Inelastic electron scattering form factors involving the second excited 2^+ level in the isotopes $^{50,52,54}Cr$

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Abstract

An expression for the transition charge density is investigated where the deformation in nuclear collective modes is taken into consideration besides the shell model transition density. The inelastic longitudinal form factors C2 calculated using this transition charge density with excitation of the levels for $^{50,52,54}Cr$ nuclei. In this work, the core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state two-body charge density distributions (2BCDD's). It is noticed that the core polarization effects which represent the collective modes are essential in obtaining a remarkable agreement between the calculated inelastic longitudinal *F(q)*'s and those of experimental data.

Key words

Collective modes, inelastic longitudinal form factors, two-body charge density.

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عوامل التشكل للاستطارة الالكترونية الغير المرنة للمستوي المتهيج 2^+ للنظائر $^{50,52,54}Cr$

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الخلاصة

تم اختبار علاقة كثافة الشحنة للحالات المثارة باخذ التشوه الحاصل في الانماط النووية التجميعية بالاضافة الى كثافة الشحنة للحالات الناتجة من انموذج القشرة. استخدمت هذه العلاقة في حساب عوامل التشكل للاستطارة الالكترونية الطولية C2 للنظائر $^{50,52,54}Cr$. ان تاثيرات استقطاب القلب لكثافة الانتقالات حسبت بالاعتماد على شكل انموذج Tassie الى جانب الصيغة الرياضية المشتقة لتوزيعات كثافة الشحنة النووية ذو صيغة الجسيمين في الحالة الارضية (2BCDD's). لقد لوحظ بان تاثير استقطاب القلب الذي يمثل انماط تجميعية يكون جوهريا" للحصول على توافق جيد بين حسابات الاستطارة الطولية غير المرنة *F(q)*'s والقيم العملية.

Introduction

Charge density distributions, transition densities and form factors are considered as fundamental characteristics of the nucleus. These quantities are usually determined experimentally from the scattering of high energy electrons by the nucleus. The information extracted from such experiments was more accurate with higher momentum transfer to the nucleus. Various theoretical methods [1, 2, 3] are used for calculations the charge density distributions, among them the Hartree-Fock method with the Skyrme effective interaction the theory of finite Fermi systems and the single particle potential method. Calculations of form factors [4] using the model space wave function alone is inadequate for reproducing the data of electron scattering. Therefore effects out of the model space, which is called core polarization effects, are necessary to be included in the calculations. These effects can be considered as a polarization of core protons by the valence protons and neutrons. Core polarization effects can be treated either by connecting the ground state to the *J*-multipole $n\hbar\omega$ giant resonances [4], where the shape of the transition densities for these excitations is given by Tassie model [5], or by using a microscopic theory [6-8] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Comparisons between theoretical and observed longitudinal electron scattering form factors have long been used as stringent test of models of nuclear structure. Inelastic Electron Scattering from fp shell nuclei had been studied by Sahu et al. [9]. They calculated form factors for $^{46,48,50}Ti$, $^{50,52,54}Cr$ and $^{54,56}Fe$ by the use of Hartree-Fock method, their results are in a good agreement with the experimental data.

The aim of the present work is to study the inelastic longitudinal form factor C2 for $^{50,52,54}Cr$ isotopes. The calculation of form factors using the many particle shell model space alone were known to be inadequate in describing electron scattering data. So effects out of the model space (core-polarization) are necessary to be included in the calculations. The shape of the transtion density for the excitation considered in this work was given by the Tassie model [5], where this model is connected with the ground state charge density, where the ground state charge density of the present work is to derive an expression for the ground state two - body charge density distributions (2BCDD), based on the use of the two - body wave functions of the harmonic oscillator and the full two–body correlation functions FC's. The size parameter b chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one-body density matrix (OBDM) elements are calculated using the shell model code OXBASH [10].

