Inelastic longitudinal electron scattering $C4_2$ form factors in ^{42}Ti nucleus

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Abstract

Inelastic longitudinal electron scattering form factors for second excited state $C4_2$ in ^{42}Ti nucleus have been calculated using shell model theory. Fp shell model space with configuration ($1f_{7/2}$ $2p_{3/2}$ $1f_{5/2}$ $2p_{1/2}$) has been adopted in order to distribute the valence particles (protons and neutrons) outside an inert core ^{40}Ca. Modern model space effective interactions like FPD6 and GXPF1 have been used to generate model space vectors and harmonic oscillator wave function as a single particle wave function. Discarder space (core orbits + higher orbits) has been included in (core polarization effect) as a first order correction in microscopic theory to measure the interested multipole form factors via the model space.

Gogny and Michigan sum of three-range Yukawa potential (M3Y-p2) have been utilized as a residual interaction to couple the (particle-hole) pair across the model space active particles and the excitation energy of the pair is ($2\hbar\omega$) and four options for the used effective and residual interactions were determined for the transitions from ($^+0$) to ($^+0_{1,2,3}$), ($^+2_{1,2,3}$) and ($^+4_{1,2,3}$).

Key words

Inelastic electron scattering form factors, ^{42}Ti nucleus.

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عوامل التشكل $C4_2$ للاستطارة الالكترونية غير المرنة لنواة ^{42}Ti

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الخلاصة

تم دراسة عوامل التشكل للاستطارة الالكترونية الطولية غير المرنة للحالة المتهيجة الثانية $C4_2$ في نواة التيتانيوم ^{42}Ti خلال نموذج القشرة النووية باعتماد القشرة Fp للفضاء الانموذجي للتشكيل ($f_{7/2}$ $2p_{3/2}$ $1f_{5/2}$ $2p_{1/2}$) للدلالة على توزيع جسيمات التكافؤ (البروتونات والنيوترونات) خارج القلب الخامل ^{40}Ca. التفاعلات المؤثرة لانموذج الفضاء مثل (FPD6) و (GXPF1) أستخدمت لتوليد متجهات أنموذج الفضاء وجهد المتذبذب التوافقي البسيط لتوليد الدوال الموجية للجسيم المنفرد. وقد تم تضمين الفضاء المستثنى بعملية تدعى "تاثير إستقطاب القلب" خلال النظرية المجهرية لتكوين التصحيح من المرتبة الاولى على حسابات عوامل التشكل. إستقطاب القلب يتحقق بطاقة إستثارة مقدارها ($2\hbar\omega$) وباستعمال التفاعل الواقعي الحديث Gogny, M3Y-2p كتفاعل بقية حيث أستخدمت أربعة خيارات بين التفاعلات المؤثرة وتفاعلات البقية للانتقالات من ($^+0$) الى ($^+0_{1,2,3}$ ، $^+2_{1,2,3}$، $^+4_{1,2,3}$).

Introduction

The obtained information from the high energy of electron scattered by the nuclei depends on the magnitude of the de-Broglie wave length that is associated with the electron compared with the range of the nuclear forces. When the incident electron has energy 100 MeV and higher, the de-Broglie wave length will be in the range of the spatial extension of the target nucleus. Thus with this energy, the electron represents a best

probe to study the nuclear structure[1,2].

The scattering cross-section was derived for relativistic electron from spinless nucleus of charge Ze. The nuclear size can be taken into account by multiplying the Mott's cross-section by a factor called "nuclear form factor" which depends on the charge, current and magnetization in the target nucleus [3].

The change in nuclear root mean square (rms) radii for ^{46}Ti - ^{50}Ti isotopes by electron scattering at 29 and 58 MeV had measured by Theissen et al. 1966 [4]. The differential cross-sections for inelastic electron scattering from ^{40,42,44}Ca, ^{46,48,50}Ti and ^{56}Fe isotopes have been studied by [5].

Inelastic electron scattering form factors for the $0^+ \rightarrow 2^+$ transitions in some even Ti isotopes were studied in terms of the projected Hartree-Fock Bogoliubov wave function in the 2p-1f shell resulting from a slightly modified version of the Kuo-Brown effective interaction [6].

