Nucleon momentum distributions and elastic electron scattering form factors for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes using the framework of coherent fluctuation model

Adel K. Hamoudi, Hasan F. Ojaimi

Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

E-mail: has_far21@yahoo.com

Abstract

The nucleon momentum distributions (NMD) and elastic electron scattering form factors of the ground state for some 1f-2p-shell nuclei, such as $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes have been calculated in the framework of the coherent fluctuation model (CFM) and expressed in terms of the weight function $|f(x)|^2$. The weight function (fluctuation function) has been related to the nucleon density distribution (NDD) of the nuclei and determined from the theory and experiment. The NDD is derived from a simple method based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states. The feature of the long-tail behavior at high momentum region of the NMD's has been obtained by both the theoretical and experimental weight functions. The calculated elastic electron scattering form factors for considered isotopes are in reasonable agreement with those of experimental data throughout all values of momentum transfer q.

Key words

Nucleon density distributions,
Nucleon momentum distributions,
Elastic electron scattering form factors of some fp-shell nuclei,
Ni-isotopes.

Article info.

Received: Apr. 2014
Accepted: May. 2014
Published: Sep. 2014

توزيعات زخم النيكليون وعوامل التشكل للاستطاره الالكترونيه المرنة لنظائر النيكل $^{58}Ni, ^{60}Ni, ^{62}Ni, ^{64}Ni$ بأستخدام أنموذج التموج المتشاكه

عادل خلف حمودي, حسن فرحان عجيمي

قسم الفيزياء, كلية العلوم, جامعة بغداد, بغداد, العراق

الخلاصة

تم حساب كل من توزيعات زخم النيكليون للحالة الأرضية وعوامل التشكل للأستطارة الألكترونية المرنة لبعض النوى الواقعة ضمن القشرة النووية 1f-2p مثل نظائر النيكل $^{64}Ni, ^{58}Ni, ^{60}Ni, ^{62}Ni$ حيث تم التعبير عنهما بدلالة دالة التموج. ترتبط دالة التموج مع توزيعات كثافة النيكليون وتم حسابها من النتائج النظرية والعملية لتوزيعات كثافة النيوكليون. ان اشتقاق توزيعات كثافة النيكليون يعتمد بالأساس على كل من اعداد اشغال الحالات النووية وعلى الدوال الموجية للجسيمة المنفردة المتواجدة في الجهد التوافقي. تميزت نتائج توزيعات زخم النيكليون (المستندة على دالة التموج النظرية والعملية) بصفة الذيل الطويل عند منطقة الزخم العالي. أظهرت هذه الدراسة بأن النتائج النظرية لعوامل التشكل للأستطارة الالكترونية المرنة لنظائر النيكل $^{58}Ni, ^{60}Ni, ^{62}Ni, ^{64}Ni$ و المحسوبة بانموذج التموج المتشاكه تتفق مع النتائج العملية و لكل قيم الزخم المنتقل .

Introduction

There is no method for directly measuring the nucleon momentum distribution (NMD) in nuclei. The quantities that are measured by particle-nucleus and nucleus-nucleus collisions are the cross sections of different reactions, and these contain information on the NMD of target nucleons. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei established the existence of long-tail behavior of the NMD at high momentum region ($k \geq 2\, fm^{-1}$) [1-6]. In principle, the mean field theories cannot describe correctly the form factors $F(q)$ and the NMD simultaneously [7] and they exhibit a steep-slope behavior of the NMD at high momentum region. In fact, the NMD depends a little on the effective mean field considered due to its sensitivity to the short-range and tensor nucleon-nucleon correlations [7, 8] which are not included in the mean field theories.