Theory

The interaction of the electron with charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the CDD through the matrix elements of multipl operators $\hat{T}_J^L(q)$ [4].

$$\left| F_J^L(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle \right|^2 \left| F_{cm}(q) \right|^2 \left| F_{fs}(q) \right|^2 \qquad (1)$$

where Z is the atomic number of the nucleus, $F_{cm}(q)$ is the center of mass correction, which remove the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [4] .

$$F_{cm}(q) = e^{q^2b^2/4A} \qquad (2)$$

where A is the nuclear mass number and b is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and takes the form [11].

$$F_{fs}(q) = \left[1 + \left(\frac{q}{4.33} \right)^2 \right]^{-2} \qquad (3)$$

The longitudinal operator is defined as [12].

$$\hat{T}_{Jt_z}^L(q) = \int dr\ j_J(qr) Y_J(\Omega)\ \rho(r,t_z) \qquad (4)$$

where $j_J(qr)$ is the spherical Bessel function, $Y_J(\Omega)$ is the spherical harmonic wave function and $\rho(r,t_z)$

is the charge density operator. The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of the one body density matrix (OBDM) elements times the single particle matrix elements of the longitudinal operator [4], i.e.

$$\left\langle f \left\| \hat{T}^{L}_{JT} \right\| i \right\rangle = \sum_{a,b} OBDM^{JT}(i,f,J,a,b) \left\langle b \left\| \hat{T}^{L}_{JT} \right\| a \right\rangle \tag{5}$$

The many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [4].

$$\left\langle f \left\| \hat{T}^{L}_{J}(\tau_Z,q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{T}^{L}_{J}}(\tau_Z,q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{T}^{L}_{J}}(\tau_Z,q) \right\| i \right\rangle \tag{6}$$

where the model space matrix element in Eq.(6) has the form [4].

$$\left\langle f \left\| \overset{ms}{\hat{T}^{L}_{J}}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{ms}{\rho}_{J,\tau_Z}(i,f,r) \tag{7}$$

where $\overset{ms}{\rho}_J(i,f,r)$ is the transition charge density of model space and given by [4].

$$\overset{ms}{\rho}_{J,\tau_Z}(i,f,r) = \sum_{jj'(ms)} OBDM(i,f,J,j,j',\tau_z) \left\langle j \left\| Y_J \right\| j' \right\rangle R_{nl}(r)\, R_{n'l'}(r) \tag{8}$$

The core- polarization matrix element is given by [4]

$$\left\langle f \left\| \overset{cor}{\hat{T}^{L}_{J}}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{core}{\rho}_J(i,f,r) \tag{9}$$

where $\overset{core}{\rho}_J$ is the core- polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core- polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J\tau_Z}(i,f,r) = \overset{ms}{\rho}_{J\tau_Z}(i,f,r) + \overset{core}{\rho}_{J\tau_Z}(i,f,r) \tag{10}$$

where $\overset{core}{\rho}_J$ is assumed to have the form of Tassie shape and given by [5].

$$\rho^{core}_{J\tau_z}(i,f,r) = N\frac{1}{2}(1+\tau_z)\, r^{J-1} \frac{d\rho(i,f,r)}{dr} \tag{11}$$

where N is a proportionality constant. It is determined by adjusting the reduced transition probability *B(CJ)* and given by [13]

$$N = \frac{\int_0^\infty dr\, r^{J+2} \overset{ms}{\rho}_{J\tau_Z}(i,f,r) - \sqrt{(2J_i+1)B(CJ)}}{(2J+1)\int_0^\infty dr\, r^{2J}\rho(i,f,r)} \tag{12}$$

$\rho(i,f,r)$ is the ground state charge density distribution. It is derived an effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the two-body charge density operator with the two-body correlation functions $\tilde{f}_{ij}$ as[14]

$$\hat{\rho}^{(2)}_{eff}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i \neq j} \tilde{f}_{ij} \left\{ \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}\right] \right\} \tilde{f}_{ij} \qquad (13)$$

where $\vec{r}_{ij}$ and $\vec{R}_{ij}$ of relative and center of mass coordinates and the form of $\tilde{f}_{ij}$ is given by [14].

