

Calculation Mars – Earth Distance and Mars Orbital Elements with Julian Date

Atared Y. Qhatan^{1a*}

¹Department of Astronomy and Space, College of Science, University of Baghdad, Baghdad, Iraq

^{a*}Corresponding Email: ataredyarub93@gmail.com

Abstract

In this paper, the Mars orbital elements were calculated. These orbital elements—the major axis, the inclination (i), the longitude of the ascending node (Ω), the argument of the perigee (ω), and the eccentricity (e)—are essential to knowing the size and shape of Mars' orbit. The quick basic program was used to calculate the orbital elements and distance of Mars from the Earth from 25/5/1950 over 10000 days. These were calculated using the empirical formula of Meeus, which depended on the Julian date, which slightly changed for 10000 days; Kepler's equation was solved to find Mars' position and its distance from the Sun. The ecliptic and equatorial coordinates of Mars were calculated. The distance between Mars and the center of the Earth, in astronomical units (A.U.), was calculated. RM-E(min) was found to be between 0.4763 and 0.5108, and RM-E(max) was found to be between 2.548 and 2.6259. Furthermore, the findings revealed that the Mars orbital elements have changed over time.

Article Info.

Keywords:

Mars orbit, Orbital elements, Mars coordinates, Mars distance, Mars position.

Article history:

Received: Nov. 24, 2022

Accepted: Feb. 12, 2023

Published: Mar.01, 2023

1. Introduction

Mars is the fourth planet from the Sun; it is 1.52366231 A.U. from the Sun, with a diameter of about 4221 miles. Mars is famous as the Red Planet; it obtains its red color from the iron constituent of its soil. Mars has a thin cloudless atmosphere that allows a clear vision of the surface [1]. A solar day on Mars is almost the same length as the Earth's [2]. The Mars orbit is the second roughly eccentric in the solar system, the first being that of Mercury. At perihelion, Mars is 206,655,215 km from the Sun, and at aphelion, it is 249,232,432 km, a change of just less than 42,600,000 km. The average distance from the Sun to Mars (called the semi-major axis) is 228 million km. Mars takes approximately 687 Earth days to complete one orbit. The orbit of Mars changes with the gravitational effect of the bodies around it so that the eccentricity can vary over time. As recently as 1.35 million years ago, Mars was in an almost circular orbit [3]. Mars has two natural satellites: Phobos (Orbital Semi major axis = 9377.2 km, orbital period= 0.318910 days and mass= 1.08×10^{16} kg) and Deimos (orbital semi-major axis equal 23463.2km, orbital period = 1.262441 days and mass = 1.80×10^{15} kg) [4]. In this work, a calculation to the Mars orbital elements was attended.

2. Calculation of Mars Coordinate and Transformation between Coordinate Systems

2.1. The Coordinate of the Sun and the Distance from the Sun

Sun longitude in the era $J_{1900.0}$ was 280.46, and the average of the Earth going about the Sun is 0.985647359^0 per day (for equinox to equinox). The Sun mean longitude was given by Qureshi [5]:

$$L_s = 279^{\circ}.69668 + 36000^{\circ}.76892 T + 0^{\circ}.0003025 T_2 \quad (1)$$

T: The number of Julian centuries elapsed since midday of the beginning of 1st January 1900 [6].

T₂: The number of Julian centuries elapsed since midday starting of 2000 as given by Montenbruck and Gill [7].

The equation of the center for the sun C_s is given by the following relationship [6]:

$$C_s = 1^\circ.914602 - 0^\circ.004817 T_2 - 0^\circ.000014T_2^2 \sin(M_s) + 0^\circ.019993 - 0^\circ.000101T_2 \sin(2M_s) + 0^\circ.000289 \sin(3M_s) \quad (2)$$

M_S is the Sun's mean anomaly [8]:

$$M_s = 357^\circ.52543 + 35999^\circ.04944 T_2 - 0.0001536T_2^2 \quad (3)$$

True longitude of the Sun (λ_s) can be calculated by the following equation [6]:

$$\lambda_s = L_s + C_s \quad (4)$$

For more accuracy some corrections are to be applied to (λ_s) [6]:

$$A = 153.23 + 22518.7541T \quad (5)$$

$$B = 213.57 + 45037.5082 T \quad (6)$$

$$C = 312.69 + 32964.3577 T \quad (7)$$

$$D = 350.74 + 445267.1142 T - 0.00144 T^2 \quad (8)$$

$$E = 231.19 + 20.20 T \quad (9)$$

$$\lambda_s' = 0.00134 \cos(A) + 0.00154 \cos(B) + 0.00200 \cos(C) + 0.00179 \sin(D) + 0.00178 \sin(E) \quad (10)$$

Applying the corrections to (λ_s) as:

$$\lambda_s = \lambda_s + \lambda_s' \quad (11)$$

Without error the Sun latitude (β_s) can be counted zero as it stays on the ecliptic.