There are several theoretical methods used to study elastic electron-nucleus scattering, such as the plan-wave Born approximation (PWBA), the eikonal approximation and the phase-shift analysis method [9-15]. The PWBA method can give qualitative results and has been used widely for its simplicity. To include the Coulomb distortion effect, which is neglected in PWBA, the other two methods may be used. In the past few years, some theoretical studies of elastic electron scattering off exotic nuclei have been performed. Wang et al. [11, 12] studied such scattering along some isotopic and isotonic chains by combining the eikonal approximation with the relativistic mean field theory. Roca-Maza et al. [13] systematically investigated elastic electron scattering off both stable and exotic nuclei with the phase-shift analysis method. Karataglidis and Amos [14] have studied the elastic electron scattering form factors, longitudinal and transverse, from exotic (*He* and *Li*) isotopes and from ^{8}B nucleus using large space shell model functions. Chu et al. [15] have studied the elastic electron scattering along *O* and *S* isotopic chains and shown that the phase-shift analysis method can reproduce the experimental data very well for both light and heavy nuclei.

In the coherent fluctuation model (CFM), which is exemplified by the work of Antonov et. al. [4, 16], the local nucleon density distribution (NDD) and the NMD are simply related and expressed in terms of an experimentally obtainable fluctuation function (weight function) $|f(x)|^2$. They [4, 16] investigated the NMD of (^{4}He and ^{16}O), ^{12}C and (^{39}K, ^{40}Ca and ^{48}Ca) nuclei using weight functions $|f(x)|^2$ expressed in terms of the experimental two parameter Fermi (2PF) NDD [17], the experimental data of Ref. [18] and the experimental model-independent NDD [17] , respectively. It is important to point out that all above calculations obtained in the framework of the CFM proved a high momentum tail in the NMD. Elastic electron scattering from ^{40}Ca nucleus was also investigated in Ref. [16], where the calculated elastic differential cross sections ($d\sigma/d\Omega$) were found to be in good agreement with those of experimental data.

Recently, Hamoudi et al. [19, 20] have studied the NMD and elastic electron scattering form factors for 1p-shell and 2s-1d shell nuclei using the framework of CFM. They [19, 20] derived an analytical form for the NDD based on the use of the single particle harmonic oscillator wave functions and the occupation number of the states. The derived NDD's, which are applicable throughout the whole 1p-shell [19] and 2s-1d shell [20] nuclei, have been used in the CFM. The calculated NMD and elastic form factors of all considered nuclei have been in very good agreement with experimental data.

The aim of the present work is to extend the calculations of Hamoudi et al. [19, 20] to higher shells (such as the 2f-1p shell nuclei) and to derive an analytical expression for the NDD based on the use of the single particle harmonic oscillator wave functions and the occupation numbers of the states. The derived NDD is employed in determining the theoretical weight function $|f(x)|^2$ which is used in the CFM to study the NMD and elastic form factors for ^{58}Ni, ^{60}Ni, ^{62}Ni and ^{64}Ni isotopes. We shall see later that the theoretical $|f(x)|^2$, based on the derived NDD, is capable to provide information about the NMD and elastic electron scattering form factors as do those of experimental NDD of Refs. [17, 21].

Theory

The nucleon density distribution (NDD) of the one body operator can be written as [22]

$$\rho(r) = \frac{1}{4\pi}\sum_{nl}\zeta_{nl}\,4(2l+1)\phi^*_{nl}(r)\phi_{nl}(r) \qquad (1)$$

where ζ_{nl} is the nucleon occupation probability of the state nl ($\zeta_{nl}=0$ or 1 for closed shell nuclei and 0< ζ_{nl} <1 for open shell nuclei) and $\phi_{nl}(r)$ is the radial part of the single particle harmonic oscillator wave function. The NDD form of *Ni*-isotopes is derived on the assumption that there are filled 1s, 1p and 1d orbitals and the nucleon occupation numbers in 2s, 1f and 2p orbitals are equal to, respectively, (4-δ_1), (A-40-δ_2) and ($\delta_1+\delta_2$) and not to 4, ($A-40$) and 0 as in the simple shell model. Using this assumption in Eq. (1), an analytical form for the ground state NDD of *Ni*-isotopes is obtained as:

$$\rho(r)=\frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{10-\frac{3}{2}\delta_1+\left(\frac{11}{3}\delta_1+\frac{5}{3}\delta_2\right)\left(\frac{r}{b}\right)^2+\left(8-2\delta_1-\frac{4}{3}\delta_2\right)\left(\frac{r}{b}\right)^4+\left(\frac{20}{105}\delta_2+\frac{8}{105}(A-40)+\frac{4}{15}\delta_1\right)\left(\frac{r}{b}\right)^6\right\}, \qquad (2)$$

where A is the nuclear mass number, b is the harmonic oscillator size parameter, the parameter δ_1 characterizes the deviation of the nucleon occupation numbers from the prediction of the simple shell model (δ_1=0). The parameter δ_2 in Eq. (2) is a assumed as a free parameter to be adjusted to obtain agreement with the experimental NDD.