$$\tilde{f}_{ij} = f(r_{ij})\Delta_1 + f(r_{ij})\left\{1+\alpha(A)S_{ij}\right\}\Delta_2 \quad (14)$$

It is clear that Eq. (14) contains two types of correlations:

1. The two body short range correlations presented in the first term of Eq. (14) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body functions with the exception of $^3S_1$ and $^1D_3$ states. It should be remarked that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances, where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of $f(r_{ij})$ is given as [14]

$$f(r_{ij}) = \begin{Bmatrix} 0 & for\ r_{ij} \le r_c \\ 1-\exp\left\{-\mu(r_{ij}-r_c)^2\right\} & for\ r_{ij} > r_c \end{Bmatrix} \quad (15)$$

where r_c (in *fm*) is the radius of a suitable hard core and $\mu = 25\ fm^{-2}$ [14] is a correlation parameter.

2. The two-body tensor correlations presented in the second term of Eq.(14) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto $^1S_3$ and $^1D_3$ states only. S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i.\vec{r}_{ij})(\vec{\sigma}_j.\vec{r}_{ij}) - \vec{\sigma}_i.\vec{\sigma}_j \qquad (16)$$

The parameter $\alpha(A)$ is the strength of tensor correlations and it is non zero only in the $^1S_3 - ^1D_3$ channels.

Results and discussion

The inelastic longitudinal electron scattering form factors C2 are calculated using an expression for the transition charge density of Eq.(10). The model space transition density is obtained using Eq.(8), where the OBDM elements required by the calculations of the form factors of open shell nuclei are taken from [10] . For considering the collective modes of the nuclei, the core polarization transition density of Eq.(11) is evaluated by adopting the Tassie model together with the calculated ground state 2BCDD of Eq.(13).

The OBDM elements for all transitions considered are calculated by using universal code OXBASH and will be referred to subsequently as each form factor is to be considered in turn. All parameters required in the following calculations of 2BCDD's, $\langle r^2 \rangle^{1/2}$ and longitudinal *F(q)*'s are presented in Table 1.

Table 1:Parameters which have been used in the calculations of the present work for the 2BCDD's, $\langle r^2 \rangle^{1/2}$ and elastic and inelastic longitudinal F(q)'s of all nuclei under study.

Nucleus	***b*** (fm)	$\alpha(A)$	$\langle r^2 \rangle^{1/2}_{Theo.}$ (fm)	$\langle r^2 \rangle^{1/2}_{Exp.}$ (fm)[15]
^{50}Cr	1.842	0.08	3.558	3.638(18)
^{52}Cr	1.853	0.085	3.476	3.613(17)
^{54}Cr	1.860	0.09	3.611	3.673(14)

1. The nucleus ^{50}Cr

For the conventional multiparticle shell-model, ^{50}Cr has ten nucleons outside the core ^{40}Ca and it is possible to perform shell-model calculations for this nucleus in $1f_{7/2}$ shell space. The transitions under investigation is C2, 0.78MeV $J^{\pi}T= 0^+ 1$ to 2^+1.

The (2^+_1 1) state at 0.78 MeV

In this transition, the electron excites the nucleus from the ground state 0^+1 to the state 2^+1 with excitation energy of 0.78 MeV. In Fig.1 the experimental data of the C2 Coulomb form factors which are taken from Ref. [16] are compared with the theoretical fp-shell model calculation. The solid curve shows the results with core-polarization effects by using Tassie model with the effective two-body charge density distribution and the dashed curve corresponding to the result without core polarization effects. In fp-shell model, the calculated form factors underpredict the data in all regions of momentum transfers (q). In this model only model space wave function are considered. The inclusion of core-polarization effect enhances the form factor. This enhancement brings the total theoretical results of the longitudinal C2 form factor very close to the experimental data which are plotted versus q as shown by solid curve of Fig. 1. The experimental data of $B(C2)$ is equal to $1.7 \pm 0.2\ e^2.fm^4$ [16]. The OBDM elements are give in Table 2.

