

Magnetic solar surface flux transport simulation

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Abstract

In this paper, the solar surface magnetic flux transport has been simulated by solving the diffusion–advection equation utilizing numerical explicit and implicit methods in 2D surface. The simulation was used to study the effect of bipolar tilted angle on the solar flux distribution with time. The results show that the tilted angle controls the magnetic distribution location on the sun's surface, especially if we know that the sun's surface velocity distribution is a dependent location. Therefore, the tilted angle parameter has distribution influence.

Keywords

Magnetic flux, diffusion-advection equation, bipole region.

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محاكاة انتقال الفيض المغناطيسي لسطح الشمس

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الخلاصة

في هذا البحث تم محاكاة انتقال الفيض المغناطيسي لسطح الشمس عن طريق حل معادلة الانتشار-الأنجراف بالاستفادة من الطرق العددية الظرنية و الصريحة لسطح ثنائية الأبعاد. استخدمت المحاكاة لدراسة تأثير زاوية الميل لثنائي القطب على توزيع الفيض المغناطيسي الشمسي مع الزمن. اظهرت النتائج أن زاوية الميل تسيطر على مواقع توزيع الفيض المغناطيسي على سطح الشمس، خاصة إذا علمنا أن توزيع سرعة سطح الشمس يعتمد على هذه المواقع. لذلك فإن عامل زاوية الميل يؤثر على التوزيع.

Introduction

Motions of magnetic flux on the surface of the Sun are characterized by three primary modes of transport: super granular flows, differential rotation, and meridional flow. Super granular flows are caused by convection in the Sun. These turbulent flows are the most complex of the surface flows, acting on multiple scales and in all directions. Both differential rotation and meridional flow are axisymmetric. Differential rotation describes the

longitudinal motion, while meridional flow describes the latitudinal motion [1]. Differential rotation produces a relative longitudinal velocity of 100-250 m s⁻¹ at the surface of the Sun. Super granular flows produce cellular velocities on the order of 500 m s⁻¹. However, meridional flow speeds are only on the order of 0-21 m s⁻¹. An ideal flux transport model should be able to reproduce the magnetic field evolution at the surface by incorporating the flows [2]. There are two aspects of this model that should

appreciate. First, it is not dynamo. It is only part of a dynamo. A complete dynamo requires a feedback mechanism to generate new bipolar magnetic regions from the expanding field. Second, the model is a way of understanding the contributions of bipolar magnetic regions to the solar surface, and our numerical simulation of the solar active region evolution is carried out after solving the transport equation for magnetic flux numerical solution in 2D, utilizing both explicit and implicit methods.

Bipolar Magnetic Regions(BMRs)

Magnetic flux appears at the solar surface in the form of (BMRs) with a wide range of values for the (unsigned) magnetic flux and lifetimes. Large (BMRs) form conspicuous sunspot groups while the smaller regions can only be detected through magnetic field measurements [3].

To define (BMRs) numerically, consider the particular initial condition of a magnetic bipole with half-separation ρ_0 and tilt angle δ_0 (the angle between the bipolar spots is equal to one half of the latitude). This is given by the magnetic field [4]:

$$B(x, y, 0) = B_0 e^{1/2} \frac{x'}{\rho_0} e^{-\xi} \quad (1)$$

where, B_0 initial magnetic field (= 1), and

$$\xi = \frac{(x')^2/2 + (y')^2}{\rho_0^2} \quad (2)$$

And the tilted coordinates (x_0, y_0) are given in terms of the untilted (x, y) as:

$$x' = (x - x_0) \cos \delta_0 - (y - y_0) \sin \delta_0 \quad (3)$$

$$y' = (x - x_0) \sin \delta_0 - (y - y_0) \cos \delta_0 \quad (4)$$

Here (x_0, y_0) is the location of the bipole center. The tilt angle of this bipole which Eq.[5]:

$$\tan \delta(t) = \frac{y_2(t) - y_1(t)}{x_1(t) - x_2(t)} \quad (5)$$

The solar magnetic flux transport model

Flux transport models describe the evolution of the flux distribution at the solar surface as a result of bipolar magnetic regions and the transport of the corresponding radial magnetic flux by the horizontal flows due to convection, differential rotation and meridional circulation [6]. The standard equation of magnetic flux transport of the large-scale magnetic field on the solar surface is described by Cartesian coordinates providing that consider only a localized region, such as that occupied by a single active region, is written as [7]:

$$\frac{\partial B_r}{\partial t} = -\nabla \cdot (vB_r) + D\nabla^2 B_r \quad (6)$$

where B_r is the radial magnetic flux, v is the plasma velocity vector (which includes the convective flows and the observed axisymmetric flows), D is the diffusivity, ∇ represents gradient, $\nabla \cdot$ represents divergence, and ∇^2 Laplace operator.