The normalization condition of the NDD is given by [4, 17]

$$A = 4\pi\int_0^\infty \rho(r)\, r^2\, dr, \qquad (3)$$

and the mean square radius (MSR) of the considered nuclei is given by [4, 17]

$$< r^2 > = \frac{4\pi}{A}\int_0^\infty \rho(r)\, r^4\, dr \qquad (4)$$

The central NDD, $\rho(r=0)$, is obtained from Eq. (2) as

$$\rho(0)=\frac{1}{\pi^{3/2}b^3}\left\{10-\frac{3}{2}\delta_1\right\} \qquad (5)$$

Then, δ_1 is obtained from Eq. (5) as

$$\delta_1 = 2(10-\rho(0)\pi^{3/2}b^3)/3 \qquad (6)$$

Substituting Eq.(2) into Eq. (4) and after simplification we get

$$< r^2 > = \frac{b^2}{A}\left\{\frac{9A-120}{2}+\delta_1\right\} \qquad (7)$$

The NMD of the considered nuclei is determined by two distinct methods. In the first method, it is determined by the shell model using the single particle harmonic oscillator wave functions in momentum representation and is derived as

$$n(k) = \frac{b^3}{\pi^{3/2}} e^{-b^2k^2}\left[10 + 8(bk)^4 + 8\frac{(A-40)}{105}(bk)^6\right] \quad (8)$$

where k is the momentum of the particle.

In the second method, the NMD is determined by the CFM, where the mixed density is given by [4, 16]

$$\rho(r,r') = \int_0^\infty |f(x)|^2 \rho_X(r,r')dx \quad (9)$$

where:

$$\rho_x(r,r') = 3\rho_0(x)\frac{j_1(k_F(x)|\vec{r}-\vec{r}'|)}{k_F(x)|\vec{r}-\vec{r}'|}\theta\left(x - \frac{|\vec{r}+\vec{r}'|}{2}\right) \quad (10)$$

is the density matrix for A nucleons uniformly distributed in a sphere with radius x and density $\rho_0(x) = 3A/4\pi x^3$. The Fermi momentum is defined as [4, 16]

$$\begin{aligned} k_F(x) &= \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \\ &= \left(\frac{9\pi A}{8}\right)^{1/3}\frac{1}{x} = \frac{\alpha}{x}; \qquad \alpha = \left(\frac{9\pi A}{8}\right)^{1/3} \end{aligned} \quad (11)$$

and the step function θ is defined by

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \quad (12)$$

The diagonal element of Eq. (9) gives the one-particle density as

$$\begin{aligned} \rho(r) &= \rho(r, r' = r) \\ &= \int_0^\infty |f(x)|^2 \rho_x(r)dx \end{aligned} \quad (13)$$

In (13), $\rho_x(r)$ and $|f(x)|^2$ have the following forms [4, 16]

$$\rho_x(r) = \rho_0(x)\theta(x - |\vec{r}|) \quad (14)$$

$$|f(x)|^2 = -\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}\Big|_{r=x} \quad (15)$$

The weight function $|f(x)|^2$ of Eq. (15), determined in terms of the NDD, satisfies the normalization condition [4, 16]

$$\int_0^\infty |f(x)|^2 dx = 1 \quad (16)$$

and holds only for monotonically decreasing NDD, i.e. $\frac{d\rho(r)}{dr} < 0$.