Table 2: The values of the OBDM elements for the longitudinal C2 transition $2^+1$of ^{50}Cr.

J_i	J_f	OBDM (ΔT=0)	OBDM (ΔT=1)
7/2	7/2	1.52630	-0.08622

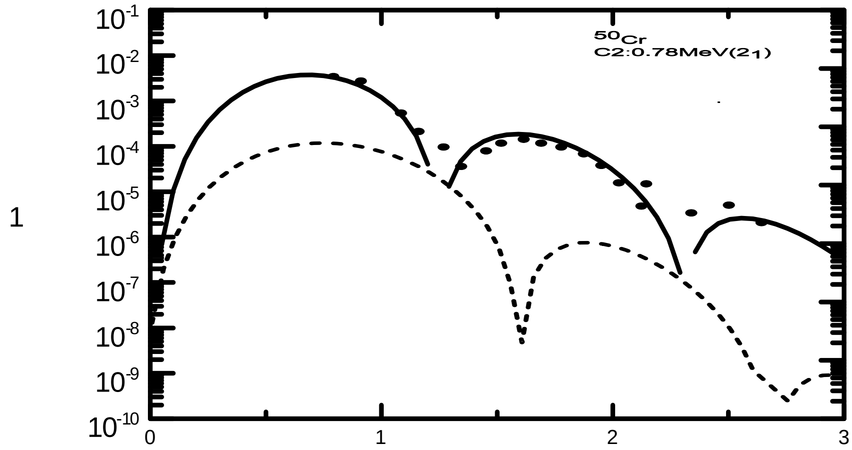

Fig.1: Inelastic longitudinal form factors for the transition to the 2^+_1 state in the ^{50}Cr with and without core-polarization effects, the experimental data are taken from Ref.[16].

2. The nucleus ^{52}Cr

The structure and properties of ^{52}Cr are experimentally and theoretically well studied. For the conventional multiparticle shell-model, ^{52}Cr has 12 nucleons outside the core ^{40}Ca and it is possible to perform shell-model calculations for this nucleus in $1f_{7/2}$ shell space. The transitions under investigation is C2, 1.43MeV $J^{\pi}T = 0^+ 2$ to 2^+2.

The 1.44 MeV (2^+ 2) state

The nucleus is excited from the ground state (0^+ 2) to the excited state (2^+2) with an excitation energy 1.43 MeV. Fig. 2 shows the relation between the longitudinal Coulomb C2 electron scattering form factors as a function of momentum transfers q. The dashed curve represents the results of the model space (1f 2p-shell), while the solid curve represents the results of 2p1f-shell with the inclusion of core polarization effects using Tassie model. The 1f 2p-shell -shell model fail to describe the data in both the transition strength and the form factors. While the core-polarization effect calculations raise the 1f 2p-shell model space calculation making the total theoretical form factor agreed with the experimental values which are taken from Ref.[16], but it fails to describe the experimental data in many regions of momentum transfer. The experimental values of $B(C2)$ is 17.4 $\pm$ 0.7 $e^2.fm^4$ [16]. The OBDM elements are give in Table 3.

Table 3: The values of the OBDM elements for the longitudinal C2 transition 2^+2of ^{52}Cr.