In this equation notice a combination of the diffusion and advection equations, which describes flux distribution at the solar surface, where the right-hand side of the equation is the sum of two contributions.

The first term $-\nabla \cdot (vB_r)$: describes advection, and The second term $D\nabla^2 B_r$: describes diffusion[8]. By numerical analytical solution to Eq. (6), in Cartesian planar surface for 2D, is [7]:

$$\frac{\partial B_z}{\partial t} = -v_x \frac{\partial B_z}{\partial x} - v_y \frac{\partial B_z}{\partial y} + D \left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} \right) \quad (7)$$

where B_z is magnetic flux through z surface, (x, y) plane represents the photosphere, $v_x(x, y)$ component represents differential rotation, and $v_y(x, y)$ component represents meridional flow [9]. To enable analytical solution to the problem, choose the incompressible, steady flow, where a formula for the differential rotation of the Sun's surface is [10]:

$$u_\theta = \Omega(\theta) R_\odot \sin \theta \quad (8)$$

Ω is the angular velocity of differential rotation, given by:

$$\Omega(\theta) = 0.18 - 2.3\cos^2\theta - 1.62\cos^4\theta \quad (9)$$

And meridional flow distribution on the other hand can be given [11]:

$$u_\theta = C\cos\left[\frac{\pi(\theta_{max} + \theta_{min} - 2\theta)}{2(\theta_{max} - \theta_{min})}\right]\cos\theta \quad (10)$$

where $\theta_{min} = 0$ and $\theta_{max} = \pi/2$, C is constant $\cong 16 \text{ ms}^{-1}$ [12].

$$\begin{aligned} \frac{B_{ij}^{n+1} - B_{ij}^n}{\Delta t} = & -v_x \left(\frac{B_{i+1,j}^n - B_{i-1,j}^n}{2\Delta x} \right) - v_y \left(\frac{B_{i,j+1}^n - B_{i,j-1}^n}{2\Delta y} \right) + D \left(\frac{B_{i+1,j}^n - 2B_{ij}^n + B_{i-1,j}^n}{\Delta x^2} \right) \\ & + D \left(\frac{B_{i,j+1}^n - 2B_{ij}^n + B_{i,j-1}^n}{\Delta y^2} \right) \end{aligned} \quad (11)$$

Stability criteria of the explicit method is:

$$\Delta t \leq \frac{\Delta x + \Delta y}{v}$$

And

$$\Delta t \leq \frac{\Delta x^2 + \Delta y^2}{2D}$$

Numerical differential scheme

The flux transport model Eq. (7) solved numerically with explicit and implicit finite differencing (first order in time and second order in space) to produce magnetic flux maps of the Sun. The explicit formulations, define as:

where the subscript i and j represents x and y coordinates in 2D, and $\Delta x, \Delta y$ is the grid spacing, v is the velocity, Δt is the time step, and D is the diffusivity ($= 450 \text{ km}^2 \text{ sec}^{-1}$).

The implicit formulations, define as:

$$\begin{aligned} \frac{B_{ij}^{n+1} - B_{ij}^n}{\Delta t} = & -v_x \left(\frac{B_{i+1,j}^{n+1} - B_{i-1,j}^{n+1}}{2\Delta x} \right) - v_y \left(\frac{B_{i,j+1}^{n+1} - B_{i,j-1}^{n+1}}{2\Delta y} \right) + D \left(\frac{B_{i+1,j}^{n+1} - 2B_{ij}^{n+1} + B_{i-1,j}^{n+1}}{\Delta x^2} \right) \\ & + D \left(\frac{B_{i,j+1}^{n+1} - 2B_{ij}^{n+1} + B_{i,j-1}^{n+1}}{\Delta y^2} \right) \end{aligned} \quad (12)$$

In this procedure, the lagged value is updated until a specified convergence criterion is reached. For the first iteration, $B_{i+1,j}^n$ is the value at the previous station, i.e., B_{ij}^n . Once $B_{i+1,j}^{n+1}$ has computed, the coefficient $B_{i+1,j}^n$ is updated and a new solution is sought. The procedure continues until a convergence criterion is met. For example, given an $n \times n$ real matrix A and a real n -vector b , the problem considered is: Find x , such that [13]:

$$Ax = b \quad (13)$$

where, A is the coefficient matrix, b is the right-hand side vector, and x is the vector of unknowns. Our matrix which used in this work is $(100^2 \times 100^2)$ grid system, there are a total of (100000000) unknowns at time level $n+1$. Therefore, a total of (100000000) simultaneous equation must be solved. To solve equations (11) and (12) using a Dell computer (Intel(R), Core i7, 3770 CPU, 3.40 GHz, and Installed memory (RAM) 16.00 GB), system type 64-bit operating system.