On the basis of Eq. (13), the NMD [$n(k)$]is expressed as [4, 16]

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k)dx \quad (17)$$

where

$$n_x(k) = \frac{4}{3}\pi x^3 \theta\left(k_F(x) - |\vec{k}|\right) \quad (18)$$

is the Fermi-momentum distribution of the system with density $\rho_0(x)$. By means of Eqs. (15), (17) and (18), an explicit form for $n(k)$ is expressed in terms of $\rho(r)$ as

$$n(k) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{A} \times \left[6\int_0^{\alpha/k} \rho(x)x^5dx - \left(\frac{\alpha}{k}\right)^6 \rho\left(\frac{\alpha}{k}\right)\right] \quad (19)$$

with normalization condition

$$\int n(k)\frac{d^3k}{(2\pi)^3} = A \quad (20)$$

The elastic monopole form factor $F(q)$ of the nucleus is also expressed in the CFM and is given by [4, 16]

$$F(q) = \frac{1}{A}\int |f(x)|^2 F(x,q)dx \quad (21)$$

where $F(x,q)$ is the form factor of uniform charge density distribution given by

$$F(x,q) = \frac{3A}{(qx)^2}\left[\frac{\sin(qx)}{qx} - \cos(qx)\right] \quad (22)$$

Inclusion of the correction of the nucleon finite size $F_{fs}(q)$ and the centre of mass $F_{cm}(q)$ corrections in the calculations requires multiplying the form factor of Eq.(21) by these corrections. Here, $F_{fs}(q)$ is assumed as the free nucleon form factor which is considered to be the same for protons and neutrons. This correction takes the form [10]

$$F_{fs}(q) = e^{(-0.43q^2/A)} \tag{23}$$

The correction $F_{cm}(q)$ removes the spurious state arising from the motion of the centre of mass when shell model wave function is used and given by [10]

$$F_{cm}(q) = e^{(q^2b^2/4A)} \tag{24}$$

It is important to point out that all physical quantities studied in the present work in the framework of the CFM, such as $n(k)$ and $F(q)$, are expressed in terms of the weight function $|f(x)|^2$. Therefore, it is worthwhile trying to obtain the weight function firstly from the NDD of three parameter Fermi (3PF) model extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The NDD of 3PF is given by [17]

$$\rho_{3PF}(r) = \rho_0\left(1+\frac{wr^2}{c^2}\right)\Big/\left(1+e^{\frac{r-c}{z}}\right); \quad \rho_0 = \frac{A}{4\pi}\frac{1}{\int_0^\infty\left(1+\frac{wr^2}{c^2}\right)\left(1+e^{\frac{r-c}{z}}\right)^{-1} r^2dr} \tag{25}$$

Introducing Eq. (25) into Eq. (15), we obtain the experimental weight function $|f(x)|^2_{3PF}$ as

$$|f(x)|^2_{3PF} = \frac{4\pi x^3\rho_0}{3A}\left[\frac{\left(1+\frac{wx^2}{c^2}\right)\left(1+e^{\frac{x-c}{z}}\right)^{-2} e^{\frac{x-c}{z}}}{z} - \frac{2wx\left(1+e^{\frac{x-c}{z}}\right)^{-1}}{c^2}\right] \tag{26}$$

Moreover, introducing the derived NDD of Eq. (2) into Eq. (15), we obtain the theoretical weight function $|f(x)|^2$ as

$$\begin{aligned} |f(x)|^2 = {} & \frac{8\pi}{3Ab^2}x^4\rho(x) - \frac{16x^4}{3A\pi^{1/2}b^5}e^{-x^2/b^2} \\ & \times\left\{\frac{11}{6}\delta_1+\frac{5}{6}\delta_2+\left(8-2\delta_1-\frac{4}{3}\delta_2\right)\frac{x^2}{b^2}-\left(\frac{4}{35}(A-40)+\frac{2}{5}\delta_1+\frac{2}{7}\delta_2\right)\frac{x^4}{b^4}\right\} \end{aligned} \tag{27}$$

Results, discussion and conclusions

The NMD and elastic electron scattering form factors, $F(q)$, for ^{58}Ni, ^{60}Ni, ^{62}Ni and ^{64}Ni isotopes are studied by means of the CFM. The NMD of Eq. (19) is calculated in terms of the NDD obtained firstly from theoretical consideration, as in Eq. (2), and secondly from the fit to the electron-nuclei scattering experiments, such as the 3PF [17].