J_i	J_f	**OBDM (ΔT=0)**
7/2	7/2	0.81449

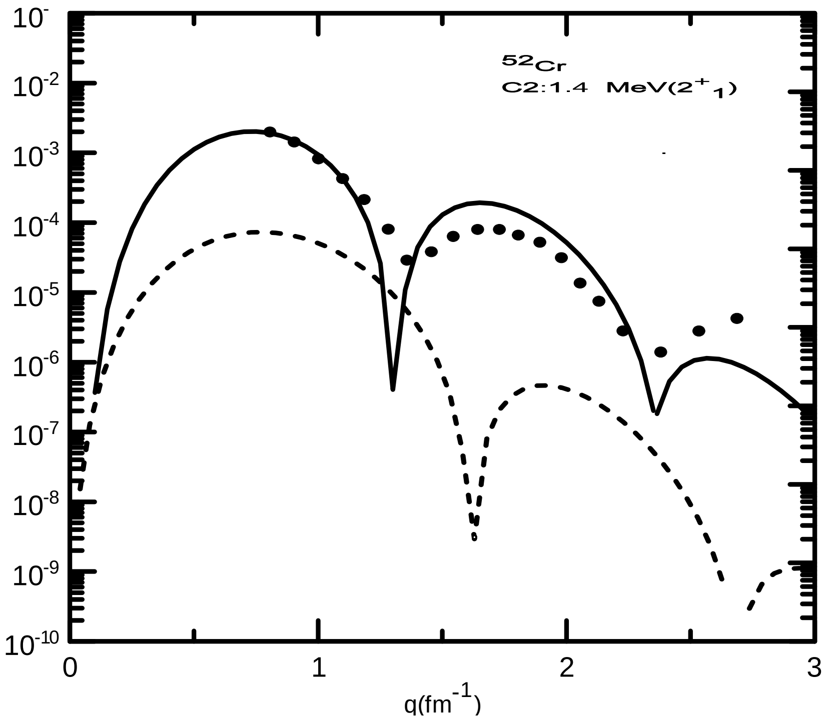

Fig. 2: Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{52}Cr with and without core-polarization effects, the experimental data are taken from Ref.[16].

3. The nucleus ^{54}Cr

Chromium ^{54}Cr has been extensively studied both theoretically and experimentally. For the conventional many particle shell model, this nucleus is considered as an inert ^{40}Ca core plus six nucleons distributed over 2p1f space.

The 0.84 MeV (2^+3) state

The form factors for C2 transition in ^{54}Cr with an excitation energy 0.84MeV,. The odel space fail to describe the form factors in all momentum transfers. A good fit to the C2 data is obtained with the 2p1f-shell model calculations including core polarization effects using Tassie model in first and third maxima, but the second maximum is overestimated as shown in Fig. 3 by solid curve. The experimental values of $B(C2)$ is 494 $e^2.fm^4$ [16]. The OBDM elements are give in Table 4.

Table 4: The values of the OBDM elements for the longitudinal C2 Transition $2^+3$of ^{54}Cr.

J_i	J_f	OBDM (ΔT=0))	OBDM (ΔT=1))
7/2	7/2	0.5998	0.0
5/2	5/2	0.0	0.0488
5/2	3/2	0.0	-0.0998
5/2	1/2	0.0	0.1199
3/2	5/2	0.0	0.0476
3/2	3/2	0.0	0.4456
3/2	1/2	1.8799	0.2047
1/2	5/2	0.0	0.0702
1/2	3/2	0.0	0.3231