Han, in 2000 [7] explained that the full paired configuration mixing calculations in the even-even deformed nucleus ^{46}Ti shows that only small parts of the configuration components were important for the case of either ground states or excited states.

Brandolini and Oliveira, in 2004 [8] discussed the experimental positive parity levels were discussed and compared with the theoretical and finding that the lowest 0^+ , 2^+ , and 4^+ levels in ^{46}Ti are considered in addition to the ground state band.

Dinca et al. 2005 [9] showed the even ^{52,54,56}Ti isotopes have been studied with intermediate-energy Coulomb excitation and absolute B(E; $0^+ \rightarrow 2^+_1$) transition rates had been obtained. These data confirm at neutron number N=32 in neutron–rich nuclei above the doubly magic nucleus ^{48}Ca. Large scale shell model calculations with the most recent effective interactions are unable to reproduce the magnitude of the measured strengths in the semi-magic Ti nuclei.

The level schemes and transition rates B(E2) of even-even ^{48,50,52,54,56}Ti isotopes have been studied by performing large scale shell model calculations with FPD6 and GXPF1 effective interactions [10].

Orbit $1f_{7/2}$ had been adopted as a model space. The use of modern realistic M3Y effective nucleon-nucleon interaction with two sets of fitting parameters (Ried fitting (M3Y-P1), and Paris fitting (M3Y-P0)) beside the use of MSDI had been done as a residual interactions within the calculation of core polarization effects in inelastic longitudinal electron scattering C6 form factor in ^{50}Ti within the framework of first order perturbation theory (microscopic theory) with $2\hbar\omega$ excitation energy[11].

Salman and Adeeb in 2013[12] studied the form factors for the inelastic electron scattering to 2^+, 4^+ states in ^{46,48,50}Ti in the framework of shell model. They used MSDI as effective interaction and M3Y as residual interaction. The calculation was performed in ($0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$) model space as well as extended $6\hbar\omega$ model space. They were noticed that the core polarization effects were essential in obtaining a remarkable agreement between the calculated inelastic longitudinal F(q)'s and experimental data.

Theory

The single particle transition operator depends on the single nucleon, which is a proton or a neutron and it can be written as [13]:

$$\hat{T}^{\eta}_{t_z}=\left\{\frac{1}{2}(1+\tau_z)\hat{T}^{\eta}_{p}+\frac{1}{2}(1-\tau_z)\hat{T}^{\eta}_{n}\right\} \quad (1)$$

where:

$\hat{T}^{\eta}_{p,n}$ is the single particle operator for proton/neutron.

By rearranging equation, the transition operator can be written as:

$$\hat{T}^{\eta}_{t_z}=\frac{1}{2}(\hat{T}^{\eta}_{p}+\hat{T}^{\eta}_{n})+\frac{1}{2}\tau_z(\hat{T}^{\eta}_{p}-\hat{T}^{\eta}_{n})=\hat{T}^{\eta}_{T=0}+\hat{T}^{\eta}_{T=1} \quad (2)$$

where $\hat{T}^{\eta}_{T=0}$ is the isoscalar part of the operator and $\hat{T}^{\eta}_{T=1}$ is the isovector part of the operator. The reduced single-particle matrix of the isoscalar and isovector parts between two single-particle states (α) and (β) are:

$$\left\langle \alpha \left\| \hat{T}^{\eta}_{JT} \right\| \beta \right\rangle = \left\langle \alpha \left\| \hat{T}^{\eta}_{JT=0} \right\| \beta \right\rangle + \left\langle \alpha \left\| \hat{T}^{\eta}_{JT=1} \right\| \beta \right\rangle \quad (3)$$

Many particle matrix elements of the electron scattering operator T_{Λ} are expressed as fallows [14]:

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP} \quad (4)$$

The first and second terms in Eq.(4) are the model space and core polarization (CP) contributions respectively.