The harmonic oscillator size parameters b are chosen in such a way as to reproduce the measured root mean square radii (rms) of nuclei. The parameters δ_1 are determined by introducing the chosen values of b and the experimental central densities $\rho_{\exp}(0)$ into Eq. (6). The values of the parameters b and δ_1 together with the other parameters employed in the present calculations for isotopes under study are listed in Table 1.

Table 1: Parameters for the NDD of considered isotopes

Nuclei	3PF [17]			$\rho_{\exp}(0)$ (fm^{-3}) [17]	$\langle r^2\rangle^{1/2}_{\exp}$ (fm) [17]	b (fm)	δ_1	δ_2
	w (fm)	c (fm)	z (fm)					
^{58}Ni	-0.1308	4.3092	0.5169	0.1710227	3.764	2.017	1.512638	1.4
^{60}Ni	-0.2668	4.4891	0.5369	0.1773539	3.769	2.023	1.249854	2
^{62}Ni	-0.2090	4.4425	0.5386	0.1770482	3.822	2.0285	1.177425	2
^{64}Ni	-0.2284	4.5211	0.5278	0.1760695	3.845	2.034	1.114692	2

The dependence of the NDD's (in fm^{-3}) on r (in fm) for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes is shown in Fig. 1. The open circle symbols are the fitted to the experimental NDD of the 3PF [17], the solid and dashed curves are the calculated NDD's of Eq. (2), when $\delta_1 \neq \delta_2 \neq 0$ and $\delta_1 = \delta_2 = 0$, respectively. This figure shows that the dashed curves are in poor agreement with the fitted to the experimental data, especially for small r. Introducing the parameter δ_1 (i.e., considering the higher orbitals) in the calculation leads to a very good agreement with the fitted to the experimental data as shown by the solid curve.

The dependence of NMD (in fm^3) on k (in fm^{-1}) for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes is shown in Fig. 2. The dashed curves are the calculated NMD of Eq. (8) obtained by the shell model calculation using the single particle harmonic oscillator wave functions in momentum representation. The open circle symbols and solid curves are the NMD obtained by the CFM of Eq. (19) using the experimental and theoretical NDD, respectively. It is clear that the behavior of the dashed curves obtained by the shell model calculations is in contrast with those reproduced by the CFM. The important feature of the dashed distributions is the steep slope behavior when k increases. This behavior is in disagreement with the studies [4, 5, 8, 16] and it is attributed to the fact that the ground state shell model wave function given in terms of a Slater determinant does not take into account the important effect of the short range dynamical correlation functions.

Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the NMD [5, 7]. It is noted that the general structure of the open circles and solid distributions at the region of high momentum components is almost the same for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes, where these distributions have the feature of long-tail behavior at momentum region $k \geq 2\,fm^{-1}$. The feature of the long-tail behavior obtained by the CFM, which is in agreement with the studies [14, 5, 8, 16], is related to the existence of high densities $\rho_x(r)$ in the decomposition of Eq. (13), though their weight functions $|f(x)|^2$ are small.