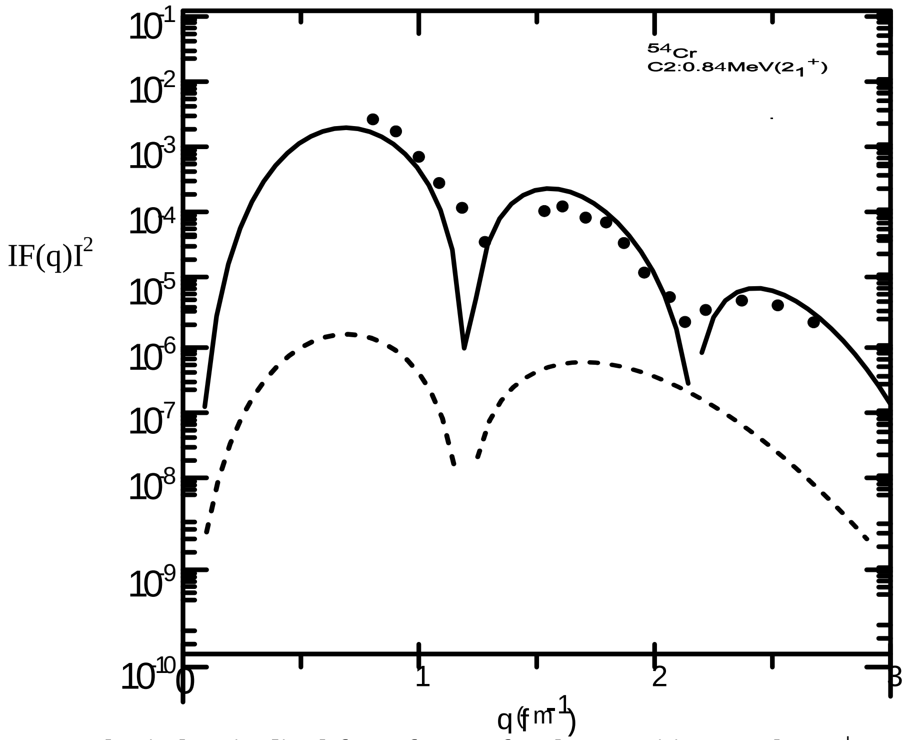

Fig. 3: Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{54}Cr with and without core-polarization effects, the experimental data are taken from Ref.[16].

Conclusions

The fp-shell models, which can describe the static properties and energy levels, are less successful for describing dynamics properties such as C2 transition rates and electron scattering form factors. The core-polarization effect enhances the form factors and makes the theoretical results of the longitudinal form factors closer to the experimental data in the C2 transition which is studied in the present work.

Considering the effect of FC's is, generally, essential in getting good agreement between the calculated results of $\langle r^2 \rangle^{1/2}$ and those of experimental data.

References

[1] A. N. Antonov, P. E. Hodgson, I. Zh. Petkov, Clarendon, Oxford, (1988).
[2] I. S. Gulkarov, M. M. Mansurov and A. A. Khomich, Sov. J. Nucl. Phys., 47 (1988) 25.
[3] B. I. Barts, Yu. L. Bolotin, E. V. Inopin, V. Yu. Gonchar, Keiv, Naukova, Dumka, (1982).
[4] B. A. Brown, R. A. Radhi, B. H. Wildenthal, Phys. Rep., 101 (1983) 313.
[5] L. J. Tassie, Aust. Jour. Phys. 9 407 (1956).
[6] R. A. Radhi, A. Al- Rahmani, A. K. Hamoudi, E. A. Salman, Iraqi J. Sci, 43C, 2 (2002) 27.
[7] A. D. Salman, N. M. Adeeb, M. H. Oleiwi, J. of Nucl. Phys. and Particle Phys., 3 (2013) 20.
[8] A. D. Salman, N. M. Adeeb, M. H. Oleiwi, J. Adv. Phys, 2 (2013) 1-5.
[9] R. Sahu, D. P. Ahalpara, K. H. Bhatt, Bull. Am. Phys. Soc. 31, 10 (1986) 1764.
[10] B. A. Brown, A. Etchgoyen, N. S. Godwin, W. D. M. Rae, W. A. Richter, W. E. Ormand, E. K. Warburton, J. S. Winfield, L. Zhao, C. H. Zimmerman, MSU- NSCL report number, 1289, (2005) and private communication.
[11] R. S. Willey, Nucl. Phys., 40 (1963) 529.
[12] H. Chandra and G. Sauer, Phys. Rev. C13 (1976) 245.
[13] G. N. Flaiyh, Ph.D Thesis, University of Baghdad, 2008.
[14] J. Fiase, A. Hamoudi, J. M. Irvine, F. Yazici, J. Phys, G14, 27 (1988) 27.
[15] H. De Vries, C. W. De Jager, C. De Vries, Atomic data and nuclear data tables, 36 (1987) 495.
[16] R. Sahu, D. P. Ahalpara, K. H. Hatt, J. Phys. G16 (1990) 733.