Study the variation of synodic month for the moon through 2000-2100

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Abstract

In this research study the synodic month for the moon and their relationship with the mean anomaly for the moon orbit and date A.D and for long periods of time (100 years), we was design a computer program that calculates the period of synodic months, and the coordinates of the moon at the moment of the new moon with high accuracy. During the 100 year, there are 1236 period of synodic months.

We found that the when New Moon occurs near perigee (mean anomaly = 0°), the length of the synodic month at a minimum. Similarly, when New Moon occurs near apogee (mean anomaly = 180°), the length of the synodic month reaches a maximum. The shortest synodic month on 2053 /1/ 16 and lasted (29.27436) days. The longest synodic month began on 2008 /11/ 27 and lasted (29.81442) days. The mean synodic month (29.53109) days. We found the relationship between synodic month with months. The shortest synodic month are correlated with date (June and July) when the Earth is near aphelion. And the longest Synodic month are correlated with date (December and January) when the Earth is near perihelion.

Key words

Moon orbit, Moon period, synodic month.

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دراسة تغير مدة الأشهر الاقترانية للقمر للفترة بين 2000 و 2100 م

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الخلاصة

تم في هذا البحث دراسة مدة الأشهر الاقترانية للقمر وعلاقتها بمعدل الانحراف المداري للقمر والتاريخ الميلادي وللفترات طويلة (100عام)، حيث تم تصميم برنامج حاسوبي لحساب مدة الأشهر الاقترانية وإحداثيات القمر لحظة ولادة الهلال وبدقة عالية، خلال مئة سنة يوجد 1236 شهرا قمريا اقترانيا.

ووجد أن مدة الأشهر الاقترانية تكون في اقل قيمها عندما تكون ولادة الهلال قريبة من الحضيض القمري (معدل الانحراف المداري قريباً من 0°)، وان مدة الأشهر الاقترانية تكون في أعلى قيمها عندما تكون ولادة الهلال قريبة من الأوج القمري (معدل الانحراف المداري قريباً من 180°). وخلال الفترة (2000 - 2100) كانت اقل مدة لشهر اقتراني هي (29.27436) يوما في 16/ 1/ 2053، و أطول مدة (29.81442) يوما في 2008/11/27. وكان معدل مدة الأشهر الاقترانية (29.53109) يوما، ووجد أن مدة الأشهر الاقترانية ترتبط بعلاقة مع الأشهر، تكون مدة الأشهر الاقترانية في اقل قيمها عند الأشهر الميلادية (حزيران وتموز) عندما الأرض قرب الأوج، وتكون مدة الاشهر الاقترانية في أعلى قيمها عند الأشهر الميلادية (كانون الأول وكانون الثاني) عندما الأرض قرب الحضيض.

Introduction

The Moon revolves around Earth in an elliptical orbit with a mean eccentricity of 0.0549. Thus, the Moon's (center-to-center) distance from Earth varies with mean values of

363,396 km at perigee to 405,504 km at apogee. The mutual gravitational force between the Sun and Moon 2.22 to the Moon and Earth for this reason, the Sun plays a dominant role in perturbing the Moon's motion. The ever changing distances and relative positions between the Sun, Moon, and Earth, the Solar radiation pressure on the Moon, the oblations of Earth less than the gravitational attraction of the other planets all act to throw the Moon's orbital parameters into a constant state of change which are less than the solar attraction. Although the Moon's position and velocity can be described by the classic Keplerian orbital elements, such osculating elements are only valid for a single instant in time [1].

There are five types of months for the Moon depending on the start point to measure the period that Moon takes it to return to the same point which starts from it. These are [2, 3]:

1. Syndic month (lunar month): Period between successive new or full moons. This is the same duration as one lunation and is equivalent to 29.53059 days of mean solar time which equals to ($29^d\ 12^h\ 44^m\ 2.8^s$).
2. Sidereal month: Time taken for the Moon to complete a single revolution around the Earth, measured relative to a fixed star; it is equivalent to 27.32166 days of mean solar time. Which equals to($27^d 7^h 43^m 11.5^s$).
3. Anomalistic month: Time taken for the Moon to complete a single orbit around the Earth, measured from perigee to perigee. An anomalistic month is shorter than the more commonly used Synodic month, being equivalent to 27.55455 days of mean solar time or ($27^d 13^h 18^m 37.4^s$).
4. Draconic month (nodical month): Time taken for the Moon to complete a single revolution around the Earth, measured relative to its ascending node; it is equivalent to 27.21222 days of mean solar time or ($27^d 5^h 5^m 34.1^s$).
5. Tropical month: Time taken for the Moon to complete a single revolution around the Earth, measured relative to the first point of Aries; it is equivalent to 27.32158 days of mean solar time or ($27^d 7^h 34^m 4.7^s$).