The Sun's true anomaly f_s is given as [6]:

$$f_s = M_s + C_s \quad (12)$$

The distance among the Earth center of the Sun, given in Astronomical Units (A.U.), as [6]:

$$R_s = \frac{1.000001018(1 - e^2)}{(1 + e \cos f_s)} \quad (13)$$

e represents the eccentricity of the Earth path computed as [6]:

$$e = 0^\circ.016708634 - 0^\circ.000042037 T_2 - 0^\circ.000000126 \quad (14)$$

For additional precision some corrections are added to (R_s) [6]:

$$R_s' = 0.00000543 \sin(A) + 0.00001575 \sin(B) + 0.00001627 \sin(C) + 0.00003076 \cos(D) + 0.00000927 \sin(H) \quad (15)$$

where:

$$H = 353.40 + 65928.7155 T \quad (16)$$

$$R_S = R_S + R_S' \quad (17)$$

2.2. Mars Distance and Position from the Sun

The distance and position of Mars from the Sun are calculated by the orbital elements required to identify any orbit, which is calculated in the classical two-body systems. Six parameters are applied in astronomy and orbital mechanics: eccentricity (e), the semi-major axis (a) that represents the size and shape of the orbit, inclination (i), which is known as the perpendicular tilt of the ellipse orbit according to the reference surface which is measured from the ascending node, longitude of ascending node (Ω) which is the point where the orbit of the object passes through the plane of reference. Argument of perigee (ω) defined as the direction of the ellipse in the orbital plane, which represents the angle measured from ascending node to the semi-major axis, and the mean anomaly (M) denoting the location of the orbiting over the ellipse at definite period [9]. The elliptical orbital elements for Mars can be calculated from the following equation [6]:

$$L = 293.737\ 334 + 19141.695\ 51 T + 0.000\ 3107 T^2 \quad (18)$$

where

L = Mars mean longitude.

$$a = 1.523\ 6883 \quad (19)$$

$$e_M = 0.093\ 312\ 90 + 0.000\ 092\ 064 T - 0.000\ 000\ 077 T^2 \quad (20)$$

$$i = 1.850\ 333 - 0.000\ 6750 T + 0.000\ 0126 T^2 \quad (21)$$

$$\omega = 285.431\ 761 + 1.069\ 7667 T + 0.000\ 1313 T^2 + 0.000\ 004\ 14 T^3 \quad (22)$$

$$\Omega = 48.786\ 442 + 0.770\ 991 T - 0.000\ 0014 T^2 - 0.000\ 005\ 33 T^3 \quad (23)$$

$$M = L - \omega - \Omega \quad (24)$$

where

M = The Mar's mean anomaly which is used to compute the eccentric anomaly (E) using Kepler equation [10]:

$$E = M + e_o \sin E \quad (25)$$

e is the transformation unit from radians into degrees when E and M in degrees.

$$e_o = e \times \frac{180}{\pi}$$

The distance from the Sun to Mars is calculated as [6]:

$$r = a(1 - e \cos E) \quad (26)$$

The true anomaly (f) of Mars is calculated using the following equation [10]:

$$\tan \frac{f}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \quad (27)$$

2.3. Coordinates of Mars from Earth

Argument latitude of Mars was computed as [6]:

$$U_M = L + f - M - \Omega \quad (28)$$

The ecliptic longitude (L') of Mars was computed as [6]:

$$\tan(L' - \Omega) = \cos i \times \tan U_M \quad (29)$$

Mars ecliptic latitude (b_M) was computed as [8]:

$$\sin b_M = \sin U_M \times \sin i \quad (30)$$

Then Mars's geocentric longitude (λ_M) is:

$$\tan(\lambda_M - \lambda_S) = \frac{r \cos b_M \sin(L' - \lambda_S)}{r \cos b_M \cos(L' - \lambda_S) + R} = \frac{N}{D} \quad (31)$$

