The effect of deformation parameter of heavy nuclei on level density parameter

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Abstract

The possible effect of the collective motion in heavy nuclei has been investigated in the framework of Nilson model. This effect has been searched realistically by calculating the level density, which plays a significant role in the description of the reaction cross sections in the statistical nuclear theory. The nuclear level density parameter for some deformed radioisotopes of (even- even) target nuclei (Dy, W and Os) is calculated, by taking into consideration the collective motion for excitation modes for the observed nuclear spectra near the neutron binding energy. The method employed in the present work assumes equidistant spacing of the collective coupled state bands of the considered isotopes. The present calculated results for first excited rotational band have been compared with the accumulated values from the literature for s-wave neutron resonance data, and were in good agreement with those data.

Key words

Rotational and vibrational, level, density, collective levels, statistical model.

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تأثير معامل التشوه للنوى الثقيلة على معامل كثافة المستوى

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الخلاصة

تم البحث في امكانية تأثير الحركة المنتظمة في النوى الثقيلة لنظام نموذج نلسن. هذا التأثير بحث بواقعية من خلال حساب كثافة المستوى, التي تلعب دور مهم في وصف المقاطع العرضية للتفاعل في النظرية الاحصائية النووية. تم حساب معامل كثافة المستوى النووية لبعض النظائر المشوهة لاهداف نووية (زوجي- زوجي) مثل (Dy, W and Os), اخذين بنظر الاعتبار الحركة المنتظمة في نظام متهيج لطيف نووي منظور قريب من طاقة الربط للنيترون. ان الطريقة المستخدمة في هذا العمل اعتمدت على نظام الفضائات المتساوية لحزم المستوى المقترن والمنتظم للنظائر المدروسة. تم مقارنة نتائج الحسابات في العمل الحالي للحزمة الدورانية المتهيجة الاولى مع القيمة المتراكمة من المصادر العلمية ولقيم موجة النيترون الرنيني-s والتي وجد تطابق جيد معها.

Introduction

Due to enormous applications of the nuclear level density in nuclear reaction database [1,2], it became a very consequential ingredient in semi-classical statistical model [3,4] besides to quantum mechanical treatment [5], calculations for nuclear reactions. Where their parameter has more influenced in calculating the energy spectrum with an acceptable comparison with experimental work [6]. Also, there is a possibility to achieve an empirical formula [7] regarding the shape and size of the target nucleus competent and energy dependence [6], whether to be odd-odd or even-even nuclei as described in detail by reference [8].

Different approaches have been implemented extensively to calculate the nuclear level density, among them the Fermi gas model (FGM), which is basically concentrated on Equidistant Space Model (ESM) [5] and Bethe formula [9], which, coupled the entropy with the average energy of the system released non-interacting particle in the Fermi gas [10]. Other contributions achieved in this aspect by adding the effect of different corrections like paring, shell effect, deformation [11,12], finite size [12], thermal and quantal effect [13] and spin cutoff parameters effect [5]. However, these contributions didn't include the collective effect, especially for the deformed nuclei. These play a good part in describing the level density and an acceptable calculation in the strength reactions, which is representing a beneficial role in identifying pre-equilibrium stage for a certain nuclear reactions [2].

Theoretical study on nuclear level density

Since the Bethe formula for Fermi gas model accounted for calculating the standard nuclear level density, that is depend on total angular momentum J of the nucleus, as a function of the excitation energy E corrected for paring effects Δ correlated with state density.

The above-mentioned Bethe theory gives also the dependence of the nuclear level density on the total angular momentum J of the nucleus. The expression used for the observable nuclear level density at any excitation energy E and momentum J can be written as [4, 5]

$$w(E,J)=\frac{1}{24\sqrt{2}}\frac{2J+1}{\sigma^{3}a^{1/4}}\frac{\exp[2(a(E-\Delta))^{1/2}-J(J+1)/2\ \sigma^{2}]}{(E-\Delta+t)^{5/4}} \qquad (1)$$

where $a=(\pi^2/6)\,g(\varepsilon_F)$ and $\sigma=\sqrt{g(\varepsilon_F)\langle m^2\rangle t}$, are the level density and spin distribution parameters, respectively. Here, the parameter $g(\varepsilon_F)$ is the sum of the neutron and proton single-particle states density at the Fermi energy (ε_F), $\langle m^2\rangle$ is the mean square magnetic quantum number of single particle states, and t is the nuclear thermodynamic temperature of an excited nucleus in the Fermi gas model.