Results and discussion

1. Generation of the initial bipole magnetic field

The first step of magnetic field simulation is the magnetic field distribution. Single magnetic bipole distributed initially generated to check the simulation components. To generate a single bipole magnetic a set of parameters should be used. This parameters represents peaks positions (x_1, y_1) and (x_2, y_2) , tilt angle δ_0 (the angle between the bipolar spots is equal to one half of the latitude), and half-separation distance between peaks ρ_0 , see Eqs. (1-5), where $\rho_0 = 7.5$ (in grid units). The initial bipole magnetic field takes the following form Fig. 1. This figure shows one of the important parameter that effect the magnetic advection which is the tilted angle. Tilted angle control the magnetic distribution location on the sun surface, especially if we

know that the sun surface velocity distribution is location dependent. Therefore the tilted angle parameter has the distribution influence. Figs.2 shows different tilted angle cases.

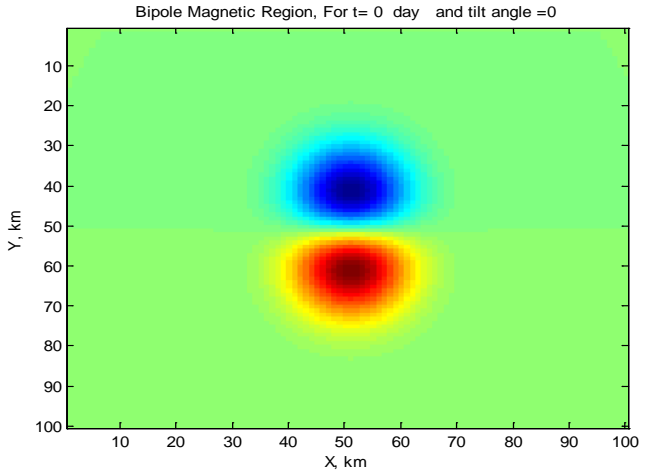


Fig. 1: Bipole magnetic regions.