Sun ecliptic coordinate (λ_S, b_S) and ecliptic coordinate of Mars (λ_M, b_M) must be transformed to equatorial coordinate (α_S, δ_S) and (α_M, δ_M), respectively using the following equation [8]:

$$\tan \alpha = \frac{\sin \lambda \cos \varepsilon - \tan \beta \sin \varepsilon}{\cos \lambda} \quad (32)$$

and

$$\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda \quad (33)$$

2.4. Mars Distance and Position from the Earth

The distance of Mars and its location from Earth can be calculated from the following equation (Eq.(34)) [8]:

$$\Delta^2 = N_1^2 + D_1^2 + (r \sin b_M)^2 \quad (34)$$

$$N_1 = r \cos b_M \sin(L' - \lambda_S) \quad (35)$$

$$D_1 = r \cos b_M \cos(L' - \lambda_S) + R_S \quad (36)$$

$$\tan(\lambda_M - \lambda_S) = \frac{N_1}{D_1} \quad (37)$$

Another method can be used to find the distance of Mars from Earth, as shown [11]:

$$x_{\text{sun}} = R_s \cos \delta_s \cos \alpha_s \quad (38)$$

$$y_{\text{sun}} = R_s \cos \delta_s \sin \alpha_s \quad (39)$$

$$z_{\text{sun}} = R_s \sin \delta_s \quad (40)$$

$$x_{\text{Mars}} = r \cos \delta_M \cos \alpha_M \quad (41)$$

$$y_{\text{Mars}} = r \cos \delta_M \sin \alpha_M \quad (42)$$

$$z_{\text{Mars}} = r \sin \delta_M \quad (43)$$

$$X_{\text{Earth}} = - (X_{\text{Sun}}) \quad (44)$$

$$Y_{\text{Earth}} = -(Y_{\text{Sun}}) \tag{45}$$

$$Z_{\text{Earth}} = -(Z_{\text{Sun}}) \tag{46}$$

$$X_{\text{Earth - Mars}} = X_{\text{Mars}} - X_{\text{Earth}} \tag{47}$$

$$Y_{\text{Earth - Mars}} = Y_{\text{Mars}} - Y_{\text{Earth}} \tag{48}$$

$$Z_{\text{Earth - Mars}} = Z_{\text{Mars}} - Z_{\text{Earth}} \tag{49}$$

$$R_{\text{Earth-Mars}} = \sqrt{(X_{\text{Earth}} - X_{\text{Mars}})^2 + (Y_{\text{Earth}} - Y_{\text{Mars}})^2 + (Z_{\text{Earth}} - Z_{\text{Mars}})^2} \tag{50}$$

3. Results and Discussion

The quick basic program was used to calculate the orbital elements and distance of Mars from the Earth from 25/5/1950 over 10000 days. The Excel program was used to draw the results. Table 1 lists the values of orbital elements and Julian date for one period. Fig.1 (A-D) illustrates the relationship between the Julian date on the X-axis, and the orbital elements, on the Y-axis. The inclination (i) decreased by 0.0005% through one period, as shown in Fig.1(A), the argument of the perigee(ω) increased by 0.006% through one period, as shown in Fig.1(B), the longitude of ascending node (Ω) increased by 0.02% through one period as shown in Fig.1(C), the mean longitude of the planet (L) increased by 2.03% through one period as shown in Fig.1(D), the eccentricity(e) increased by 0.001% through one period as shown in Fig.1(E), while the semi-major axis (a) was constant throughout the period. The orbital elements of Mars change periodically with time because the eccentricity of Mars is greater than that of other planets, except for Mercury, which causes a difference between the aphelion and perihelion distances; also, the distance of Mars from other planets affects the orbit of Mars. The equatorial coordinates were calculated. The declination (δ) of Mars was plotted with Julian date over 10000 days, as shown in Fig.2. The results showed that the values of were $\delta_{\text{min}} = (-25.9486, -26.9484)$, and that of $\delta_{\text{max}} = (23.1163, 24.82)$. The results of the declination of the Earth's effect on Mars were not cumulative, but after several cycles, it returned to its actual orbit; peaks occurred when the Earth passed in front of Mars. The right ascension (α) for Mars was plotted against Julian date during 10000 days, as shown in Fig.3. The results showed that $\alpha_{\text{min}} = (0.0, 0.733)$, $\alpha_{\text{max}} = (23.2667, 23.8)$. The results showed that the angle of right ascension changes not only because of the rotation of the planet but also because of the point of the vernal equinox and the occurrence of secondary peaks in the passage of the Earth in front of the planet Mars. This was noticed by backing off because of the speed of the Earth. The distance between the centre of the Earth and Mars was calculated with a mean value of (2.60375 A.U.), as shown in Fig.4. When the Earth is in the aphelion and Mars near the perihelion, the closest distance to Mars from Earth was (0.4763 A.U.) at the date (2440123.5).