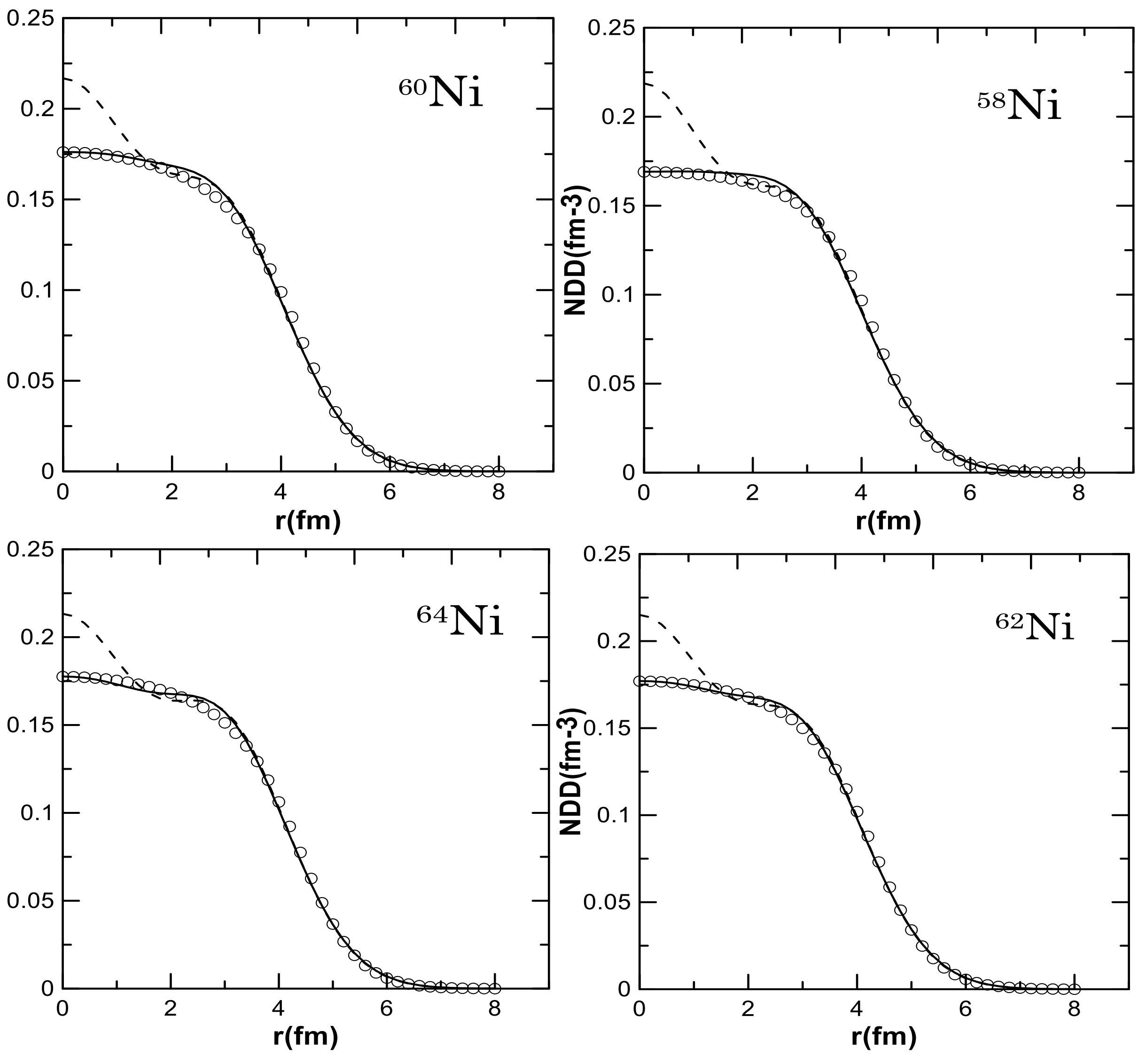

Fig.1. The dependence of the NDD on r (fm) for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes. The dashed and solid curves are the calculated NDD of eq.(2) when $\delta_1 = \delta_2 = 0$, and $\delta_1 \neq \delta_2 \neq 0$, respectively. The open circle symbols are the fitted to the experimental data [17].