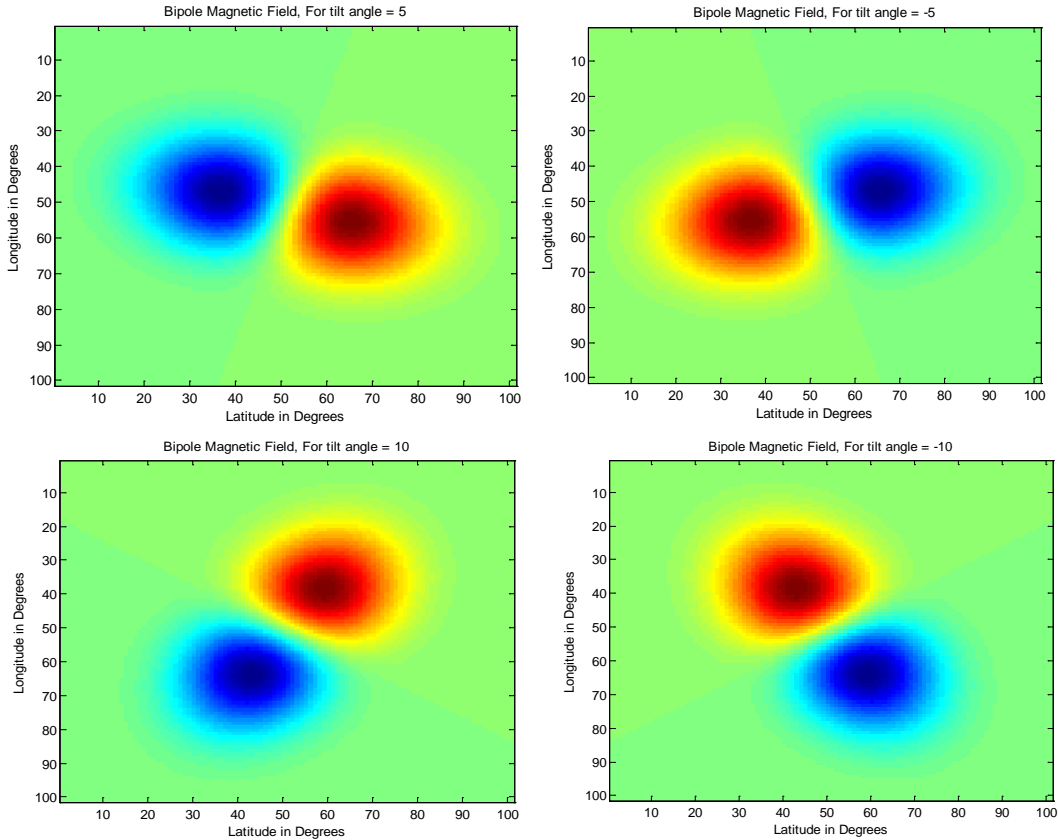


Fig. 2: Simulation the Bipole Magnetic Regions with a positive and negative tilt angles (in degree).

2.Surface flow profiles

In this subsection will illustrates the measurement of the flows that transport flux on the surface of the Sun. The flux include: the differential rotation, Eq. (8), angular velocity (Eq. (9)), and meridional flow, Eq. (10) which are the main components of the transport flux. Fig. 3, shows differential rotation distribution on the solar surface, and Fig. 4, shows the angular velocity on solar surface (the meridional flow is constant is equal $(15.27 \times 10^{-3} kmsec^{-1})$).

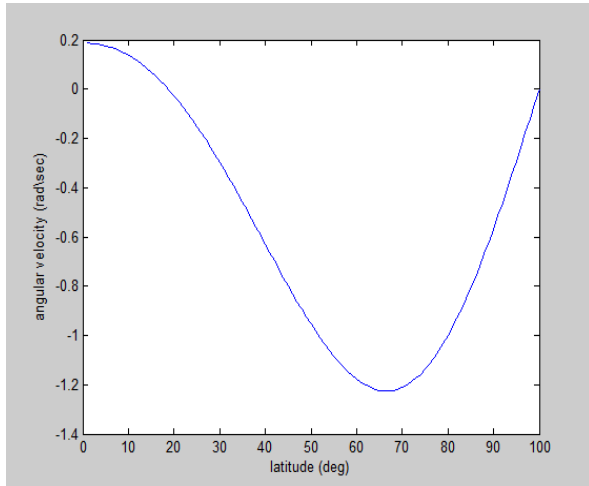


Fig. 3: Differential rotation distribution on sun surface.

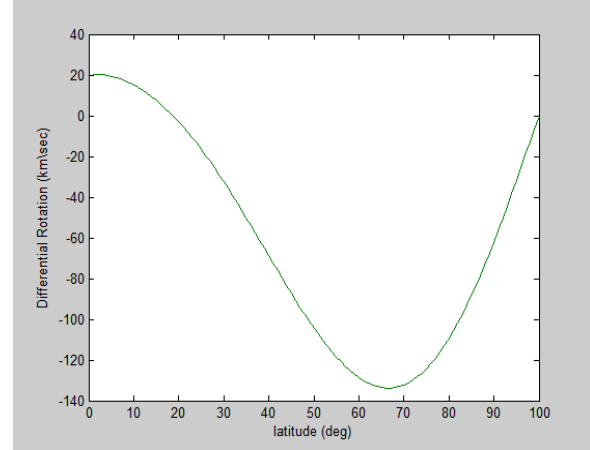


Fig. 4: Angular velocity distribution on sun surface.

In these figures, note a simple model does not include fluctuations in the meridional flow, and this is acceptable because we are ultimately interested in the large-scale coronal magnetic field structure rather than precise local details in the photosphere. In any case, the meridional flow is an order of magnitude weaker than the differential rotation.

3. Global surface flux transport simulation

We have created a surface flux transport model to simulate the magnetic fields over the entire surface of the Sun. As a reference model, taken the transport Eq.(7), Table 1 gives an overview of the parameters used in this simulation.

Table 1: Gives an overview of the parameters used in this simulation.

Parameter	Standard case	Range
Diffusion coefficient $km^2 sec^{-1}$	450	
Meridional flow $km sec^{-1}$	15.27×10^{-3}	0 – 21
Δt (sec)	600	
$\Delta x, \Delta y$ (Km)	$\frac{2\pi R_{*}}{\text{no. of iterations}}$	44×10^5
R_{*} (Km)	7×10^5	
ρ_0 (grid unit)	7.5	
t (day)	$\frac{\text{no. of iteration} \times \Delta t}{24 \times 60 \times 60}$	0 - 60
Tilt angle (δ) (degree)	$\delta = \tan^{-1} \frac{y(t)}{x(t)}$	-5, 0, and 5,
nMatrix Dimensions	For explicit $n = 100 \times 100$ For implicit $n = 100^2 \times 100^2$	

- Explicit Finite Difference Schemes: implementation of explicit method can be

given by Figs. 5-7 for different case of tilted angle and different times.