Table 1: The values of Orbital elements and Julian Date for one period.

JD	L	a	i	ω	Ω	$\text{ex}10^{-2}$
2433443.5	228.9609375	1.52368832	1.85375762	285.3963928	49.99798203	0.93359
2433444.5	229.484375	1.52368832	1.8537575	285.3963928	49.9979744	0.933593
2433445.5	230.0087891	1.52368832	1.85375726	285.3964233	49.99796677	0.93359
2433446.5	230.5332031	1.52368832	1.85375702	285.3964539	49.99795914	0.933593
2433447.5	231.0566406	1.52368832	1.85375679	285.3964539	49.99795151	0.93359
2433448.5	231.5810547	1.52368832	1.85375655	285.3964844	49.99794388	0.933593
2433449.5	232.1054688	1.52368832	1.85375631	285.3965149	49.99793243	0.93359
2433450.5	232.6289063	1.52368832	1.85375607	285.3965149	49.9979248	0.933593
2433451.5	233.1533203	1.52368832	1.85375583	285.3965454	49.99791718	0.93359
2433452.5	233.6777344	1.52368832	1.85375571	285.3965759	49.99790955	0.933593

2433453.5	234.2011719	1.52368832	1.85375547	285.3965759	49.99790192	0.93359
2433454.5	234.7255859	1.52368832	1.85375524	285.3966065	49.99789429	0.933593
2433455.5	235.2490234	1.52368832	1.853755	285.396637	49.99788666	0.93359
2433456.5	235.7734375	1.52368832	1.85375476	285.396637	49.99787903	0.933593
2433457.5	236.2978516	1.52368832	1.85375452	285.3966675	49.9978714	0.93359
2433458.5	236.8212891	1.52368832	1.85375428	285.396698	49.99785995	0.933593