Synodic month

The most familiar lunar cycle is the synodic month because it governs the well-known cycle of the Moon's phases. The Moon has no light of its own but shines by reflected sunlight. As a consequence, the geometry of its orbital position relative to the Sun and Earth determines the Moon's apparent phase.

The mean length of the synodic month is 29.53059 days ($29^d\ 12^h\ 44^m\ 2.8^s$). This is nearly 2.21 days longer than the sidereal month. As the Moon revolves around Earth, both objects also progress in orbit around the Sun. After completing one revolution with respect to the stars, the Moon must continue a little farther along its orbit to catch up to the same position it started from relative to the Sun and Earth. This explains why the mean synodic month is longer than the sidereal month [4].

We can find the Julian date for new moon (JDE) using the equation[5]:

$$JDE = 2451550.09766 + 29.53058861K + 0.00015437T'^2 - 0.000000150T'^3 + 0.00000000073T^4 \quad (1)$$

These instants are expressed in Ephemeris Time (Julian Ephemeris Days). In the formula above, an integer value of k gives a New Moon, an integer value increased by:

0.25 gives a First Quarter, 0.50 gives a Full Moon, 0.75 gives a Last Quarter.
An approximate value of K is given by:
K = (year – 2000) × 12.3685
where the "year" should be taken with decimals and T' is the time in Julian centuries since the epoch 2000, which calculated with a sufficient accuracy from:
T' = K / 1236.85
To obtain the time of the true phase, the following corrections should be added to the time of the mean phase given by Eq. (1).
For New and Full Moon[5]:

$$+ (0.1734 - 0.000393\ T')\sin Ms + 0.0021 \sin 2Ms - 0.4068 \sin Mm + 0.0161 \sin 2Mm - 0.0004 \sin 3Mm + 0.0104 \sin 2Fm - 0.0051 \sin(Ms + Mm) - 0.0074 \sin(Ms - Mm) + 0.0004 \sin(2Fm + Ms) - 0.0004 \sin(2Fm - Ms) - 0.0006 \sin(2Fm + Mm) + 0.0010 \sin(2Fm - Mm) + 0.0005 \sin(Ms + 2Mm) \quad (2)$$

where:
(Ms):Sun's mean anomaly, (Mm): Moon's mean anomaly at JD for year 2000 A.D as a function (K,T') are calculated as [5]:

$$Ms = 2.5534 + 29.10535670\ K - 0.0000014\ T'^2 - 0.00000011\ T'^3 \quad (3)$$

$$Mm = 201.5643 + 385.816935\ K + 0.0107582\ T'^2 + 0.00001238\ T'^3 - 0.0000000538\ T'^4 \quad (4)$$

(Fm): Moon's argument of latitude as a function (K,T')

$$Fm = 160.7108 + 390.67050284\ K - 0.0016118\ T'^2 - 0.00000227\ T'^3 + 0.000000011 T'^4 \quad (5)$$

which are expressed in degrees and decimals and may be reduced to the interval (0 – 360) degrees.
To obtain the time of JDE with high accuracy must be add corrections of planetary arguments and true phase the equations described in [5].
The distances Earth - Moon and Sun-Moon were calculated as ref. [6] to determine the perigee date.

Results and discussion

The program was designed using Quick-Basic language to calculate the time of New Moon, period of synodic month, and orbital elements for the Moon. The results obtained from the program have been compared with [4, 5, 7,8] and proved high accuracy.

Table 1 contains details for all synodic month in 2015. The first to third columns lists the decimal date of every new Moon throughout the year (Universal Time), while the fourth column gives the duration of each synodic month. The fifth column is the difference between the actual and mean of synodic month (29.53059) day. The first synodic month of the year (20.Jan) was 2.175^h shorter than the mean. Continuing through 2015, the length of each synodic month drops and reaches a minimum of 3.461^h shorter than the mean value (18.Apr). The duration now increases with each succeeding synodic month until the maximum value of the year is reached of 4.954^h longer than the mean (13.Oct).

The last column in Table 1 gives the Moon's mean anomaly at the instant of New Moon. The mean anomaly is the angle between the Moon's position and the point of perigee along its orbit. Table 1 shows that when New Moon occurs near perigee (mean anomaly = 0°), the length of the synodic month at a minimum value (e.g., 20. Mar. and 18.Apr.). Similarly, when New Moon occurs near apogee (mean anomaly = 180°), the length of the synodic month reaches a maximum value (e.g., 13.Sepand 13.Oct).