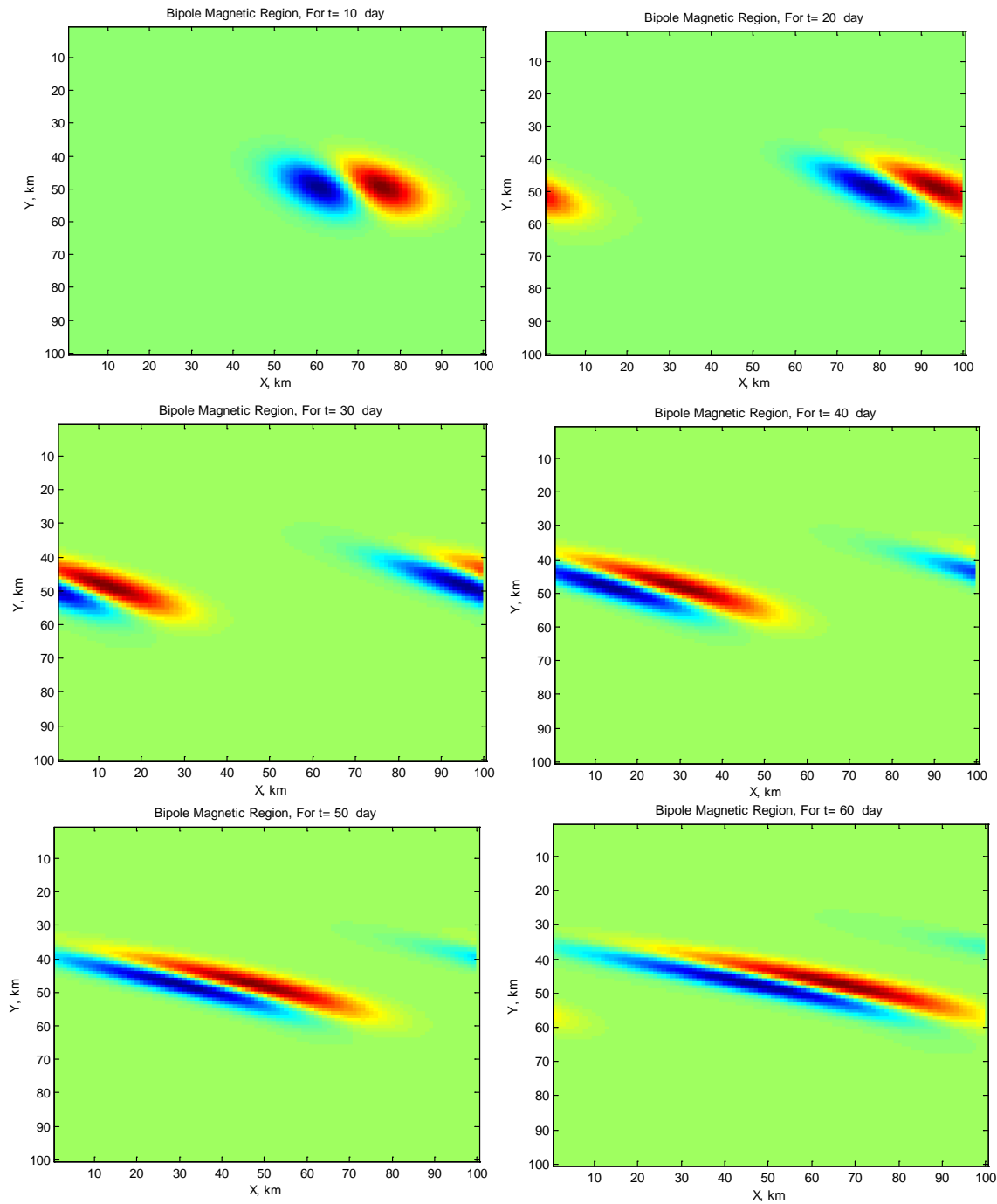


Fig. 5: Explicit numerical solution of equation of magnetic transport flux for 60 days and $\delta = 0^\circ$.