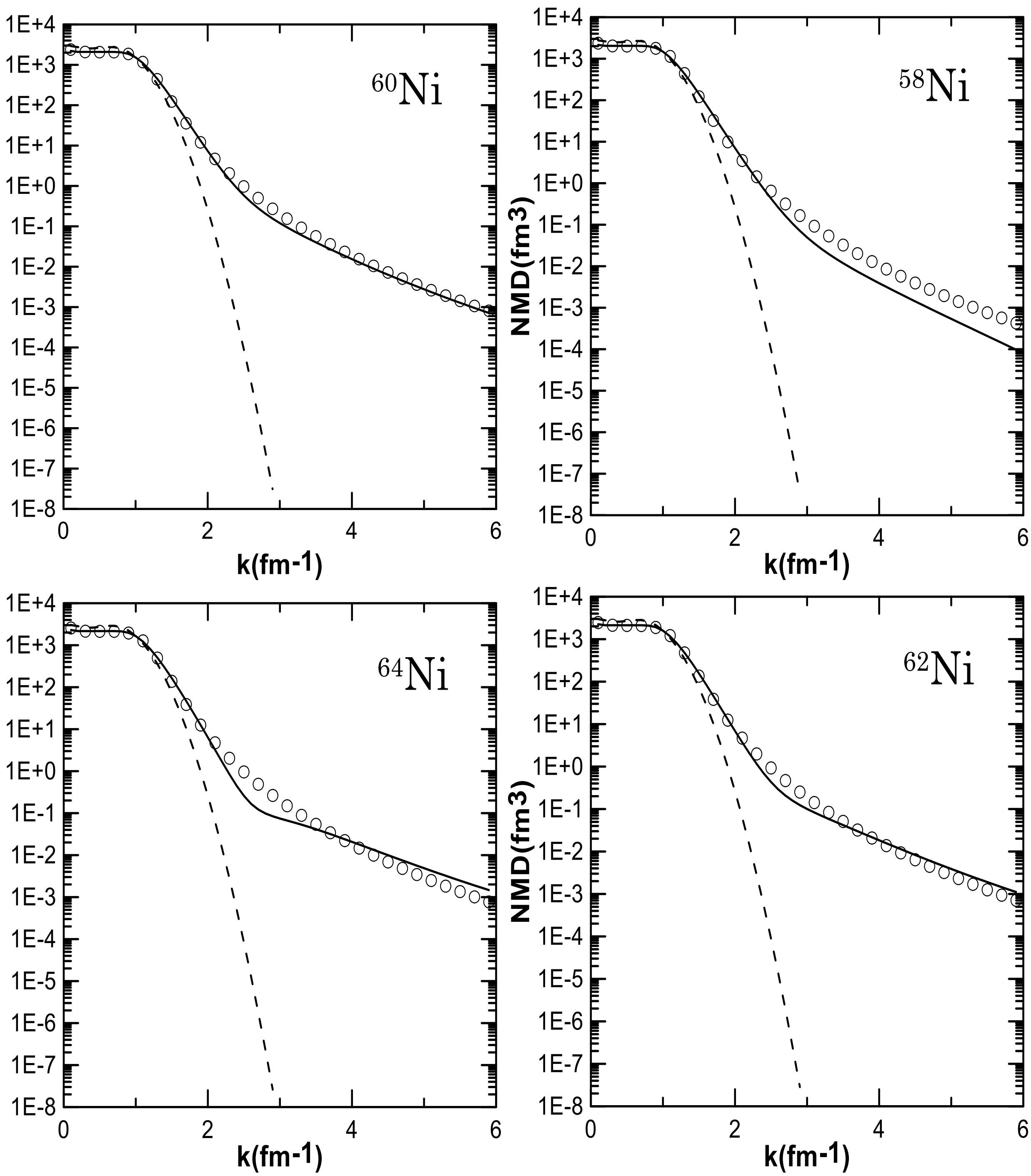

Fig.2. The dependence of NMD on k for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes. The dashed distributions are the results obtained by the shell model calculation of Eq.(8) using the single particle harmonic oscillator wave functions in the momentum representation. The open circle symbols and solid curves are the calculated NMD expressed by the CFM of Eq.(19) using the experimental NDD of Eq.(25) and theoretical NDD of Eq.(2), respectively.

The dependence of $F(q)$ on q (in fm^{-1}) for considered isotopes is shown in Fig. 3. The calculated form factors (solid curves) of $^{58}Ni, ^{60}Ni, ^{62}Ni$ isotopes, obtained in the framework of CFM using the theoretical weight function of Eq. (27), are compared with those of experimental data (open circle symbols) [23, 24]. As there is no data available for ^{62}Ni isotope, we have compared the calculated form factors of this isotope with those obtained by the Fourier transform of the 3PF density (triangles).

This figure shows that the diffraction minima and maxima of considered isotopes are reproduced in the correct places. However, both the behavior and the magnitudes of the calculated form factors of these isotopes are in reasonable agreement with those of the experimental data.

It is concluded that the derived form of NDD of Eq.(2) employed in the determination of theoretical weight function of Eq. (27) is capable to reproduce information about the NMD and elastic form factors as do those of the experimental data.