Table 1: Synodic month length in 2015 (sample of results through 100 years).

Date of New Moon (U.T)			Length of Synodic Month	Difference From Mean Synodic Month	Moon's mean Anomaly
Day	**h**	**m**	**(day)**	**(hour)**	**(deg)**
20.Jan	16	11	29.48461	-2.17537	324
18.Feb	2	45	29.43995	-2.91832	349
20.Mar	12	33	29.40899	-3.38948	15
18.Apr	21	54	29.38936	-3.46191	41
18.May	7	10	29.38634	-2.86527	67
16.Jun	17	3	29.4112	-1.41571	93
16.Jul	4	22	29.4716	0.750988	118
14.Aug	17	51	29.56188	3.064489	144
13.Sep	9	39	29.65828	4.673194	170
13.Oct	3	3	29.72531	4.954473	196
11.Nov	20	45	29.73703	3.968462	222
11.Dec	13	27	29.69594	2.284763	248

The relationship between synodic month with mean anomaly through 2000-2100 is quite apparent in Fig.1. The shortest synodic month are clearly correlated with New Moon at perigee (mean anomaly = 0°), while the longest lunations occur at apogee (mean anomaly = 180°).

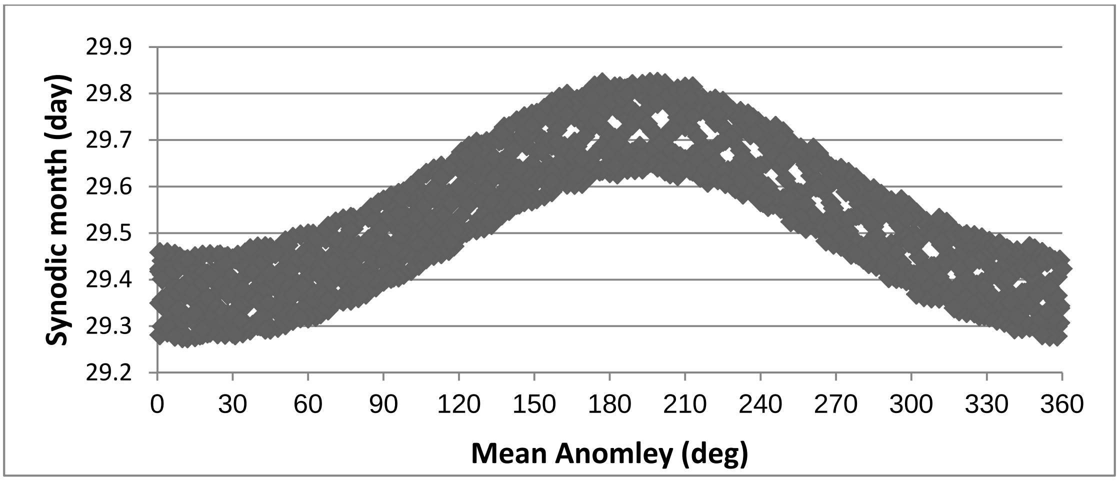

Fig.1: Period of the synodic month with mean anomaly through 2000-2100.

And the interpretation of this relationship between synodic month with mean anomaly, the Moon's orbital velocity is slower at apogee so it takes longer to travel a given distance. Thus, the length of the synodic month is shorter than average when New Moon occurs near perigee and longer than average when New Moon occurs near apogee. Earth's elliptical orbit around the Sun also factors into the length of the synodic month. With an eccentricity of 0.0167, Earth's orbit is about one third as elliptical as the Moon's orbit. Nevertheless, it affects the length of the synodic month by producing shorter synodic month near aphelion and longer synodic month near perihelion.

During the hundred year (2000 - 2100), there are 1236 period of synodic months. The Fig.2 shows the distribution in the length of the synodic month over 100 years. We can obtain a new important results:

1- There is a strong periodic pattern that repeats about every 111 synodic months (almost 8.85 years), which the time required for the lunar orbital perigee to advance eastward 360° with respect to the Earth orbital perihelion.
2- The shortest synodic month on 2053/1/16 and lasted 29.27436 days ($29^d\ 06^h35^m\ 4^s$). and The longest synodic month began on 2008/11/27 and lasted 29.81442 days ($29^d19^h32^m45^s$). Thus, the duration of the synodic month varies over a range of $12^h57^m41^s$ during this time interval.