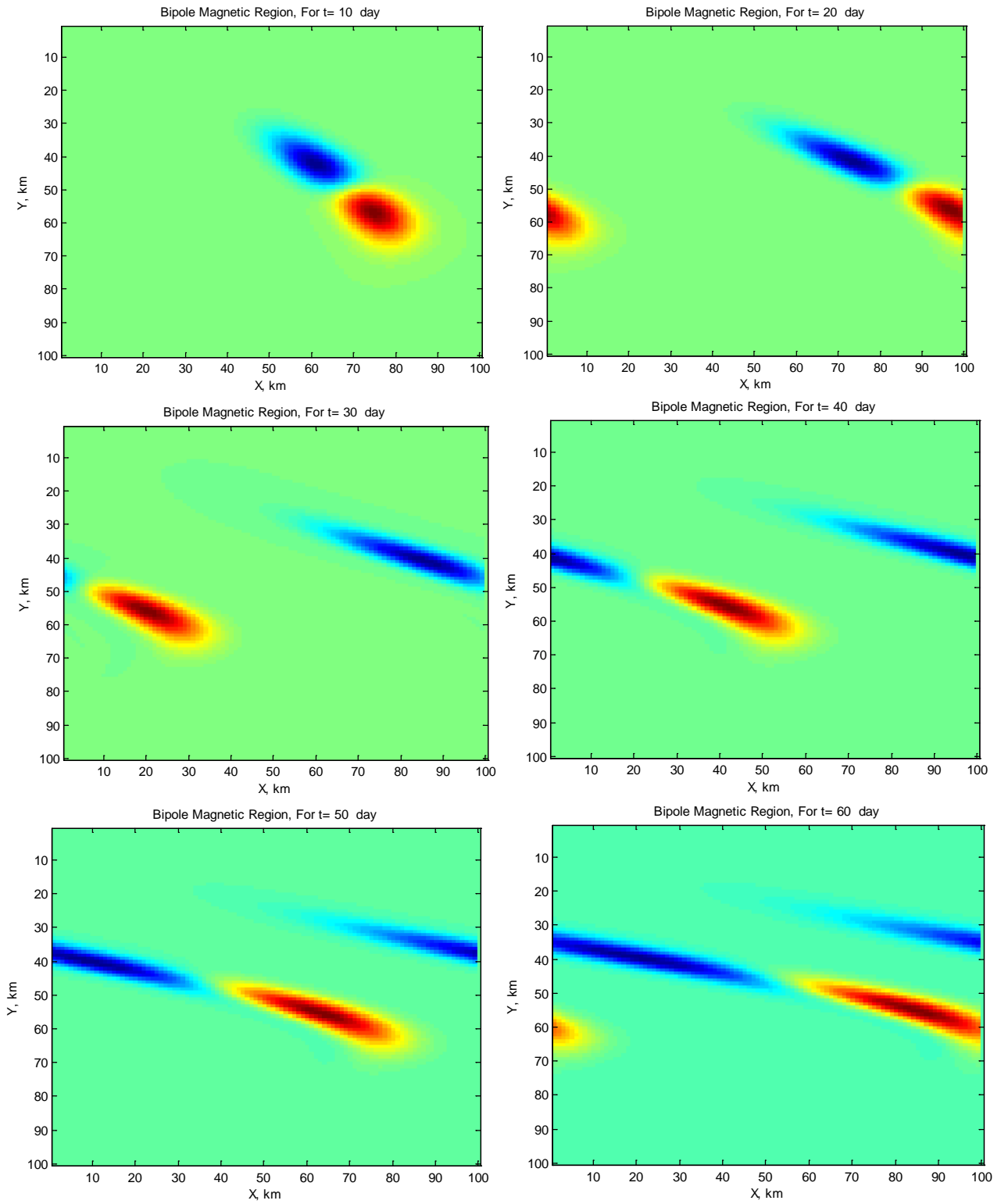


Fig. 6: Explicit numerical solution of equation of magnetic transport flux for 60 days and $\delta = 5^\circ$.