For a selected operator T^{η}_{JT}, the reduced matrix elements are written as the sum of the product of the one-body transition density matrix elements (OBDM) times the single-particle transition matrix elements [14]:

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle = \sum_{\alpha,\beta} OBDM(\Gamma_i,\Gamma_f,\alpha,\beta) \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle \quad (5)$$

where $\Lambda = JT$ is the multipolarity and the states $\Gamma_i \equiv J_i T_i$ and $\Gamma_f \equiv J_f T_f$ are initial and final states of the nucleus. While α and β denote to the final and initial single-particle states, respectively (isospin is included).

The OBDM used in the present work are calculated by generating the wave functions of a given transition in the known nucleus using the shell model code OXBASH v. 2005 [15] which contains a complete library of shell model effective interactions. All the information about transitions of given multipolarities are contained in the OBDM which represents the combination of the model space wave functions.

The realistic M3Y effective nn interaction, which is used in electron scattering ($V_{res} = v_{12}$) is expressed as a sum of the central potential part $v_{12}^{(C)}$, spin-orbit potential part $v_{12}^{(LS)}$, long range tensor part $v_{12}^{(TN)}$, and density dependence $v_{12}^{(DD)}$, as follows [16,17] :

$$v_{12} = v_{12}^{(c)} + v_{12}^{(LS)} + v_{12}^{(TN)} + v_{12}^{(DD)} \quad (6)$$

The three potentials are expressed as:[17]

$$v_{12}^{(c)} = \sum_n (t_n^{(SE)} P_{SE} + t_n^{TE} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO}) f_n^{(C)}(r_{12})$$
$$v_{12}^{(LS)} = \sum_n (t_n^{(LSE)} P_{TE} + t_n^{(LSO)} P_{TO}) f_n^{(C)}(r_{12}) L_{12} \cdot (\vec{S}_1 + \vec{S}_2)$$
$$v_{12}^{(TN)} = \sum_n (t_n^{(TNE)} P_{TE} + t_n^{(TNO)} P_{TO}) f_n^{(TN)}(r_{12}) r_{12}^2 S_{12}$$
$$v_{12}^{(DD)} = t^{(DD)}(1 + x^{(DD)} P_O)[\rho(r_1)]^{\alpha} \delta(r_{12}) \quad (7)$$

$f_n(r) = e^{-\mu_n r} / \mu_n r$ for M3Y-type int.

μ_n : range parameter

$f_n^c(r) = e^{-(\mu_n r)^2}$, $f_n^{LS} = \nabla^2 \delta(r)$ for Gogny int.

$v_{12}^{(DD)}$ introduced with contact form (originasl M3Y ….. $v_{12}^{(DD)} = 0$ unable to produce saturation) Mainly focuse on central part;

LS/ tensor part enhanced/quenched only by an overall factor(future problem) Logest-range term of $v_{12}^{(c)}$ form.

For M3Y-type int. Eq. (7) can be rewrite as follow:

$$v_{12}^{(c)}=\sum_n (t_n^{(SE)}P_{SE}+t_n^{TE}P_{TE}+t_n^{(SO)}P_{SO}+t_n^{(TO)}P_{TO})\ \ e^{-\mu_n r}/\mu_n r$$
$$v_{12}^{(LS)}=\sum_n (t_n^{(LSE)}P_{TE}+t_n^{(LSO)}P_{TO})f_n^{(C)}(r_{12})L_{12}.(\vec{S}_1+\vec{S}_2)$$
$$v_{12}^{(TN)}=\sum_n (t_n^{(TNE)}P_{TE}+t_n^{(TNO)}P_{TO})f_n^{(TN)}(r_{12})r_{12}^2 S_{12}$$
$$v_{12}^{(DD)}=t^{(DD)}(1+x^{(DD)}P_O)[\rho(r_1)]^{\alpha}\delta(r_{12}) \qquad (7a)$$

and for Gogny Eq. (7) becomes:

$$v_{12}^{(c)}=\sum_n (t_n^{(SE)}P_{SE}+t_n^{TE}P_{TE}+t_n^{(SO)}P_{SO}+t_n^{(TO)}P_{TO})e^{-(\mu_n r_{12})^2}$$
$$v_{12}^{(LS)}=\sum_n (t_n^{(LSE)}P_{TE}+t_n^{(LSO)}P_{TO})\nabla^2\delta(r_{12})L_{12}.(\vec{S}_1+\vec{S}_2)$$
$$v_{12}^{(TN)}=\sum_n (t_n^{(TNE)}P_{TE}+t_n^{(TNO)}P_{TO})f_n^{(TN)}(r_{12})r_{12}^2 S_{12}$$
$$v_{12}^{(DD)}=t^{(DD)}(1+x^{(DD)}P_O)[\rho(r_1)]^{\alpha}\delta(r_{12}) \qquad (7b)$$

The relative coordinate is denoted by $\vec{r}_{12}=\vec{r}_1-\vec{r}_2$ and $|\vec{r}_{12}|=r_{12}$ correspondingly, the relative momentum is defined by $\vec{P}_{12}=\frac{(\vec{P}_1-\vec{P}_2)}{2}$, $\vec{L}_{12}$ is the relative orbital angular momentum, $\vec{L}_{12}=\vec{r}_{12}\times\vec{P}_{12}$, $\vec{S}_1$ and $\vec{S}_2$ are the nucleon spin operators, and S_{12} is the tensor operator which is defined as [16, 17].

The values of the best fit to the potential parameters ($t_n^{(SE)}, t_n^{(SO)}, t_n^{(TO)}, t_n^{(TE)}, t_n^{(LSE)}, t_n^{(LSO)}, t_n^{(TNE)}, t_n^{(TNO)}$) are shown in Table 1 [17].

Table 1: The values of the best fit to the potential parameters [17].

parameters	Unit	M3Y-P2
$R_1^{(c)}$	fm	0.25
$t_1^{(SE)}$	MeV	8027
$t_1^{(TE)}$	MeV	6080
$t_1^{(SO)}$	MeV	-11900
$t_1^{(TO)}$	MeV	3800
$R_2^{(c)}$	fm	0.4
$t_2^{(SE)}$	MeV	-2880
$t_2^{(SO)}$	MeV	-4266
$t_2^{(TE)}$	MeV	2730
$t_2^{(TO)}$	MeV	-780
$R_3^{(c)}$	fm	1.414
$t_3^{(SE)}$	MeV	-10.463
$t_3^{(SO)}$	MeV	-10.463
$t_3^{(TE)}$	MeV	31.389
$t_3^{(TO)}$	MeV	3.488
$R_1^{(LS)}$	fm	0.25
$t_1^{(LSE)}$	MeV	-9181.8
$t_1^{(LSO)}$	MeV	-3414.6
$R_2^{(LS)}$	fm	0.4
$t_2^{(LSE)}$	MeV	-606.6

Results and discussion

In the present work, ^{40}Ca is used as inert core and the model space is the fp shell. The Titanium nucleus ^{42}Ti is considered in the fp-shell region. The OBDM for all transitions considered are calculated using the effective interactions FPD6 and GXPF1. The core-polarization effects has been included in order to account for the contribution of configurations from outside of the model space in the transition as shown in Table 2. Using a realistic effective nucleon-nucleon (nn) interaction as a residual interaction to calculate the CP effects through a (perturbation) microscopic theory, with a selection of model space effective interaction which generates the shell

model space wave functions and highly excited states. Harmonic oscillator wave function was adopted as a single particle wave function. We shall discuss the cp effects on the inelastic electron scattering form factors for the low lying states of 2p1f-shell nuclei. M3Y and Gogny interactions of Nakada [17] is to be adopted as a residual interaction for the core polarization matrix elements.