These factors are expressed as follows:

$$g(\varepsilon_F)=\frac{3}{2}\frac{A}{\varepsilon_F},\langle m^2\rangle=0.146A^{2/3},\qquad t^2=\frac{U}{a}\quad,\quad U=E-\Delta \qquad (2)$$

where A is the mass number of a nucleus.

Due to the limited resolution of the experimental technique to identify clearly different orientations of nuclear angular momentum J, through the scattering process, the level density formula that simulate the observable values can predict with an acceptable accuracy at certain excited energy by using the following formula [6,9]:

$$\rho(E)=\sum_J\rho(E,J)=\frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{a(E-\Delta)})}{a^{1/4}(E-\Delta+t)^{5/4}}\frac{1}{\sqrt{2\pi\sigma}} \qquad (3)$$

Hence, the observable level density becomes:

$$\rho(U) = \frac{a}{12\sqrt{2}} \frac{\exp(2\sqrt{aU})}{*0.298A^{1/3}(aU)^{3/2}} \qquad (4)$$

Collective excitation modes of deformed nuclei

Different studies take in consideration the level density parameters in the Bethe theory for s-wave neutron resonance for different mass nuclei[4,14,15], where this theory does not consider the collective effects of the projectile particles on the excitation of the target nuclei. In addition, the measured magnetic and quadrupole moments of the nuclei found to be deviating from the calculated values by using the single-particle shell model, where the closed shells forming the nuclear core play no part. Nevertheless, the results of the collective motion of many nucleons, not just of those nucleons that are outside the closed shell, in the nuclei mainly came from the excited states, magnetic and quadrupole moments. In this case the collective motion of the nucleons may be described as a vibrational motion about the equilibrium position and a rotational motion that maintains the deformed shape of the nucleus.

For many deformed nuclei the existence of collective energy level bands of rotational and vibrational types can now easily be identified from nuclear spectra data [2]. For instant, the contribution of the collective motion of nucleons to the energy level density has been investigated by [15, 16], where model, that is used, is complex in calculate the level density parameters for deformed nuclei. A simpler description of collective model was first suggested by Rainwater [17], who made clear the relationship between the motion of individual nuclear particles and the collective nuclear deformation. Later a quantitative development of the nuclear collective model taking into consideration the collective motion of the nuclear particles was given [18, 19, 20]. Also, a considerable attention has been introduced by [5,12,13,21], where the nuclear level density parameters identified for some region of the light nuclei and largely the deformed nuclei by using a simple model of nuclear collective excitation mechanism. Almost all data on the estimated level density parameters of these deformed nuclei are well identified on a base of collective rotational and collective vibrational bands such as a ground state band, β band, octupole band, γ-band, and so forth.

The rotational energy of an axially symmetric deformed even-even nucleus is given by Bohr et al. [18].

$$E_{rot}(I,K) = \frac{\hbar^2}{2}\left[\frac{I(I+1)}{J_0} + (\frac{1}{J_3} - \frac{1}{J_0})K^2\right] \qquad (5)$$

where I and K are the total angular momentum and its projection on the axis of symmetry (where K= ± I, ±(I-1), ± (I-2) etc.), respectively, of a nucleus and J_3 and J_0 are moments of inertia about a symmetry axis and an arbitrary axis perpendicular to the symmetry axis, respectively.

Bohr et al. [18] have considered the only quadrupole term that caused the deformation in nuclear surface, where in this model one can admit J_3 = 0, which requires the value of K in Eq.(5) to be identically zero. Then, the rotational energy equation becomes:

$$E_{rot} = \frac{\hbar^2}{2J_0} I(I+1), K = 0 \qquad (6)$$

Eq. (6) is in a good agreement with the observed low-lying energy levels of the even-even largely deformed nuclei, which are the values of angular momentum I,I =0, 2, 4, 6, . (the odd value of I is neglected due to the presence of reflection symmetry, so the allowed values of J are 0, 2, 4, etc). As mentioned above the energy level sequence in such a case is called ground state rotational band having positive parity.