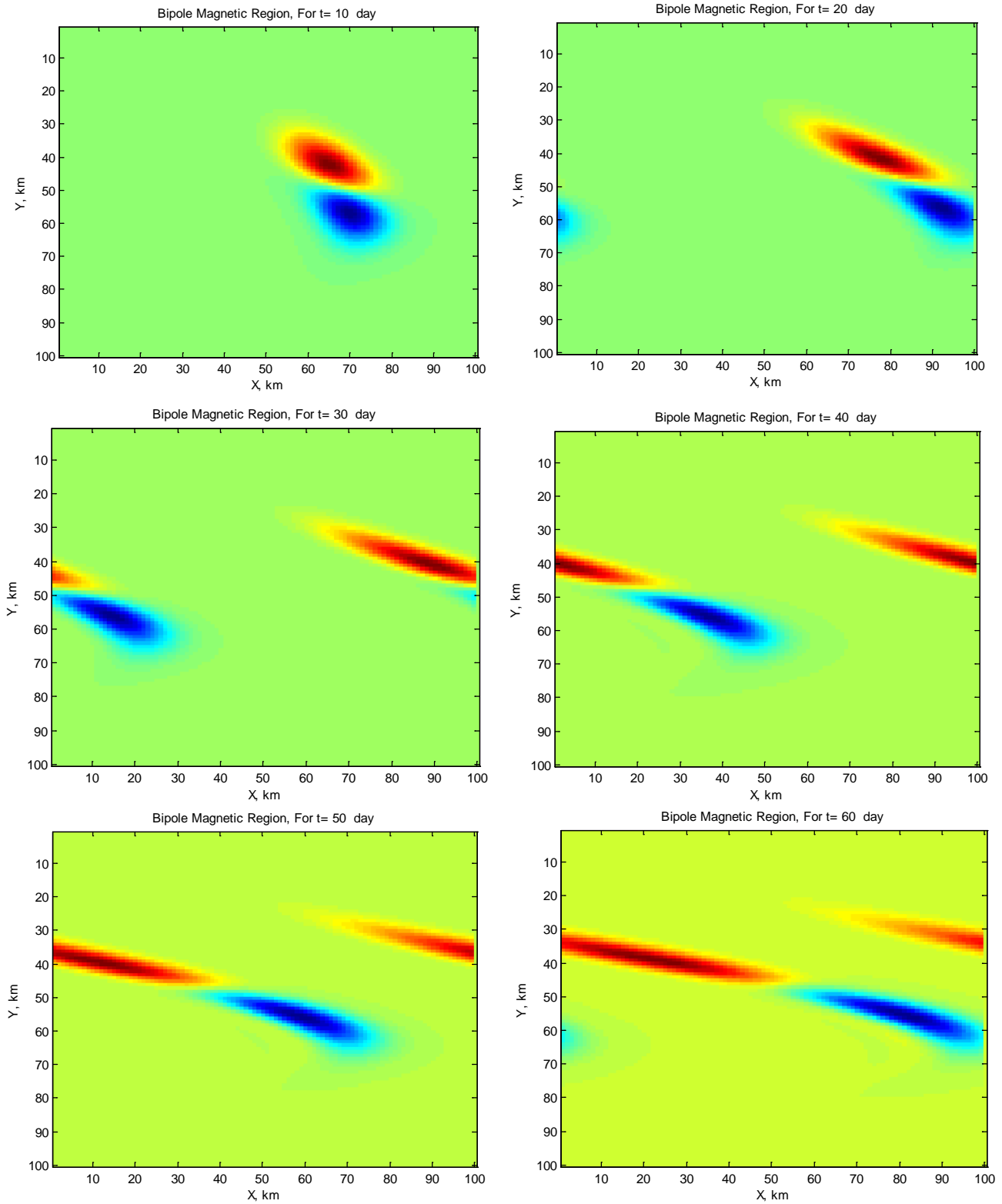


Fig. 7: Explicit numerical solution of equation of magnetic transport flux for 60 days and $\delta = -5^\circ$.

• Implicit Finite Difference Schemes: implementation of implicit method can be

given by Figs.8-10 for different case of tilted angle and different times.