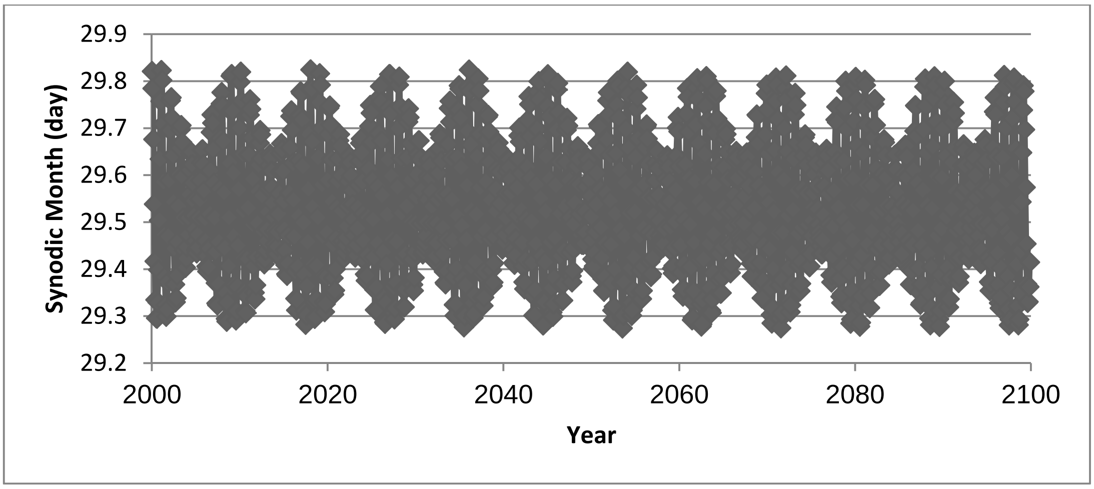

Fig.2: Period of the synodic month through 2000-2100.

The length of each synodic month is plotted in Fig.3 for the 20-year period from 2000to 2020. The time from top to top of synodic month length is 412 days meaning period of this cycle corresponds to the time between two consecutive alignments of the major axis in the direction of the Sun. It is slightly longer than a year because of the slow eastward shift of the Moon's major axis.

An interesting feature revealed in Fig.3 is how the extremes in the synodic month slowly vary over a period of nearly 8.85 years. The envelope defined by the minima and maxima appears to oscillate over a range of values from $\pm 2^h$ to $\pm 6^h$. This behavior is evidence revealing the influence of the 8.85 years cycle in the alignment of the major axes of the orbits of the Moon and Earth. In Fig.3, There is a strong periodic pattern that repeats about every 14 synodic month (about 412 days), which due to the cyclic eastward advance of the lunar orbital perigee.

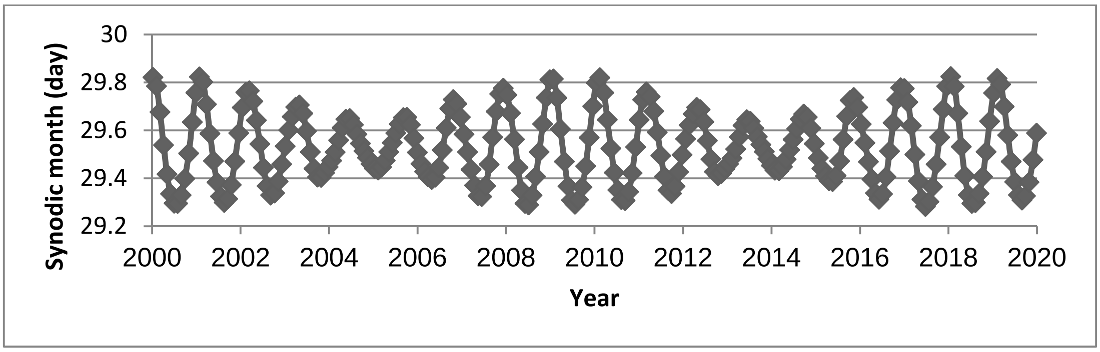

Fig.3: Period for the synodic month trough 2000-2020.

The relationship between synodic month with date in year 2000 is illustrated in Fig.4 which show that:

- The longest synodic month are clearly correlated with days 330 -30 (month Dec and Jan) when Earth is near perihelion (moving fastest) and the lunar conjunction is near apogee (Moon moving slowest).When Earth is at perihelion, its orbital velocity is at its maximum value so Earth travels a larger distance around its orbit in a given time as compared to aphelion. Thus, the Moon must travel a greater distance to align with the Sun, which results in a longer synodic month. Near aphelion, the opposite conditions produce a shorter synodic month.
- The shortest synodic month are clearly correlated with days 180 - 210 (month Jun and July) when Earth is near aphelion, because the Earth is moving slowest in orbit around the sun (Moon moving fastest).