Two modes are considered in the collective vibrartional modes the quadrupole and the octupole vibrational modes. The quadrupole mode, also called β-vibrational band, carries two units of angular momentum and even parity (0+, 2+, 4+, 6+,...), while the octupole vibrational band carries two units of angular momentum and negative parity (1−, 3−, 5−, 7−,...). Here, the β band is associated with vibrations that preserve the axis of symmetry and therefore is K = 0 bands with the level sequence and the band head $\hbar\omega_\beta$.
Another excited band is often called the gamma band γ and is associated with the vibrations not preserving the symmetry axis and having the levels given by Eq.(5). The spin sequence of γ band with K = 2 is I = 2+, 3+, 4+, 5+, ...

The nuclear level density parameter, method of calculation

The method used in calculation of nuclear level density parameters for some deformed target isotopes has been given in detail in other studies [5,12, 13, 22]. Similarly, the mentioned method, in this study, can also be applied to deformed target isotopes of interest. In any case, the nuclear energy level density depends on the effective excitation energy, U, by taking into account different excitation modes that can be expressed in the following form:

$$\rho(U)=\sum_i a_i\rho_i(U) \qquad (7)$$

where ρ_i (U) is the partial energy level density at the excitation U for the ith excitation mode and a_i is the weighting coefficient satisfying the condition $\sum_i a_i = 1$.

In the present work, a simple expression for the energy level density has been used, which considers the collective excitation modes. Here, in our determination of the nuclear level density due to excitation bands the “equidistant” condition between energy levels, which is the important property of the observed energy spectrum of isotopes considered, should be satisfied. These properties can approximately be verified for the energies of the coupled state bands in deformed isotopes considered as being the ratios given by:

$R1:R2:R3:R4:...=1:r:2r:3r:4r...$

Here, *R1, R2, R3, R4,*. . . are the ratios of the sequential level energies to the appropriate energy unit of a corresponding band. When the above relation is satisfied, the nuclear level density formula introduced depending on the excitation energy U and energy unit ε_o for the ith excitation band can be represented as:

$$\rho(U,\varepsilon_{oi})\cong\frac{\pi^2 a_{oi}}{24\sqrt{3}(a_{oi})^{3/2}}\exp(2\sqrt{a_{oi}U}) \qquad (8)$$

which is fairly simple and contains only one parameter defined as:

$$a_{oi}=\frac{\pi^2}{6\varepsilon_{oi}} \qquad (9)$$

and represents a collective level density parameter corresponding to the ith band with the unit energy ε_{oi}. The unit energies are ε_{0GS} = E(2+), $\varepsilon_{0\beta}$ = E(2^+) − E(0^+), and ε_{0oct} = E(3^-) − E(1^-) for the ground state, β, and octupole bands, respectively. Similarly, the other excitation bands can be included. In the even-even and odd-A isotopes it has been shown that the unit energy is either energy of the first excited state (for ground state bands) or the energy separation between the second and first excited states (for excited bands) of the corresponding band with the given projection of the total angular momentum K.

In the present work, this approach takes into consideration the different collective excitation modes in deformed target isotopes of interest, and the nuclear level density parameters a_{oi} defined by Eq. (12) can easily be obtained from nuclear spectra data given by Ignatyuk et al. [16] regarding nuclear level spectra of collective rotational and collective vibrational bands.

Finally, the moment of inertia as a function of the dimension of nucleus [23] can be defined as:

$$j_3 = \frac{2}{5} MA(\Delta R)^2 = I / \omega \qquad (10)$$

Results and discussion

The level density parameter of rare-earth and actinide elements determined by using different collective excitation modes of observed nuclear spectra and by refs [24-26] to estimate a value and the data are given using Table 1.

Table 1: Level density parameter for deformed nuclei by using different methods.

e-e nuclei	A	E_{1st} (keV)	a_{0i} (pw) MeV^{-1}	a_{EXP} MeV^{-1}[25]	$a=(\pi^2/6)g(\varepsilon_f)$ $g(\varepsilon_f)=3A/2\varepsilon_f$
66-Dy	162	82	20.06	16.44	10.51
66-Dy	164	74	22.22	15.23	10.68
74-W	182	100	16.44	-	11.81
74-W	184	113	14.54	18.08	11.93
74-W	186	124	13.25	-	12.07
76-Os	186	137	12.01	-	12.07
76-Os	188	155	10.61	12.19	12.2

The table lists the available data of a in the region of 162≤ A≤188 for even-even nuclei, column three lists the energies of the first excited (2+) rotational state, while the fourth column gives the a_{0i} values deduced from the Eq. (9). The a estimated from Eq. (4) are given in column 6 but the experimental value of a is presented in column5and one can observe the difference between these values and this can be justified by the accurate formula of Eq. (9).