Table 2: The values of the OBDM elements for the longitudinal C4 transition of the $4^+_2 1$ ground state of ^{42}Ti using the FPD6 (Ex=4.924 MeV) and GXPF1 (Ex=5.408 MeV) as effective interactions with M3Y and Gogny as residual interactions.

j_i	j_f	OBDM (ΔT=0)		OBDM (ΔT=1)	
		FPD6	**GXPF1**	**FPD6**	**GXPF1**
7/2	7/2	0.21381	-0.10934	0.17457	-0.08928
7/2	3/2	-0.18682	0.10243	-0.15254	0.08363
7/2	5/2	-0.00468	0.00226	-0.00382	0.00185
3/2	7/2	-0.03318	0.01463	-0.02709	0.01195
3/2	3/2	-0.53728	0.58281	-0.43869	0.47586
3/2	5/2	-0.01220	0.00535	-0.00996	0.00437
3/2	1/2	0.02269	-0.01974	0.01853	-0.01612
5/2	7/2	0.02056	-0.00820	0.01679	-0.00669
5/2	3/2	-0.00282	0.00242	-0.00230	0.00198
5/2	5/2	0.15528	-0.12701	0.12678	-0.10370

In the present work we will concentrate attention for the transition $4^+_2 1$ in ^{42}Ti isotope, where ^{40}Ca was used as a closed core and two protons freely distributed in fp shell model space. We have chooses (GXPF1 and FPD6) as a model space effective interaction to generate the model space wave functions with (Gogny and M3Y) as a residual interaction. The total form factors (MS+CP) for the transition $C4_2$ is calculated from the core polarization (CP) contribution and model space (MS), where the protons contribute to the charge form factors.

For Fig. 1, when using GXPF1 as an effective interaction at Ex=5.408MeV with Gogny as a residual interaction will see that the total form factors is larger than the two contributions in the first and second lobe respectively and they were in the same phase approximately also the diffraction minima (the point of intersection between the first and Second lobe) with respect to q (momentum transfer) values are CP (at 1.85fm^{-1}) > total (at 1.75fm^{-1}) > MS (at 1.7fm^{-1}) and the values for the same contributions with respect to $|F(q)|^2$ are total (at $1.5*10^{-7}$) > CP (at $2.5*10^{-8}$) > MS (at $2.2*10^{-8}$). Here the core part is in positive with respect to model space.

For Fig. 2, when using FPD6 as an effective interaction at Ex=5.961MeV with M3Y as a residual interaction will see that the total form factors lies between the two contributions in the first and second lobe respectively and they were in the same phase for (total and MS) except (CP) also the diffraction minima with respect to q (momentum transfer) the values are CP (at 2.2fm^{-1}) > total (at 1.7fm^{-1}) > MS (at 1.65 fm^{-1}) and for the same with respect to $|F(q)|^2$ values are CP(at $1*10^{-7}$) > total (at $4*10^{-8}$) > MS(at $3.9*10^{-8}$). Here the core part is in negative with respect to model space.

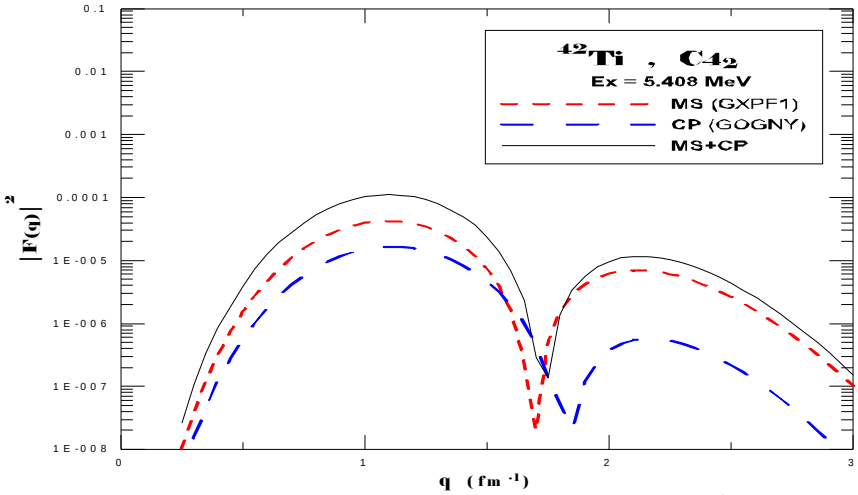

Fig. 1: Inelastic longitudinal form factors for the transition to the 4^+_2 state in ^{42}Ti with and without core-polarization effects by using Gogny as a residual interaction with GXPF1 as an effective interaction at Ex=5.408 MeV.

