why it is often described as having a ^{12}Be + 2n structure [45, 46].is bound with respect to two neutron emissions by 1.26 MeV.

The neutron drip line ^{14}Be $(J^{\pi}T = 0^{+}3)$ which coupled a ^{12}Be core $(J^{\pi}T = 0^{+}2)$ plustwo neutron system $(J^{\pi}T = 0^{+}1)$ is forming the ^{14}Be nucleus halo. The configurations $(1s1/2)^4$, $(1p)^8$ are performed for ^{12}Be and the configurations $(1s1/2)^4$, $(1p)^8$,$(1d_{5/2})^2$are performed for the ^{14}Be halo nucleus. The different model spaces are chosen for the core and the extra two neutrons. Three different configurations are considered for the description of the halo neutrons in^{14}Bethese two neutrons are assumed to be in a pure$1d_{3/2}$, or a pure$1d_{5/2}$, a pure $2s_{1/2}$ and mixing of the *sd* orbits.

In the *sd* model space all orbits in *2s-1d* shells are considered where the universal shell model (USD) [47] is used for the *sd*-shell orbits. The calculations are performed with *PSD*-model space $(0\hbar\omega)$ with *PSDMK*-interaction. The size parameter *b* =1.76 fm is chosen for ^{12}Be to reproduce the (rms) matter value R_m=2.59$\pm$0.06fm [27]. And *b*=2.91 fm for the two neutron halo to reproduce the rms matter R_m =3.16$\pm$0.38fmfor ^{14}Be[48].

The calculated *B*(E2) for the transition $0^{+}3 \rightarrow 2^{+}3$ with no core polarization effects is 13.67e^2fm^4. Including core-polarization effects gives the value *B*(E2)= 218.92 $e^2fm^4$for $e_p^{eff.}$ =1.178e, $e_n^{eff.} = 0.408e$. Using the standard effective charges $e_p^{eff.}$ =1.3e, $e_n^{eff.} = 0.5e$ the *B*(E2) becomes 309.798 e^2fm^4.These values are compared with the predicted value 44 e^2fm^4[27].The configuration given above that gives this large value of *B*(E2) which do not agree with the predicted value.

For non-halo ^{14}Be, the size parameter of the HO potential is *b*=2.058 fmto reproduce the rms matter radius Rm =3.16±0.38fm[48].The calculated *B*(E2) with no core polarization effects is 25.59 e^2fm^4.The core-polarization calculation gives the quadrupole transition value *B*(E2) =52.29 e^2 fm^4 for $e_p^{eff.}$=1.05e, $e_n^{eff.} = 0.15e$ which is in a good agreement with 59.1 e^2fm^4[49].The results of *B*(E2) and effective charges are tabulated in Table 4.

Table 4: The calculated effective charges and B(E2) values of ^{14}Be.

A $J^{\pi}\ T(g.s)$ $\tau_{1/2}$	J^{π}_{f}	**b** (fm)	R_m (fm) [48]	$e_p^{eff.}$, $e_n^{eff.}$ (e)	**B(E2)** (e^2fm^4) Calc.
14 $0^{+}3$ **4.35m** **Halo nucleus** $(^{12}_{4}Be(core) + 2n^{h})$	2^{+}	$b_c = 1.76$ $b_h = 2.911$	3.16$\pm$0.38	1.0,0.0 1.178,0.408 1.3,0.5	13.671 218.916 309.798
14 $0^{+}3$ **4.35ms** **Non-halo**	2^{+}	*b*= 2.058	3.16$\pm$0.38	1.05 , 0.15	52.29

Conclusions

Shell model calculations are performed for Beryllium isotopes (9,10,12,14) including core-polarization effects through first-order perturbation theory, where 1*p*–1*h* excitations are considered.The $0\hbar\omega$ and $(0 + 2)\hbar\omega$ calculations which succeed in describing energy levels and other static properties, are less successful for describing dynamical properties such as transition strengths B(E2). The core contributions cannot be ignored in such transitions and the core polarizationeffects play a major role for describing such dynamical property. The size parameters of the harmonic oscillator potential chosen for this work almost reproduce all the rms matter radii for ^{9,10,12,14}Be isotopes.

The main conclusions are briefly summarized as:

1. The calculations include couplings between the $3/2^-$, $5/2^-$, and $7/2^-$ states in the K = 3/2 ground state of ^{9}Be. It is shown that the *B*(E2) values for the excitation of these states are accurately described in the *p*-shell model space .The values for the ground state and {first, second} excited states appear to be in a little agreement with experimental data.
2. The calculations showed that the major contribution to the transition strength comes from the core polarization where excitations are considered from the ^{4}He core and the valence nucleons in *p*-shell orbits. The core-polarization effects give a strong modification to the quadrupole transition strength (J=2) where the core polarization effects enhance the quadrupole transition strength and bring the calculated values very close to the experimental data.
3. In our calculations the ^{12}Be nucleus calculated as:
   A. Non-halo or normal nucleus: the *B*(E2) value for this nucleus which agrees very well with the experimental value.
   B. Halo nucleus: a simple ^{10}Be+n+n structure are suggested for ^{12}Be nucleus,these two neutrons form a halo around the^{10}Be nucleus. The 8 neutrons (magic number) form a closed shell, and the value of *B*(E2) in this case is greater than the value of *B* (E2) in the normal nucleus case.
4. The matter density of ^{14}Be exhibits the most extended halo component among all isotopes being investigated, which is reflected in large matter radius. The value of *B*(E2) in ^{14}Be halo nucleus is greater than those for ^{9,10,11,12}Be and this value of the transitionstrength shows a clear exotic behavior for this neutron rich isotope. And in non-halo case the value of *B*(E2) which is in a good agreement with the experimental value.

It is found that the structure of the halo neutrons for^{12}Be and ^{14}Be have dominant $(1d)^2$configurations Also, it is found that the difference between the transition strengths of unstable exotic ^{12}Be, ^{14}Be nuclei and those of the stable ^{9}Be nucleus is the difference in the center of mass correction which depends on the mass number and the size parameter *b*.

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