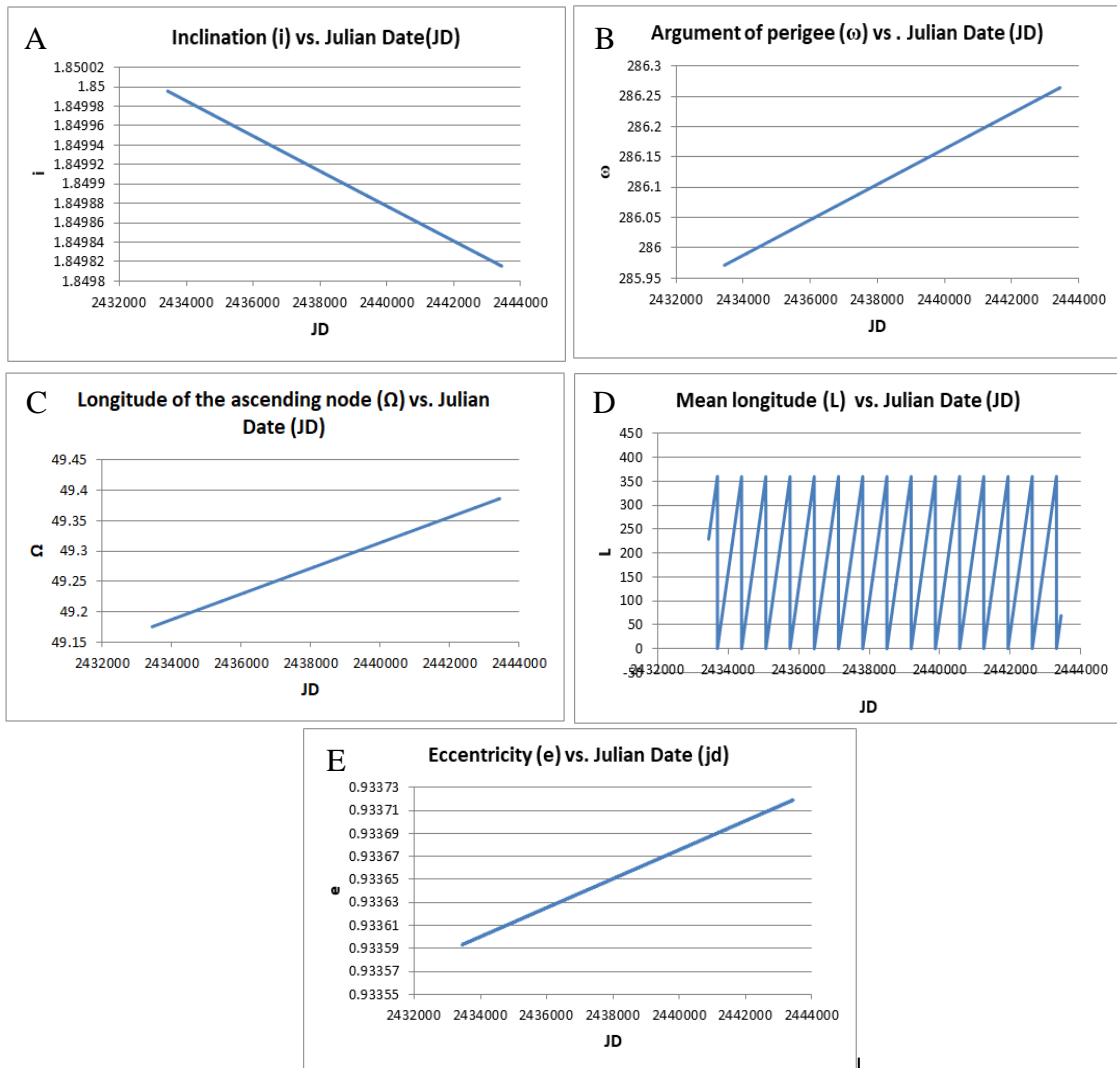


Figure 1: (A,B,C,D and E) The orbital element (i, ω, Ω, L, e) as a function of Julia day for 10000 days .

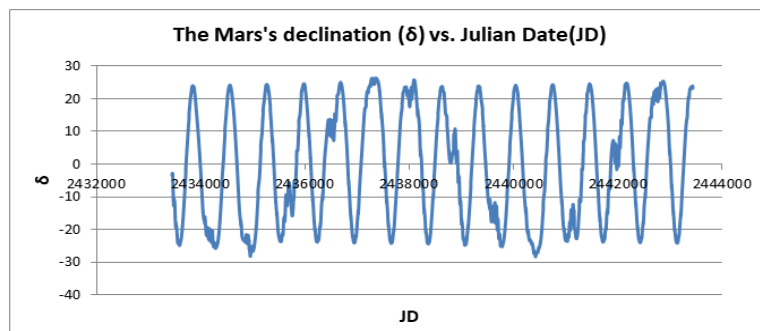


Figure 2: The change of the Mars declination with Julia date during 10000 days .

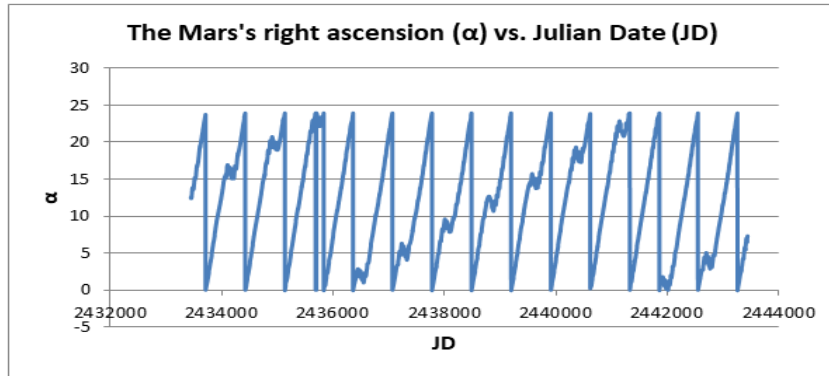


Figure 3: The change of the Mars right ascension with Julia date during 10000 days.

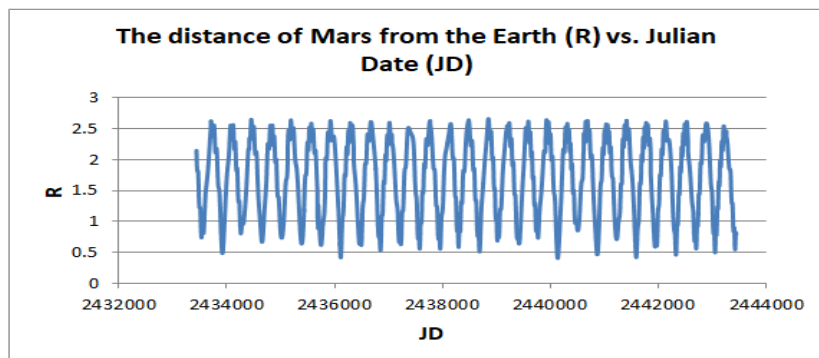


Figure 4: The change of the Mars distance from the Earth with Julia date during 10000 days.