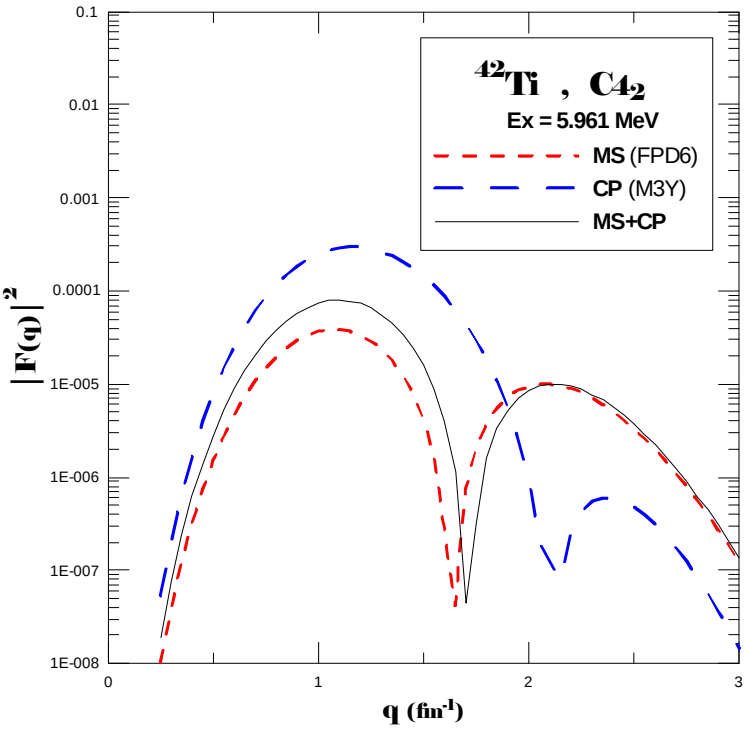

Fig. 2: Inelastic longitudinal form factors for the transition to the 4^+_2 state in ^{42}Ti with and without core-polarization effects by using M3Y as a residual interaction with FPD6 as an effective interaction at Ex=5.961 MeV.

For Fig. 3, when using GXPF1 as an effective interaction at Ex=5.408 MeV with M3Y as a residual interaction will see that the total form factors is in negative (between CP and MS) in the two contributions at the first and second lobe respectively and they were in the same phase for the (total and MS) except CP also the diffraction minima with respect to q values are CP (at 2.3 fm^{-1}) > total (at 1.75fm^{-1}) > MS (at 1.7 fm^{-1}) and for the same with respect to $|F(q)|^2$ values are total (at $3.5*10^{-8}$) > MS (at $2.2*10^{-8}$) > CP (at $1*10^{-8}$). Here the core part is in negative with respect to model space.