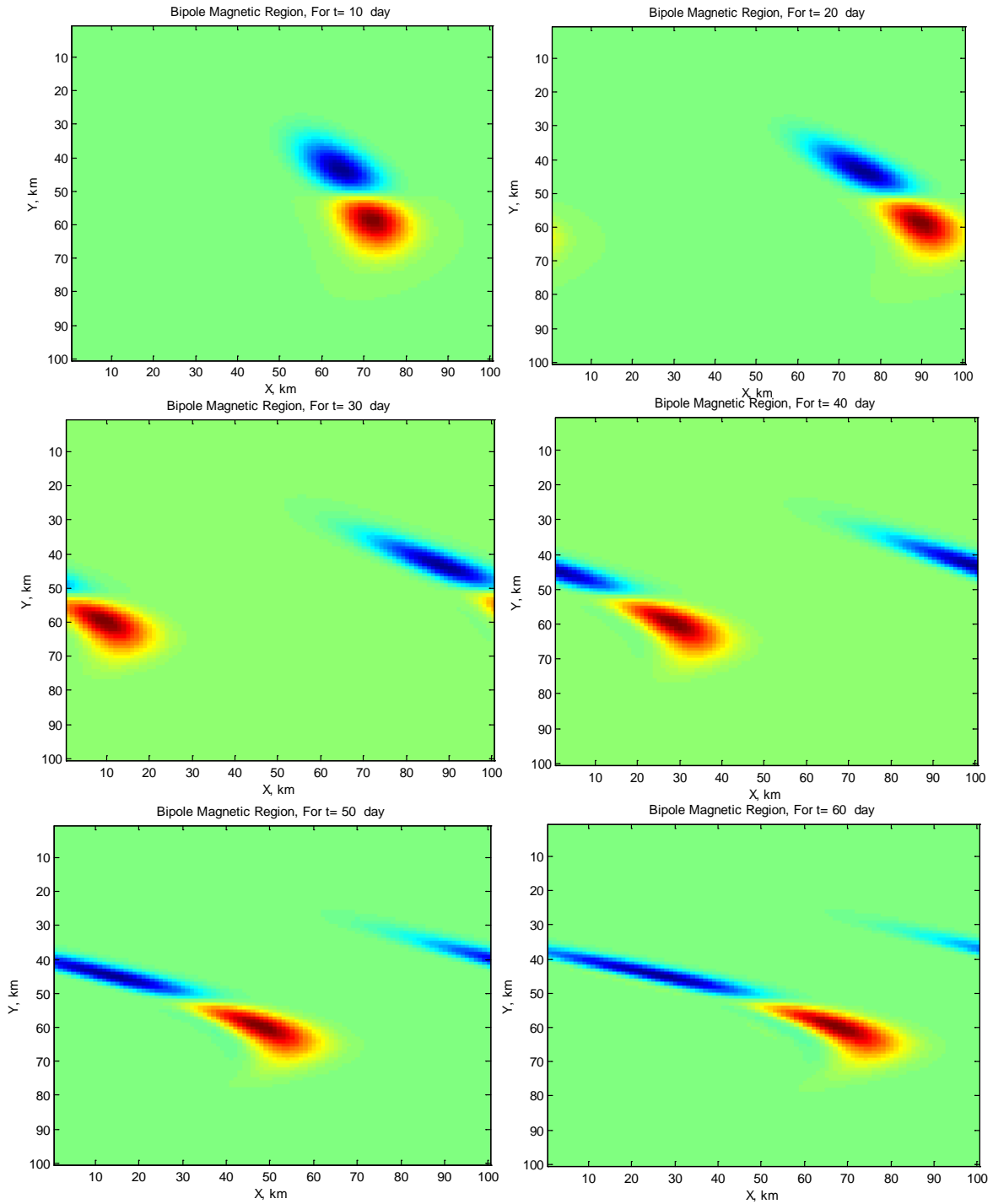


Fig. 8: Implicit numerical solution of equation of magnetic transport flux for many 60 days and $\delta = 0^\circ$.

Figs.5-10 show images which are taken at various time steps within the test simulations. Figs. 6 and 9 are from a simulation with a positive tilt angle while Figs. 7 and 10 are from a simulation with a negative tilt angle, in both implicit and explicit methods. The images are taken for (0, 10, 20, 30, 40, 50, and 60) day of the simulation. When the bipole has a positive tilt angle, the leading, negative polarity is closer to the equator and the trailing positive polarity is closer to the polar region. As the effects of differential rotation and meridional flow evolve the bipole forward, the trailing polarity heads polewards at a faster rate than the leading polarity. These results in more of a latitudinal shear in the trailing polarity than the leading polarity as when its latitude increases it moves into a region of greater gradient of differential rotation. It is also clarify, as the bipole is emerged at a lower latitude, some of the leading negative polarity is transported over the equator. This portion of the polarity will eventually become trapped in the South Pole and forming the magnetic field in this region. When the bipole has a negative pole tilt angle the above mechanism is the same but the polarity (opposite). This will then form the polar magnetic field within this region. This set of images show how the tilt angle of the bipole can affect the eventual evolution of the surface magnetic field on the sun.

Conclusions

The main conclusion could be summarized by the following obtained points:

1. A simple numerical method is shown for evolution of the physical bipole parameters, including the basic physics of advection and diffusion of the solar surface field.
2. It was found the effect of the negative and positive tilt angle of a bipole on the solar flux distribution with time.
3. Make precise measurements of the Sun's surface flows (i.e., differential

rotation and angular velocity). Where it can be seen that the characteristics of the bipole, as well as that of the profiles of the various flows on the solar surface, can have a significant effect on the evolution of the surface field configuration, and hence the evolution of magnetic flux on the solar surface.

4. The implicit method is unconditionally stable and accurate in many circumstances than explicit methods. The advantage of implicit scheme is that it removes the stability limitation associated with the diffusion operator. But, the disadvantage is that the problem becomes more computationally expensive to solve numerically.

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