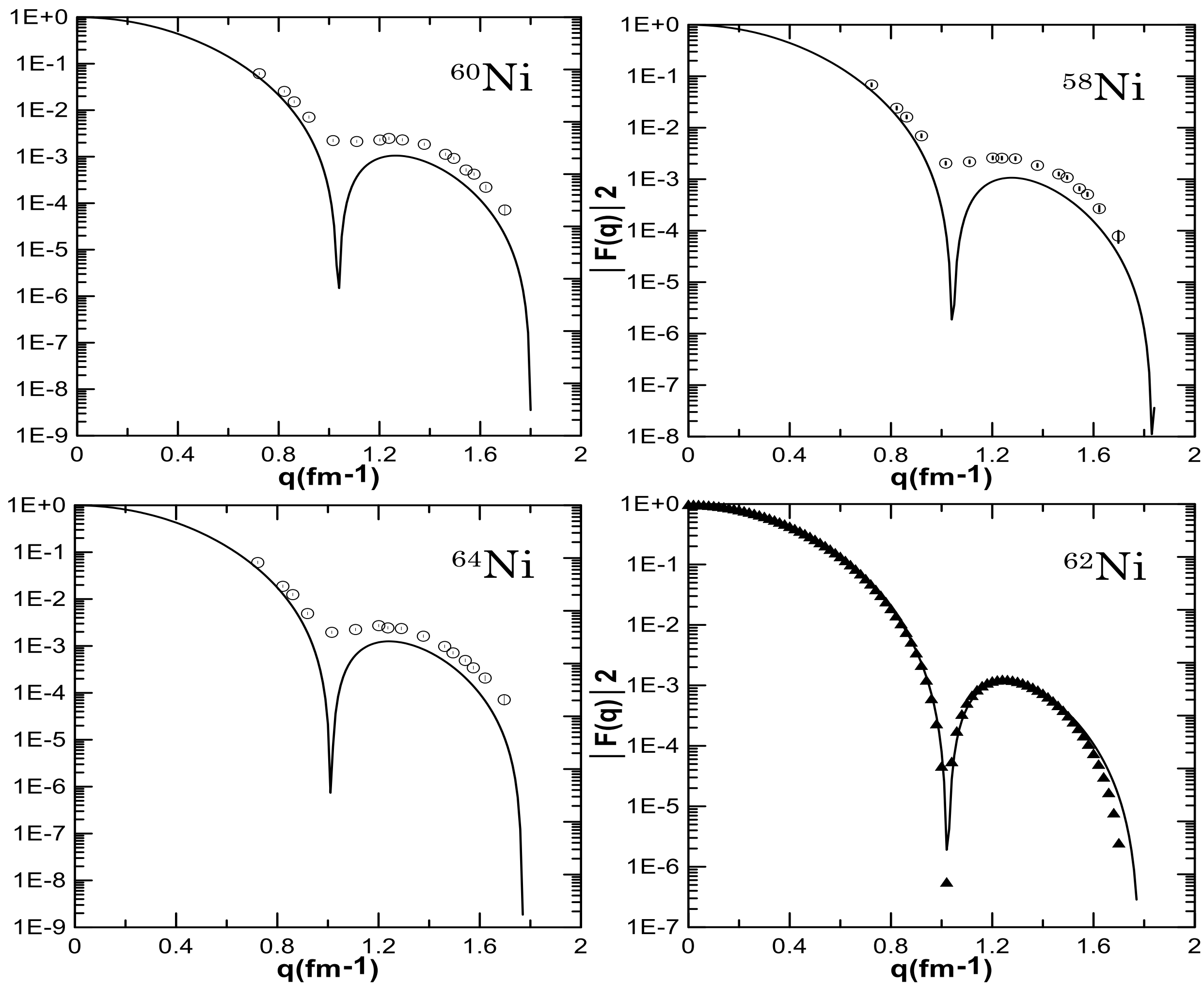

Fig.3: The dependence of form factors on q for $^{58}Ni, ^{60}Ni, ^{62}Ni$ and ^{64}Ni isotopes. The solid curves are the form factors calculated using Eq.(21). The open circle symbols are the experimental data, taken from Refs. [23,24] . The triangles are the form factors obtained by the Fourier transform of the $\rho_{3PF}(r)$.

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