Consequently, one can remark that the nuclear collective excitation modes are quite meaningful in order to obtain the level density parameters of different isotopes. The calculation of these parameters based on the properties of the measured nuclear low-lying level spectra should prove a productive area of study that should override the inherent experimental difficulties involved.

Conclusions

The present calculating results show the influence of the collection motion for the deformed nuclei in the calculation the level density parameters. It concluded that the collection motion for excitation modes in Dy, W and Os concentrated near the neutron binding energy. This conclusion based on the properties of the low lying level spectrum, especially the first excited rotational band of these nuclei.

References

[1] S. Shaleen, Calculation of nuclear level densities near the drip lines, Ph.D. thesis, Faculty of the College of Arts and Sciences of Ohio University, Physics and Astronomy (2008).

[2] Nuclear Structure and Decay Data, National Nuclear Data Center. Brookhaven National Laboratory. ENSDF (Evaluated Nuclear Structure Data File), Upton, NY, USA (2001).

[3] T. Ericson, Nuclear Physics, 8, C (1958) 265–283.

[4] A. Gilbert and A. G. W. Cameron, Canadian Journal of Physics, 43 (1965) 1446–1496.

[5] S¸. Okuducu, N. N. Akti, and E. Eser, Annals of Nuclear Energy, 38, 8 (2011) 1769–1774.
[6] S. Hilaire, Physics Letters B, 583, 3-4 (2004) 264–268.
[7] S. K. Kataria, V. S. Ramamurthy, and S. S. Kapoor, Physical Review C, 18, 1(1978) 549–563.
[8] T. Von Egidy and D. Bucurescu, Physical Review C, 72 (2005) 4.
[9] H. A. Bethe, Physical Review, 50, 4 (1936) 332–341.
[10] A. V. Ignatyuk and N. Yu Shubin, Soviet Journal of Nuclear Physics, 8 (1969) 660.
[11] S. E.Woosley, Theory and Applications of Moments Methods in Many-Fermion Systems, Plenum, New York, USA (1980).
[12] S. Okuducu, N. N. Akti, H. SaraC¸, M. H. B¨ol¨ukdemir, and E. Tel, Modern Physics Letters A, 24, 33(2009) 2681–2691.
[13] S. Okuducu and N. N. Akti, Kerntechnik, 75, 3 (2010) 109–116.
[14] H. Baba, Nuclear Physics A, 159, 2 (1970) 625–641.
[15].G. Rohr, Zeitschrift f¨ur Physik A, 318, 3 (1984) 299–308.
[16] A. V. Ignatyuk, K. K. Istekov and G. N. Smirenkin, Soviet Journal of Nuclear Physics, 29 (1979) 450–454.
[17] J. Rainwater, Physical Review, 79, 3 (1950)432–43.
[18] A. Bohr and B.Mottelson, Nuclear Structure, Vol. II, Benjamin, New York, USA (1969).
[19] A. Bohr, Physical Review, 81, 1(1951) 134–138.
[20] A. S. Davydov and A. A. Chaban, Nuclear Physics, 20, C (1960) 499–508.
[21] H. Ahmadov, I. Zorba, M. Yilmaz, and B. G¨on¨ul, Nuclear Physics A, 706, 3-4 (2002) 313–321.
[22] S. Okuducu, S. S¨onmezo˘glu, and E. Eser, Physical Review C, 74 (2006) 3.
[23] A. Bohr and B. Mottelson, Dan. Mat. Fys. Medd. 30(1955) 1.
[24] A. E. Calik, C. Deniz and M. Gercekliogl u, PRAMANA Journal of Physics, 73 (2009) 5.
[25] S. Raman, C. W. Nestor, JR., and P. Tikkaneny, Atomic Data and Nuclear Data Tables 78 (2001) 1–128.
[26] B. Pritychenko, M. Birch, M. Horoi and B. Singh, arXiv:1302.6881v1 [nucl-th] 27 Feb 2013.