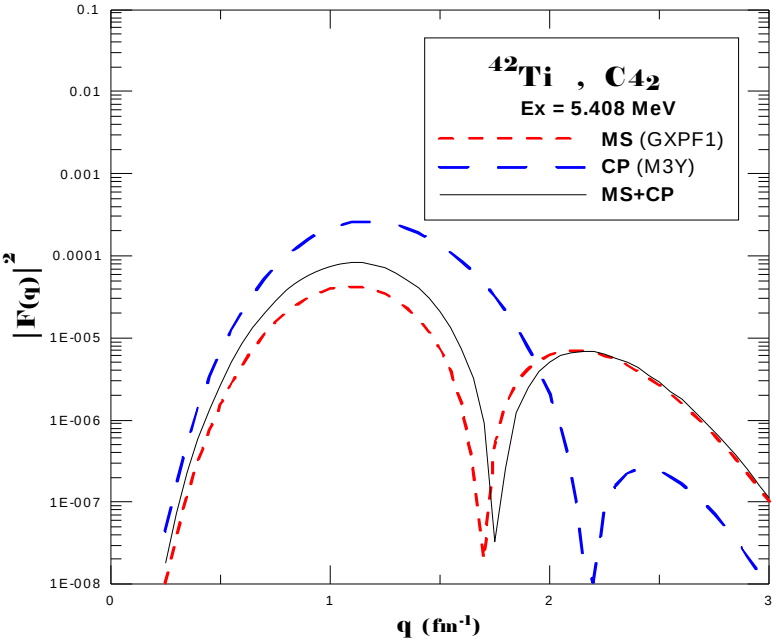

Fig. 3: Inelastic longitudinal form factors for the transition to the 4^+_2 state in ^{42}Ti with and without core-polarization effects by using M3Y as a residual interaction with GXPF1 as an effective interaction at Ex=5.408 MeV.

For Fig. 4, when using FPD6 as an effective interaction at Ex=5.961 MeV with Gogny as a residual interaction will see that the total form factors is larger than the two contributions in the first and second lobe respectively and they are in phase. The diffraction minima with respect to q values are CP (at 1.8 fm^{-1}) > total (at 1.7 fm^{-1}) > MS (at 1.65 fm^{-1}) and for the same with respect to|F(q)| 2 values are total(at 1.5*10^{-7}) > MS (at 4*10^{-8}) > CP (at 1.9*10^{-8}). Here the core part is in positive with respect to model space.

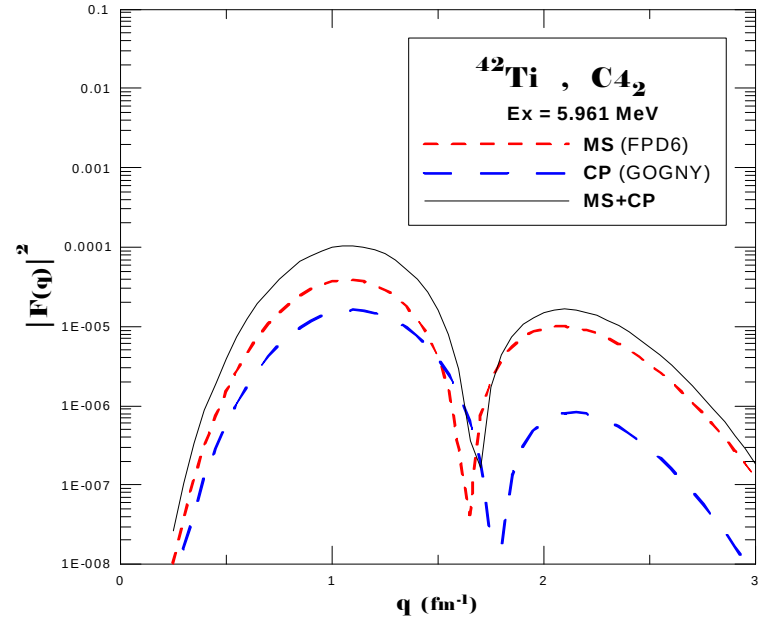

Fig. 4: Inelastic longitudinal form factors for the transition to the 4^+_2 state in ^{42}Ti with and without core-polarization effects by using Gogny as a residual interaction with FPD6 as an effective interaction at Ex=5.961 MeV.

Conclusions

The diffraction minima is near with each of them for two transitions when we used the residual interaction Gogny with effective interaction FPD6 or GXPF1 but the deviation in diffraction minima in the other transitions appear when we used the residual interaction M3Y with the two effective interaction.

M3Y as a residual interaction makes the C.P part in negative contribution with M.S for two cases when using it with effective interactions FPD6 or GXPF1. Gogny as a residual interaction makes the C.P part in positive contribution with respect to M.S for another two cases when using it with effective interactions. FPD6 or GXPF1.

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These residual interactions were modified by Asst. Prof. Dr. F. Z. Majeed. The OXBASH code was using to produce OBDM files by B. A. Brown et al. The excitation energy of core was 2ħω and the form factors results was produced by CVT. for programe by Prof. Dr. R. A. Radhi in FORTRAN 90.

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