4. Conclusions

The orbital elements of Mars change with time and may have an effect on its orbit in the future. The equatorial coordinates of Mars were computed to be $\delta_{\min} = (-25.9486, -26.9484)$, $\delta_{\max} = (23.1163, 24.82)$ and $\alpha_{\min} = (0.0, 0.733)$, $\alpha_{\max} = (23.2667, 23.8)$. The distance of Mars from the Earth, with a mean value of (2.60375 A.U.), changed after many periods under the influence of the perturbation of other bodies.

Acknowledgements

The author would like to thanks University of Baghdad /College of Science/ Department Astronomy & Space / Baghdad-Iraq.

Conflict of interest

Author declares that they have no conflict of interest.

References

1. D. Banfield, A. Spiga, C. Newman, F. Forget, M. Lemmon, R. Lorenz, N. Murdoch, D. Viudez-Moreiras, J. Pla-Garcia, and R. F. Garcia, *Nature Geosci.* **13**, 190 (2020).
2. M. G. Trainer, M. H. Wong, T. H. Mcconnochie, H. B. Franz, S. K. Atreya, P. G. Conrad, F. Lefèvre, P. R. Mahaffy, C. A. Malespin, and H. L. Manning, *J. Geophys. Res.: Planets* **124**, 3000 (2019).
3. S. Kosambe, *J. Aircraft Spacecraft Tech.* **3**, 154 (2019).
4. A. N. Deutsch, J. W. Head, K. R. Ramsley, C. M. Pieters, R. W. Potter, A. M. Palumbo, M. S. Bramble, J. P. Cassanelli, E. R. Jawin, and L. M. Jozwiak, *Advan. Space Res.* **62**, 2174 (2018).

5. M. S. Qureshi, In Charge Assistant Professor Institute of Space Planetary Astrophysics, University of Karachi, 1 (2005).
6. J. Meeus, *Astronomical Formulae for Calculators*. 4th Ed. Revised. (Richmond, United States of America, Willmann-Bell, 1988).
7. O. Montenbruck and E. Gill, *Satellite Orbits: Models, Methods and Applications*. (Berlin, Heidelberg, Springer 2012).
8. J. Meeus, *Astronomical Algorithms*. 2nd Ed. (Virginia, Willmann-Bell, 1998).
9. R. H. Ibrahim and A.-R. H. Saleh, Iraqi J. Sci. **60**, 2269 (2019).
10. B. Gerhard, *Methods of Celestial Mechanics*. Vol. I. (Verlage Berlin Heidelberg, Springer, 2005).
11. J. Katzer, C. Suchocki, W. Błaszczak-Bąk, and M. Damińska-Suchocka, Data in Brief **39**, 107616 (2021).

حساب المسافة بين كوكب المريخ والارض والعناصر المدارية لكوكب المريخ مع التاريخ الجولياني

عطارد يعرب قحطان¹

¹قسم الفلك والفضاء، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

تم في هذا البحث حساب العناصر المدارية لكوكب المريخ وهذه العناصر ضرورية لمعرفة شكل وحجم مدار المريخ. العناصر هي المحور الرئيسي، الميل، خط طول العقدة الصاعدة، دالة الحضيض، والشذوذ المركزي التي تم حسابها باستخدام المعادلات التجريبية التابعة لميوز التي تعتمد على التاريخ الجولياني والتي تغيرت بشكل طفيف ولمدة 1000 يوم. تم حل معادلة كبلر لإيجاد موقع و بعد المريخ عن الشمس. وتم حساب الاحداثيات البروجية والاستوائية لكوكب المريخ. وحساب المسافة بين مركز الارض وكوكب المريخ بالوحدة الفلكية (A.U) و اظهرت النتائج ان القيم تتراوح بين $R_{M-E}(\min)$ (0.5108 و 0.4763) وبين $R_{M-E}(\max)$ (2.6259 و 2.548) . كما بينت النتائج أن العناصر المدارية للمريخ تتغير بمرور الوقت عبر سنوات عديدة.