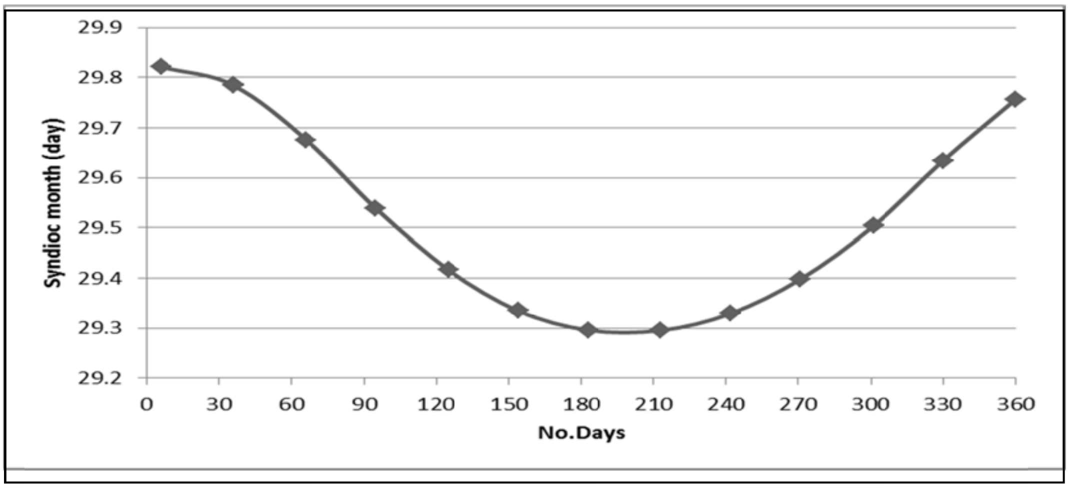

Fig.4: Period 0f the synodic month with days of year in 2000.

To ensure the validity the relationship above for all years, the relationship between synodic month with date through 2000 – 2100 is illustrated in Fig.5, where behavior shows the same relationship for all the years.

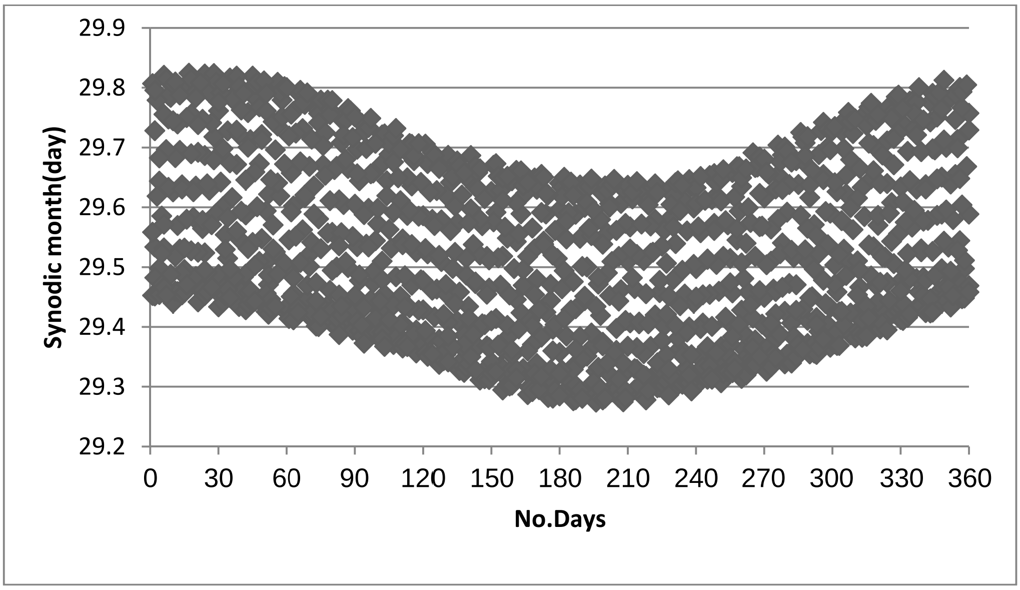

Fig.5: Period for the synodic with days trough 2000-2100.

Conclusions

1- The synodic month at a minimum When New Moon occurs near perigee of it's orbit, and have maximum value when New Moon occurs near apogee.
2- The shortest synodic (29.27436) days and the longest synodic month (29.81442) days. Therefore the mean synodic month (29.53109) days.
3- The short synodic months are correlated with months (June and July) when the Earth is near aphelion. And the long Synodic month are correlated with months (December and January) when the Earth is near perihelion.
4- The required time for the lunar orbital perigee to advance eastward 360° respect to the Earth orbital perihelion, was almost 8.85 years.

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